

# Expressiveness of Deterministic Single-Clock Timed Automata

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From FORMATS 20 and LATA 20

# Plan

1 Definitions and Motivation

2 Expressiveness

3 Conciseness

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2 Expressiveness

3 Conciseness

# Single-Clock Timed Automata

- A set of *locations*  $L$  with an initial location  $\ell_0$  and a final location  $\ell_f$
- A set of *propositions*  $AP$  and a set of *actions*  $Act$
- A set of *synchronized transitions*  $\{\ell \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} \ell'\}$  with:
  - $\varphi^-$  and  $\varphi^+$  the pre and post conditions, boolean formulas over  $AP$ ;
  - the guard  $\gamma \in \{\alpha \bowtie x \bowtie' \beta\}$  with  $\bowtie, \bowtie' \in \{<, \leq, \}, \alpha \in \mathbb{N}$  and  $\beta \in \mathbb{N} \cup \{\infty\}$ ;
  - $B \subseteq Act$  the subset of possible actions;
  - $r \in \{\emptyset, \downarrow\}$ , the resetting action on clock  $x$ .
- A set of *autonomous transitions*  $\{\ell \xrightarrow{\varphi^-, \gamma, \#, r} \ell'\}$  (or  $\{\ell \xrightarrow{\varphi^-, \gamma, \{\#\}, r, \varphi^-} \ell'\}$ ) with:
  - $\varphi^-$  the pre condition;
  - the guard  $\gamma \in \{x = \alpha\}$ ;
  - $r \in \{\emptyset, \downarrow\}$ , the resetting action on clock  $x$ .

# Deterministic Single-Clock Timed Automata (DTA)

## Determinism on synchronized transitions.

For all  $l \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} l'$  and  $l \xrightarrow{\varphi'^-, \gamma', B', r', \varphi'^+} l''$ ,

- either  $\varphi^- \wedge \varphi'^- \Leftrightarrow \text{false}$ ;
- or  $\gamma \wedge \gamma' \Leftrightarrow \text{false}$ ;
- or  $B \cap B' = \emptyset$ ;
- or  $\varphi^+ \wedge \varphi'^+ \Leftrightarrow \text{false}$ .

## Determinism on autonomous transitions.

For all  $l \xrightarrow{\varphi^-, x=\alpha, \#, r} l'$  and  $l \xrightarrow{\varphi'^-, x=\alpha', \#, r'} l''$ ,  $\varphi^- \wedge \varphi'^- \Leftrightarrow \text{false}$  or  $\alpha \neq \alpha'$ .

## Condition on the final location.

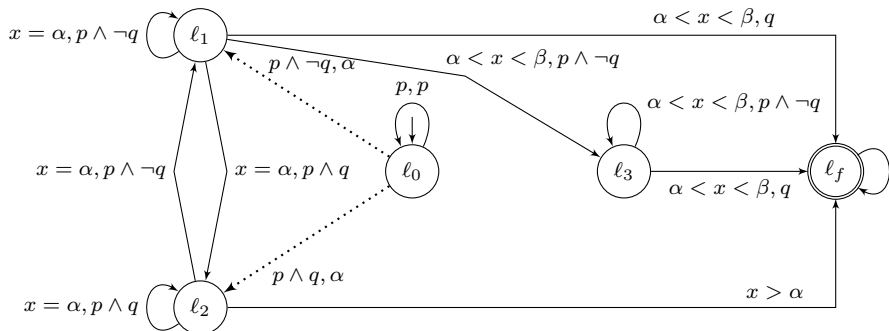
There is a single transition outgoing from the final location:

$$l_f \xrightarrow{\text{true}, \text{true}, \text{Act}, \emptyset, \text{true}} l_f$$

# A First Motivation with Illustration

A DTA is a way to express a linear timed temporal formula on infinite timed paths.

$$p\mathbf{U}^{[\alpha, \beta]}q \text{ with } \alpha > 0$$



# Timed Paths

A *timed path* is a sequence  $(v_0, \delta_0) \xrightarrow{a_0} (v_1, \delta_1) \xrightarrow{a_1} \dots (v_i, \delta_i) \xrightarrow{a_i} \dots$   
such that for all  $i \in \mathbb{N} : v_i \in \{\text{true}, \text{false}\}^{AP}$ ,  $a_i \in Act$  and  $\delta_i \in \mathbb{R}_{\geq 0}$ .

$\delta_0$  is the time of the occurrence of the first transition.

For  $i > 0$ ,  $\delta_i$  is the duration between the occurrence of the  $i^{th}$  transition and the occurrence of the  $i + 1^{th}$  transition.

## Illustration.

$$\sigma = (\{p\}, 0.4) \xrightarrow{a} (\{p, q\}, 5.6) \xrightarrow{b} (\{p\}, 2.4) \xrightarrow{c} (\emptyset, 0.7) \dots$$

## Observation.

$$\sigma \models p\mathbf{U}^{1,2}q$$

# Runs

A *run* of a DTA is a sequence:

$(\ell_0, v'_0, \bar{x}_0, \delta'_0) \xrightarrow{\varphi_0^-, \gamma_0, B_0, r_0, \varphi_0^+} (\ell_1, v'_1, \bar{x}_1, \delta'_1) \cdots (\ell_i, v'_i, \bar{x}_i, \delta'_i) \xrightarrow{\varphi_i^-, \gamma_i, B_i, r_i, \varphi_i^+} \cdots$  such that

- $\ell_i \xrightarrow{\varphi_i^-, \gamma_i, B_i, r_i, \varphi_i^+} \ell_{i+1}$  is a synchronized or autonomous transition.
- $v'_0$  satisfies the precondition of the first transition.  
For  $i > 0$ ,  $v'_i$  satisfies the postcondition of the  $i^{th}$  transition and the precondition of the  $i + 1^{th}$  transition.
- $\delta'_0$  is the time of the occurrence of the first transition.  
For  $i > 0$ ,  $\delta'_i$  is the duration between the occurrence of the  $i^{th}$  transition and the occurrence of the  $i + 1^{th}$  transition.
- $\bar{x}_0 = 0$  and for  $i > 0$ ,  $\bar{x}_i = \mathbf{1}_{r_{i-1}=\emptyset}(\bar{x}_{i-1} + \delta_{i-1})$  is the the clock value after the  $i^{th}$  transition.  
For all  $i$ ,  $\bar{x}_i + \delta'_i$  satisfies the guard of the  $i + 1^{th}$  transition.
- If the first transition is synchronized, no autonomous transition could take place before it.  
If the  $i + 1^{th}$  transition is synchronized, no autonomous transition could take place between the  $i^{th}$  and the  $i + 1^{th}$  transition.



# Paths Recognized by Runs

Let  $\sigma$  be the path

$$(v_0, \delta_0) \xrightarrow{a_0} (v_1, \delta_1) \xrightarrow{a_1} \dots (v_i, \delta_i) \xrightarrow{a_i} \dots$$

Let  $\rho$  be the run

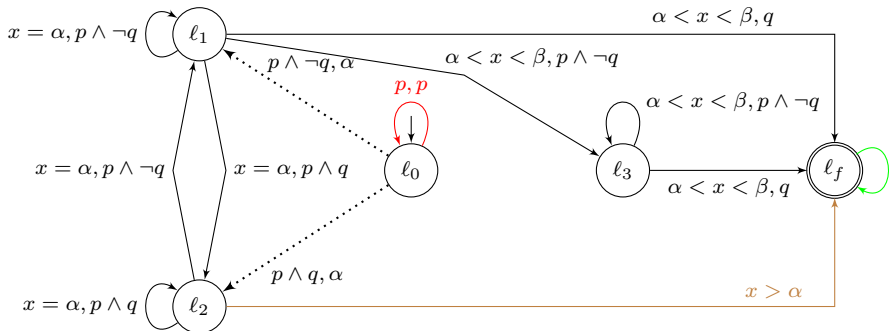
$$(\ell_0, v'_0, \bar{x}_0, \delta'_0) \xrightarrow{\varphi_0^-, \gamma_0, B_0, r_0, \varphi_0^+} (\ell_1, v'_1, \bar{x}_1, \delta'_1) \dots (\ell_i, v'_i, \bar{x}_i, \delta'_i) \xrightarrow{\varphi_i^-, \gamma_i, B_i, r_i, \varphi_i^+} \dots$$

$\sigma$  is *recognized* by  $\rho$  if there exists an increasing  $\kappa : \mathbb{N} \rightarrow \mathbb{N}$  such that for all  $i$ :

- $a_i \in B_{\kappa(i)}$ ,  $\delta_i = \sum_{\kappa(i-1) < h \leq \kappa(i)} \delta'_h$  and  $v_i = v'_{\kappa(i)}$ ;
- for all  $h \notin \kappa(\mathbb{N})$ ,  $B_h = \{\#\}$ .

$\sigma$  is *accepted* by  $\rho$  if  $\rho$  visits  $\ell_f$ .

# Illustration ( $p\mathbf{U}^{1,2}[q]$ )



Let  $\sigma = (\{p\}, 0.4) \xrightarrow{a} (\{p, q\}, 5.6) \xrightarrow{b} (\{p\}, 2.4) \xrightarrow{c} (\emptyset, 0.7) \dots$

Let  $\rho$  be:

$$\begin{array}{lll}
 (\ell_0, \{p\}, 0.0, 0.4) \rightarrow & (\ell_0, \{p, q\}, 0.4, 0.6) \dots\dots\dots \rightarrow & (\ell_2, \{p, q\}, 1.0, 5.0) \rightarrow \\
 (\ell_f, \{p\}, 6.0, 2.4) \rightarrow & (\ell_f, \emptyset, 8.4, 0.7) \dots & \dots
 \end{array}$$

Then  $\sigma$  is accepted by  $\rho$  with  $\kappa(0) = 0$  and for all  $i > 0$ ,  $\kappa(i) = i + 1$ .

# Continuous Time Markov Chains (CTMC)

A *continuous time Markov chain*  $\mathcal{M}$  is defined by:

- $S$ , a finite set of *states* with  $s_0$  the initial state;
- $lab : S \rightarrow \{\text{true}, \text{false}\}^{AP}$ , a valuation over  $S$ ;
- $R : S \times Act \times S \rightarrow \mathbb{R}_{\geq 0}$ , the rate function ;
- Let  $R_s = \sum R(s, a, s')$ . Then for all  $s$ ,  $R_s > 0$ .

$\mathcal{M}$  generates a path  $\sigma_{\mathcal{M}} = (v_0, \delta_0) \xrightarrow{a_0} (v_1, \delta_1) \xrightarrow{a_1} \dots (v_i, \delta_i) \xrightarrow{a_i} \dots$  as follows:

- the initial state is  $s_0$ ;
- $v_i = lab(s_i)$  and  $\delta_i$  is sampled w.r.t. distribution  $F_{s_i}(\tau) = 1 - e^{-R_{s_i}\tau}$ ;
- $(a_i, s_{i+1})$  is selected with probability  $\frac{R(s_i, a_i, s_{i+1})}{R_{s_i}}$ .

Let  $\mathcal{A}$  be a DTA. Then  $\mathbf{Pr}_{\mathcal{M}}(\mathcal{A}) \stackrel{\text{def}}{=} \mathbf{Pr}(\sigma_{\mathcal{M}} \models \mathcal{A})$ .

# Why Deterministic Single-Clock Timed Automata?

There is an efficient way to evaluate  $\Pr_{\mathcal{M}}(\mathcal{A})$ .

## Principles of the evaluation.

- The synchronized product of  $\mathcal{M}$  and  $\mathcal{A}$ ,  $\mathcal{M} \otimes \mathcal{A}$ , is a semi-Markovian process.
- $\Pr_{\mathcal{M}}(\mathcal{A})$  is the probability to reach  $\ell_f$  in  $\mathcal{M} \otimes \mathcal{A}$ .
- This probability only depends on  $\mathcal{M}_d$  the discrete-time Markov chain of transition probabilities between *regeneration points* of  $\mathcal{M} \otimes \mathcal{A}$ .
- $\mathcal{M}_d$  is computed via a transient analysis of *subordinated* CTMCs.

As soon as  $\mathcal{A}$  has two clocks,  
there could be no more regeneration points in  $\mathcal{M} \otimes \mathcal{A}$ .

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# Relating Families of DTA

Let  $\mathbb{A}_1$  and  $\mathbb{A}_2$  be families of DTA. Then:

- $\mathbb{A}_2$  is *at least as expressive as*  $\mathbb{A}_1$  w.r.t. timed language, denoted  $\mathbb{A}_1 \prec_{\mathcal{L}} \mathbb{A}_2$ , if for all  $\mathcal{A}_1 \in \mathbb{A}_1$  there exists  $\mathcal{A}_2 \in \mathbb{A}_2$  such that  $\mathcal{L}(\mathcal{A}_2) = \mathcal{L}(\mathcal{A}_1)$ ;
- $\mathbb{A}_2$  is *at least as expressive as*  $\mathbb{A}_1$  w.r.t. CTMCs, denoted  $\mathbb{A}_1 \prec_{\mathcal{M}} \mathbb{A}_2$ , if for all  $\mathcal{A}_1 \in \mathbb{A}_1$  there exists  $\mathcal{A}_2 \in \mathbb{A}_2$  s.t. for all  $\mathcal{M}$ ,  $\mathbf{Pr}_{\mathcal{M}}(\mathcal{A}_2) = \mathbf{Pr}_{\mathcal{M}}(\mathcal{A}_1)$ .

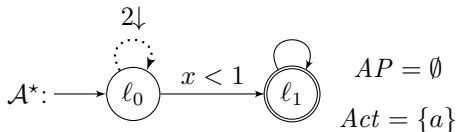
$\prec_{\mathcal{L}}$  is more discriminative than  $\prec_{\mathcal{M}}$ .

- Let  $\mathbb{A}$  be the general family of DTA;
- Let  $\mathbb{A}^{na}$  be the family of DTA without autonomous transitions;
- Let  $\mathbb{A}_{ne}^{na}$  be the family of DTA without autonomous transitions and without equality constraints on synchronized transitions.

Then  $\mathbb{A}_{ne}^{na} \sim_{\mathcal{M}} \mathbb{A}^{na}$  but  $\mathbb{A}_{ne}^{na} \not\prec_{\mathcal{L}} \mathbb{A}^{na}$ .

Are autonomous transitions necessary w.r.t. CTMC:  $\mathbb{A} \prec_{\mathcal{M}} \mathbb{A}^{na}$ ?

## Necessity of Autonomous Transitions w.r.t. $\prec_{\mathcal{L}}$



$$\mathcal{L}(A^*) = \{(\delta_0) \xrightarrow{a} (\delta_1) \xrightarrow{a} \dots \mid \exists n \in \mathbb{N} \ 2n \leq \delta_0 < 2n + 1\}$$

Assume there exists  $\mathcal{A}' \in \mathbb{A}^{na}$  such that  $\mathcal{L}(\mathcal{A}') = \mathcal{L}(A^*)$ .

Let  $C$  be the maximal constant occurring in  $\mathcal{A}'$  that we assume w.l.o.g. to be odd.

The timed path  $(C + 1) \xrightarrow{a} (0) \xrightarrow{a} (0) \dots \xrightarrow{a} (0) \dots$  belongs to  $\mathcal{L}(A^*) = \mathcal{L}(\mathcal{A}')$ .

Let  $\rho = (\ell_0, 0, C + 1) \xrightarrow{\gamma_0, \{a\}, r_0} (\ell_1, \bar{x}_1, 0) \dots$

be the corresponding accepting run in  $\mathcal{A}'$ .

Let  $\rho' = (\ell_0, 0, C + 2) \xrightarrow{\gamma_0, \{a\}, r_0} (\ell_1, \bar{x}'_1, 0) \dots$

where  $\bar{x}'_i = C + 2 = \bar{x}_i + 1$  up to the first reset and  $\bar{x}'_i = \bar{x}_i$  after it.

Then  $\rho'$  is accepting.

Thus  $(C + 2) \xrightarrow{a} (0) \xrightarrow{a} (0) \dots \xrightarrow{a} (0) \dots$  belongs to  $\mathcal{L}(\mathcal{A}')$

which contradicts  $\mathcal{L}(\mathcal{A}') = \mathcal{L}(A^*)$ .

# Laplace Transform

## From time domain to complex domain.

Let  $f(t)$  be a real function defined for  $t \geq 0$ .

When it exists  $F(s)$  with  $s \in \mathbb{C}$ , its *Laplace transform*, is defined by:

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

## Some properties.

- For all region  $Re(s) > \alpha$  included in the region of convergence,  $F$  is analytical.
- Two functions have the same Laplace transform only if they differ on a set of null Lebesgue measure.

## A property of analytical functions.

Let  $U$  be a connected open set of  $\mathbb{C}$  and  $f, g$  be analytical over  $U$ .

If  $f$  and  $g$  coincide over  $A \subseteq U$  such that  $A$  has an accumulation point in  $U$  then  $f$  and  $g$  coincide over  $U$ .



# A 0-1 Law for DTA (1)

Let  $\mathcal{A} \in \mathbb{A}^{na}$  and  $z \in [0, 1]$  such that for all  $\mathcal{M}$ ,  $\Pr_{\mathcal{M}}(\mathcal{A}) = z$  then  $z \in \{0, 1\}$ .

## Sketch of proof.

We establish by induction on the transition relation of the region graph  $G_{\mathcal{A}}$  that:

For all configuration  $(\ell, t)$  in some region of  $G_{\mathcal{A}}$  reachable from  $(\ell_0, 0)$  and all  $\mathcal{M}$  and state  $s$  of  $\mathcal{M}$ , the probability that  $\mathcal{M}$  starting in  $s$  satisfies  $\mathcal{A}$  with initial state  $\ell$  and initial clock value  $t$  is equal to  $z$ .

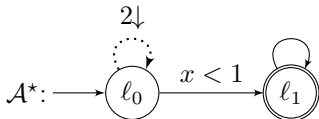
The base case of the induction follows from the hypothesis.

The induction step consists in showing that:

- the Laplace transform of this probability, is the Laplace transform of  $z$ ,
- using appropriate Markov chains and the properties of Laplace transforms.

The conclusion follows since either a region with  $\ell_f$  is reachable and  $z = 1$  or no region with  $\ell_f$  is reachable and  $z = 0$ .

## Necessity of Autonomous Transitions w.r.t. $\prec_{\mathcal{M}}$



Assume there exists  $\mathcal{A}' \in \mathcal{A}^{na}$  such that for all  $\mathcal{M}$ ,  $\mathbf{Pr}_{\mathcal{M}}(\mathcal{A}') = \mathbf{Pr}_{\mathcal{M}}(\mathcal{A}^*)$ .

Pick an arbitrary  $\mathcal{M}$ .

Let  $\mathcal{M}_{\lambda, s_0}$  be the Markov chain starting in  $s_0$  and entering  $\mathcal{M}$  at rate  $\lambda$ .

$\mathbf{Pr}_{\mathcal{M}_{\lambda, s_0}}(\mathcal{A}^*) = \frac{1 - e^{-\lambda}}{1 - e^{-2\lambda}}$  and so  $\lim_{\lambda \rightarrow 0} \mathbf{Pr}_{\mathcal{M}_{\lambda, s_0}}(\mathcal{A}) = \frac{1}{2}$ .

$\mathbf{Pr}_{\mathcal{M}_{\lambda, s_0}}(\mathcal{A}') = p_{1, \lambda} + p_{2, \lambda}$  where:

- $p_{1, \lambda}$  is the probability to accept the random timed path and that the first action takes place at most at time  $C$ ;
- $p_{2, \lambda}$  is the probability to accept the random timed path and that the first action takes place after  $C$ .

Since  $\lim_{\lambda \rightarrow 0} p_{1, \lambda} = 0$ ,  $\lim_{\lambda \rightarrow 0} p_{2, \lambda} = \frac{1}{2}$

which can be shown to contradict the 0-1 law (since  $\mathcal{M}$  is arbitrary).

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# A Hierarchy of Families of DTA

Let  $\mathbb{A}^{rc}$  be the family of DTA such that every circuit that includes a resetting autonomous transition also includes a synchronized transition,

- either resetting  $x$ ;
- or with guard  $x > C$  (the maximal constant).

Let  $\mathbb{A}^{nra}$  be the family of DTA in which no autonomous transition resets the clock.

Let  $\mathbb{A}^{nc}$  be the family of DTA in which:

- no autonomous transition resets the clock;
- and there is no cycle of autonomous transitions.

Then:

$$\mathbb{A}^{na} \sim_{\mathcal{L}} \mathbb{A}^{nc} \sim_{\mathcal{L}} \mathbb{A}^{nra} \sim_{\mathcal{L}} \mathbb{A}^{rc} \xrightarrow{\mathcal{M}} \mathbb{A}$$

## From $\mathbb{A}^{rc}$ to $\mathbb{A}^{nra}$

Consider an elementary path of  $\mathcal{A} \in \mathbb{A}^{rc}$

not including synchronized transitions with reset or with guard  $x > C$ .

Its *delay* is the sum of constants occurring in its resetting autonomous transitions.

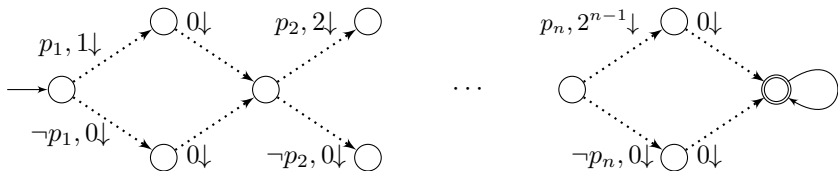
Let  $K$  be the maximal delay of such paths.

The locations of  $\mathcal{A}' \in \mathbb{A}^{nra}$  are:  $L' = \{\langle \ell, i \rangle \mid 0 \leq i \leq K \wedge \ell \in L \setminus \{\ell_f\}\} \cup \{\ell_f\}$   
with its initial location  $\langle \ell_0, 0 \rangle$  and final location  $\ell_f$ .

- For all transition outgoing  $\ell$ , there is a corresponding transition outgoing  $\langle \ell, i \rangle$  except for resetting autonomous transition with guard  $x = c$  such that  $i + c > K$ .
- The guard of the transitions outgoing  $\langle \ell, i \rangle$  is “incremented” by  $i$ .
- The corresponding transition of a resetting autonomous transition with guard  $x = c$  such that  $i + c \leq K$  does not reset but leads to  $\langle \ell, i + c \rangle$ .
- The corresponding transition of a synchronized transition with reset or with guard  $x > C$  leads to  $\langle \ell, 0 \rangle$ .

# The Exponential Blow-up is Unavoidable from $\mathbb{A}^{rc}$ to $\mathbb{A}^{nra}$

Let  $\mathcal{A}_n \in \mathbb{A}^{rc}$  be the following DTA.



Then for all  $\mathcal{A}'_n \in \mathbb{A}^{nra}$  such that  $\mathcal{L}(\mathcal{A}'_n) = \mathcal{L}(\mathcal{A}_n)$ ,  $(|Aut| + 1)|Synch| \geq 2^n$ .

- where  $Aut$  is the set of autonomous transitions
- and  $Synch$  is the set of synchronized transitions.

## From $\mathbb{A}^{nra}$ to $\mathbb{A}^{nc}$ in quadratic time

**Idea.** Visiting a circuit of autonomous transitions means non acceptance.

Let  $\mathcal{A} \in \mathbb{A}^{nra}$  and  $K$  be the number of autonomous transitions.

The set of locations of  $\mathcal{A}'$  is  $L' = \{(l, i) \mid 0 \leq i \leq K \wedge l \in L \setminus \{\ell_f\}\} \cup \{\ell_f, \ell_\perp\}$  with initial location  $(l_0, 0)$  and final location  $\ell_f$ .

For all synchronized transition  $l \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} l'$  of  $\mathcal{A}$  and  $i \leq K$ :

- if  $l' = \ell_f$  then there is a synchronized transition  $(l, i) \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} \ell_f$ ;
- otherwise there is a synchronized transition  $(l, i) \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} (l', 0)$ .

For all autonomous transition  $l \xrightarrow{\varphi^-, x=c, \#, \emptyset} l'$  of  $\mathcal{A}$  and  $i \leq K$ :

- if  $i = K$  then there is an autonomous transition  $(l, i) \xrightarrow{\varphi^-, x=c, \#, \emptyset} \ell_\perp$ ;
- else if  $l' = \ell_f$  then there is an autonomous transition  $(l, i) \xrightarrow{\varphi^-, x=c, \#, \emptyset} \ell_f$ ;
- otherwise there is an autonomous transition  $(l, i) \xrightarrow{\varphi^-, x=c, \#, \emptyset} (l', i + 1)$ .

# From $\mathbb{A}^{nc}$ to $\mathbb{A}^{na}$ in polynomial time (1)

**First stage.** Duplicating locations w.r.t. clock regions:  $\ell$  yields  $\{\langle \ell, rg \rangle\}$ .

For all synchronized transition  $\ell \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} \ell'$  and regions  $rg$  and  $rg'$   
a transition  $\langle \ell, rg \rangle \xrightarrow{\varphi^-, \gamma \wedge x \in rg', B, r, \varphi^+} \langle \ell', rg' \rangle$ .

For all autonomous transition  $\ell \xrightarrow{\varphi, x=i, \#, \emptyset} \ell'$  and region  $rg$ ,  
a transition  $\langle \ell, rg \rangle \xrightarrow{\varphi, x=i, \#, \emptyset} \langle \ell', \{i\} \rangle$ .

**Second stage.** Making explicit the priority of the autonomous transitions.

Let  $\langle \ell, rg \rangle$  be a location and  $\{\langle \ell, rg \rangle \xrightarrow{\varphi_k, x=\alpha_k, \#, \emptyset} \langle \ell_k, \{\alpha_k\} \rangle\}_{k \leq K}$   
be the autonomous transitions outgoing from  $\langle \ell, rg \rangle$  with  $rg \leq \alpha_1 \leq \dots \leq \alpha_K$ .

For all  $k$ , an autonomous transition  $\langle \ell, rg \rangle \xrightarrow{\varphi_k \wedge \bigwedge_{k' < k} \neg \varphi_{k'}, x=\alpha_k, \#, \emptyset} \langle \ell_k, \{\alpha_k\} \rangle$ .

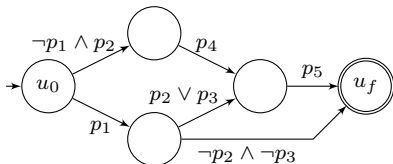
For all synchronized transition  $\langle \ell, rg \rangle \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} \langle \ell', rg' \rangle$ ,

a transition  $\langle \ell, rg \rangle \xrightarrow{\varphi^- \wedge \bigwedge_{\alpha_k \leq rg'} \neg \varphi_k, \gamma, B, r, \varphi^+} \langle \ell', rg' \rangle$ .



## From $\mathbb{A}^{nc}$ to $\mathbb{A}^{na}$ in polynomial time (2)

**Third stage.** Eliminating autonomous transitions using decision diagrams (DD):



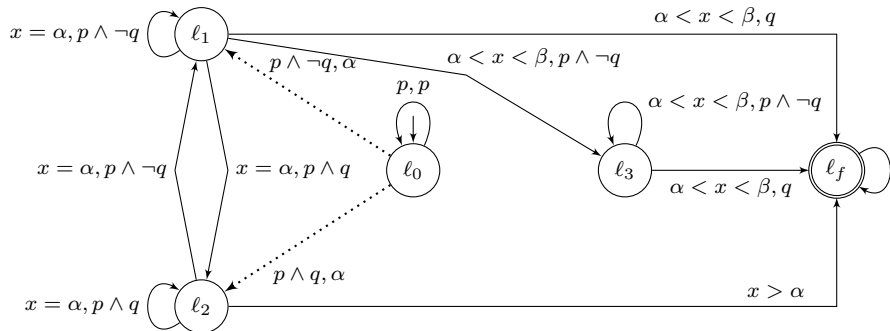
For all  $\langle \ell, rg \rangle$  and  $\langle \ell', \{i\} \rangle$  such that there is a path of autonomous transitions from  $\langle \ell, rg \rangle$  to  $\langle \ell', \{i\} \rangle$ . Let the formula  $\varphi_{\ell, rg}^{\ell', i}$  be the DD:

- whose vertices are locations both reachable from  $\langle \ell, rg \rangle$  and can reach  $\langle \ell', \{i\} \rangle$  (both by autonomous transitions);
- whose edges of the DD are the autonomous transitions between such vertices labelled by their formula.

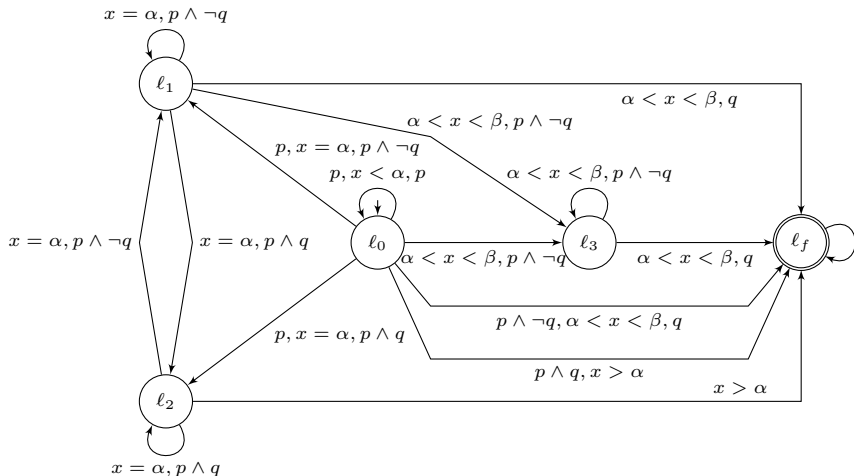
For all synchronized transition  $\langle \ell', \{i\} \rangle \xrightarrow{\varphi^-, \gamma, B, r, \varphi^+} \langle \ell'', rg'' \rangle$ ,

a transition  $\langle \ell, rg \rangle \xrightarrow{\varphi_{\ell, rg}^{\ell', i} \wedge \varphi^-, x \geq i \wedge \gamma, B, r, \varphi^+} \langle \ell'', rg'' \rangle$ .

# Illustration (1)



# Illustration (2)



# Conclusion and Perspectives

## Contributions

- Analysis of the interest of autonomous transitions in single-clock DTA w.r.t. expressiveness and conciseness;
- Analysis of the relation between state formulas and transition formulas w.r.t. conciseness (*not presented here*).

## Perspectives

- From a practical point of view, efficiency increasing of tools using logics like CSL or extensions;
- From a theoretical point of view, decision problems like “given a DTA, does there exist an equivalent DTA without autonomous transitions?”