Exercise 1: Transition monoid

1. Give the transition monoid of the following automaton $\mathcal{A}$:

2. Give a word representing each class of the syntactic congruence of the language recognized by $\mathcal{A}$.

Exercise 2: Iterating factors

Let $\Sigma$ be an alphabet and $L \subseteq \Sigma^*$ a language. We define the iterating factors of $L$ the set $\mathcal{I}(L) = \{ w \in \Sigma^* \mid \exists u, v \in \Sigma^* . uw^*v \subseteq L \}$. Prove that if $L$ is regular, then $\mathcal{I}(L)$ is also regular.

Hint: use a monoid $M$, $P \subseteq M$ and morphism $\mu$ recognizing $L$.

Exercise 3: Syntactic congruence

Let $P$ be the language of balanced strings of parentheses over alphabet $\Sigma = \{ (, ) \}$, e.g. $(())() \in P$ but $(()) \notin P$ and $)( \notin P$.

1. What are the equivalence classes of the relation $\equiv_P$?
2. Is $P$ recognizable?