Initiation à la vérification
Basics of Verification

http://mpri.master.univ-paris7.fr/C-1-22.html

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Outline

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Models

Specifications

Linear Time Specifications

Branching Time Specifications

Need for formal verifications methods

Critical systems
  - Transport
  - Energy
  - Medicine
  - Communication
  - Finance
  - Embedded systems
  - ...

Disastrous software bugs

Mariner 1 probe, 1962
See http://en.wikipedia.org/wiki/Mariner_1
- Destroyed 293 seconds after launch
- Missing hyphen in the data or program? No!
- Overbar missing in the mathematical specification:
  \( \bar{R}_n \): \( n \)th smoothed value of the time derivative of a radius.
Without the smoothing function indicated by the bar, the program treated normal minor variations of velocity as if they were serious, causing spurious corrections that sent the rocket off course.
Disastrous software bugs

Ariane 5 flight 501, 1996
See http://en.wikipedia.org/wiki/Ariane_5_Flight_501
- Destroyed 37 seconds after launch (cost: 370 millions dollars).
- Data conversion from a 64-bit floating point to 16-bit signed integer value caused a hardware exception (arithmetic overflow).
- Efficiency considerations had led to the disabling of the software handler (in Ada code) for this error trap.
- The fault occurred in the inertial reference system of Ariane 5. The software from Ariane 4 was re-used for Ariane 5 without re-testing.
- On the basis of those calculations the main computer commanded the booster nozzles, and somewhat later the main engine nozzle also, to make a large correction for an attitude deviation that had not occurred.
- The error occurred in a realignment function which was not useful for Ariane 5.

Disastrous software bugs

Spirit Rover (Mars Exploration), 2004
- Ceased communicating on January 21.
- Flash memory management anomaly: too many files on the file system.
- Resumed to working condition on February 6.

Disastrous software bugs

Other well-known bugs
- Therac-25, at least 3 death by massive overdoses of radiation.
  Race condition in accessing shared resources.
- Electricity blackout, USA and Canada, 2003, 55 millions people.
  Race condition in accessing shared resources.
  Flaw in the division algorithm, discovered by Thomas Nicely.
  See http://en.wikipedia.org/wiki/Pentium_FDIV_bug
- Needham-Schroeder, authentication protocol based on symmetric encryption.
  Published in 1978 by Needham and Schroeder
  Proved correct by Burrows, Abadi and Needham in 1989
  Flaw found by Lowe in 1995 (man in the middle)
  Automatically proved incorrect in 1996.
  See http://en.wikipedia.org/wiki/Needham-Schroeder_protocol

Formal verifications methods

Complementary approaches
- Theorem prover
- Model checking
- Static analysis
- Test
Model Checking

- Purpose 1: automatically finding software or hardware bugs.
- Purpose 2: prove correctness of abstract models.
- Should be applied during design.
- Real systems can be analysed with abstractions.

- Model Checking
  3 steps
  - Constructing the model $M$ (transition systems)
  - Formalizing the specification $\varphi$ (temporal logics)
  - Checking whether $M \models \varphi$ (algorithmics)

- Main difficulties
  - Size of models (combinatorial explosion)
  - Expressivity of models or logics
  - Decidability and complexity of the model-checking problem
  - Efficiency of tools

- Challenges
  - Extend models and algorithms to cope with more systems.
    Infinite systems, parameterized systems, probabilistic systems, concurrent systems, timed systems, hybrid systems, . . .
  - Scale current tools to cope with real-size systems.
    Needs for modularity, abstractions, symmetries, . . .

References

Bibliography

Constructing the model

Example: Men, Wolf, Goat, Cabbage

Model = Transition system
- State = who is on which side of the river
- Transition = crossing the river
- Specification
  Safety: Never leave WG or GC alone
  Liveness: Take everyone to the other side of the river.

Transition system or Kripke structure

Definition: TS
\[ M = (S, \Sigma, T, I, AP, \ell) \]
- \( S \): set of states (finite or infinite)
- \( \Sigma \): set of actions
- \( T \subseteq S \times \Sigma \times S \): set of transitions
- \( I \subseteq S \): set of initial states
- \( AP \): set of atomic propositions
- \( \ell : S \rightarrow 2^{AP} \): labelling function.

Example: Digicode ABA

Every discrete system may be described with a TS.

Description Languages

Pb: How can we easily describe big systems?

Description Languages (high level)
- Programming languages
- Boolean circuits
- Modular description, e.g., parallel compositions
- Problems: concurrency, synchronization, communication, atomicity, fairness, ...
- Petri nets (intermediate level)
- Transition systems (intermediate level)
  - with variables, stacks, channels, ...
  - Synchronized products
- Logical formulae (low level)

Operational semantics
High level descriptions are translated (compiled) to low level (infinite) TS.
Transition systems with variables

**Definition:** TSV  
\[ M = (S, \Sigma, V, (D_v)_{v \in V}, T, I, AP, \ell) \]

- **V:** set of (typed) variables, e.g., boolean, [0..4], ...
- Each variable \( v \in V \) has a domain \( D_v \) (finite or infinite)
- Guard or Condition: unary predicate over \( D = \prod_{v \in V} D_v \)
  - Symbolic descriptions: \( x < 5 \), \( x + y = 10 \), ...
- Instruction or Update: map \( f : D \rightarrow D \)
  - Symbolic descriptions: \( x := 0 \), \( x := (y+1)^2 \), ...
- **T** \( \subseteq S \times (\mathcal{P}(D) \times \Sigma \times \mathcal{P}(D)) \times S \)
  - Symbolic descriptions: \( s \xrightarrow{a,f} s' \) \( \land \nu \models g \)

### Example: Vending machine
- coffee: 50 cents, orange juice: 1 euro, ...
- possible coins: 10, 20, 50 cents
- we may shuffle coin insertions and drink selection

### Example: Digicode
- **OPEN**
- **ERROR**

... and its semantics \((n = 2)\)

- Program counter = states
- Instructions = transitions
- Variables = variables

### Example: GCD
- **OPEN**
- **ERROR**
Only variables

The state is nothing but a special variable: \( s \in V \) with domain \( D_s = S \).

Definition: TSV \( M = (V, (D_v)_{v \in V}, T, I, AP, \ell) \)

- \( D = \prod_{v \in V} D_v \)
- \( I \subseteq D, T \subseteq D \times D \)

Symbolic representations with logic formulae

- \( I \) given by a formula \( \psi(\nu) \)
- \( T \) given by a formula \( \varphi(\nu, \nu') \)
- \( \nu \): values before the transition
- \( \nu' \): values after the transition
- Often we use boolean variables only: \( D_v = \{0, 1\} \)
- Concise descriptions of boolean formulae with Binary Decision Diagrams.

Example: Boolean circuit: modulo 8 counter

\[
\begin{align*}
b'_0 &= \neg b_0 \\
b'_1 &= b_0 \oplus b_1 \\
b'_2 &= (b_0 \land b_1) \oplus b_2
\end{align*}
\]

Modular description of concurrent systems

\[ M = M_1 \parallel M_2 \parallel \cdots \parallel M_n \]

Semantics

- Various semantics for the parallel composition \( \parallel \)
- Various communication mechanisms between components:
  - Shared variables, FIFO channels, Rendez-vous, ...
- Various synchronization mechanisms

Example: Elevator with 1 cabin, 3 doors, 3 calling devices

Symbolic representation

Example: Logical representation

\[
\begin{align*}
\delta_B &= s = 1 \land \text{cpt} < n \land s' = 1 \land \text{cpt}' = \text{cpt} + 1 \\
\lor & s = 1 \land \text{cpt} = n \land s' = 5 \land \text{cpt}' = \text{cpt} + 1 \\
\lor & s = 2 \land s' = 3 \land \text{cpt}' = \text{cpt} \\
\lor & s = 3 \land \text{cpt} < n \land s' = 1 \land \text{cpt}' = \text{cpt} + 1 \\
\lor & s = 3 \land \text{cpt} = n \land s' = 5 \land \text{cpt}' = \text{cpt} + 1
\end{align*}
\]

Modular description of concurrent systems

Example: Elevator

- Cabin:
- Door for level \( i \):
- Call for level \( i \):

The actual system is a synchronized product of all these automata. It consists of (at most) \( 3 \times 2^3 \times 2^3 = 192 \) states.
Synchronizations

Synchronization by transitions
Example: Parallel compositions
Synchronized products: restrictions of the general product.
Definition: General product
- Components: \( M_i = (S_i, \Sigma_i, T_i, I_i, AP_i, \ell_i) \)
- Product: \( M = (S, \Sigma, T, I, AP, \ell) \) with
  \( S = \prod_i S_i \)
  \( \Sigma = \prod_i (\Sigma_i \cup \{\varepsilon\}) \)
  and \( I = \prod_i I_i \)
  \( T = \{(p_1, \ldots, p_n) \mid (a_1, \ldots, a_n) \rightarrow (q_1, \ldots, q_n) \text{ for all } i, (p_i, a_i, q_i) \in T_i \text{ or } p_i = q_i \text{ and } a_i = \varepsilon\} \)
  \( AP = \bigcup_i AP_i \) and \( \ell(p_1, \ldots, p_n) = \bigcup_i \ell(p_i) \)
Synchronized products: restrictions of the general product.
Parallel compositions
- Synchronous: \( \Sigma^\text{sync} = \prod_i \Sigma_i \)
- Asynchronous: \( \Sigma^\text{sync} = \bigcup_i \Sigma_i' \) with \( \Sigma_i' = \{\varepsilon\}^{i-1} \times \Sigma_i \times \{\varepsilon\}^{n-i} \)
Synchronizations
- By states: \( \Sigma^\text{sync} \subseteq S \)
- By labels: \( \Sigma^\text{sync} \subseteq \Sigma \)
- By transitions: \( T^\text{sync} \subseteq T \)

Example: Printer manager
Example: Asynchronous product
Synchronization by states: \((P, P)\) is forbidden

Example: digicode
Example: Synchronous product
Synchronization by transitions

Synchronization by Rendez-vous
Definition: Rendez-vous
- \( !m \) sending message \( m \)
- \( ?m \) receiving message \( m \)
- SOS: Structural Operational Semantics
- Local actions
  \[
  s_1 \xrightarrow{a_1} s'_1, \quad s_2, \quad s_2 \xrightarrow{a_2} s'_2
  \]
- Rendez-vous
  \[
  s_1 \xrightarrow{m_1} s'_1 \land s_2 \xrightarrow{m_2} s'_2, \quad s_1 \xrightarrow{m_1} s'_1 \land s_2 \xrightarrow{m_2} s'_2
  \]
- It is a kind of synchronization by actions.
- Essential feature of process algebra.

Example: Elevator with 1 cabin, 3 doors, 3 calling devices
- \( ?\text{up} \) is uncontrollable for the cabin
- \( ?\text{leave}_i \) is uncontrollable for door \( i \)
- \( ?\text{call}_0 \) is uncontrollable for the system
**Example: Elevator**

- Cabin:
  - 0
  - 1
  - 2
  - ?down
  - !leave, !reach
  - ?up

- Door for level $i$:
  - Closed
  - Opened
  - ?leave,$i$
  - ?reach,$i$

We should design the controller.

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**Example: Synchronization by Rendez-vous**

- Shared variables
  - Definition: Asynchronous product + shared variables
  - $\bar{s} = (s_1, \ldots, s_n)$ denotes a tuple of states
  - $\nu \in D = \prod_{v \in V} D_v$ is a valuation of variables.

**Semantics (SOS)**

- Safety: never simultaneously in critical section (CS).
- Liveness: if a process wants to enter its CS, it eventually does.
- Fairness: if process 1 wants to enter its CS, then process 2 will enter it at most once before process 1 does.

Using shared variables but no synchronization mechanisms: the **atomicity** is

- testing or reading or writing a single variable at a time
- no test-and-set: $\{x = 0; x := 1\}$

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**Peterson’s algorithm (1981)**

- Process $i$:
  - loop forever
  - req[$i$] := true; turn := 1-i
  - wait until (turn = i or req[1-i] = false)

**Critical section**

- req[$i$] := false

**Example: Mutual exclusion for 2 processes satisfying**

- Safety: never simultaneously in critical section (CS).
- Liveness: if a process wants to enter its CS, it eventually does.
- Fairness: if process 1 wants to enter its CS, then process 2 will enter it at most once before process 1 does.

**Peterson’s algorithm (1981)**

- Process $i$:
  - loop forever
  - req[$i$] := true; turn := 1-i
    - wait until (turn = i or req[1-i] = false)
    - Critical section
      - req[$i$] := false

**Exercise:**

- Draw the concrete TS assuming the first two assignments are atomic.
- Is the algorithm still correct if we swap the first two assignments?

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**Atomicsity**

- Initially $x = 1 \land y = 2$

**Program $P_1$:**

\[
\begin{align*}
\text{LoadR}_1, x &\parallel \text{AddR}_1, y \\
\text{StoreR}_1, x &\parallel \text{AddR}_2, y
\end{align*}
\]

**Program $P_2$:**

\[
\begin{align*}
\text{LoadR}_1, x &\parallel \text{AddR}_1, y \\
\text{StoreR}_1, x &\parallel \text{AddR}_2, y
\end{align*}
\]

Assuming each instruction is atomic, what are the possible results of $P_1$ and $P_2$?
Atomicity

Definition: Atomic statements: \texttt{atomic}(ES)

Elementary statements (no loops, no communications, no synchronizations)

\[
ES ::= \text{skip} \mid \text{await} \ c \mid x := e \mid ES \mid ES \square ES \\
\mid \text{when} \ c \ \text{do} \ ES \mid \text{if} \ c \ \text{then} \ ES \ \text{else} \ ES
\]

Atomic statements: if the ES can be fully executed then it is executed in one step.

\[
(\bar{s}, \nu) \xrightarrow{ES} (\bar{s}', \nu') \\
(\bar{s}, \nu) \xrightarrow{\text{atomic(ES)}} (\bar{s}', \nu')
\]

Example: Atomic statements

- atomic\((x = 0; x := 1)\) (Test and set)
- atomic\((y := y - 1; \text{await}(y = 0); y := 1)\) is equivalent to await\((y = 1)\)