A+ YEARS OF WEIGHTED LOGICS FOR WEIGHTED AUTOMATA

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Manfred is 3C

Workshop MoTAGWA, Dresden 2016

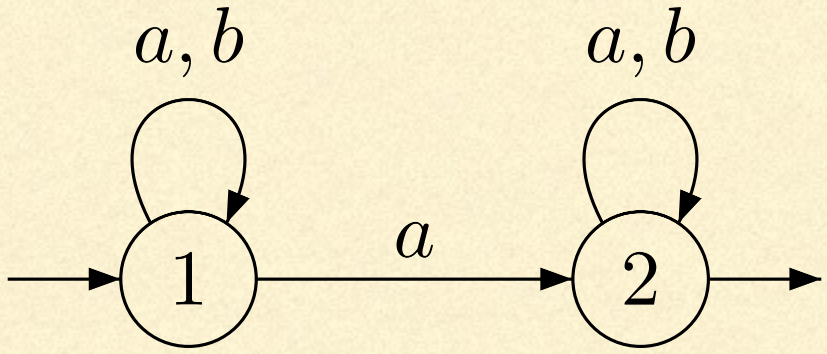
GOAL

- Qualitative, Boolean: [Büchi'60], [Elgot'61], [Trakhtenbrot'61]

- Quantitative, weights

KEEP SYNTAX CHANGE SEMANTICS

- Inspired by weighted automata on semirings

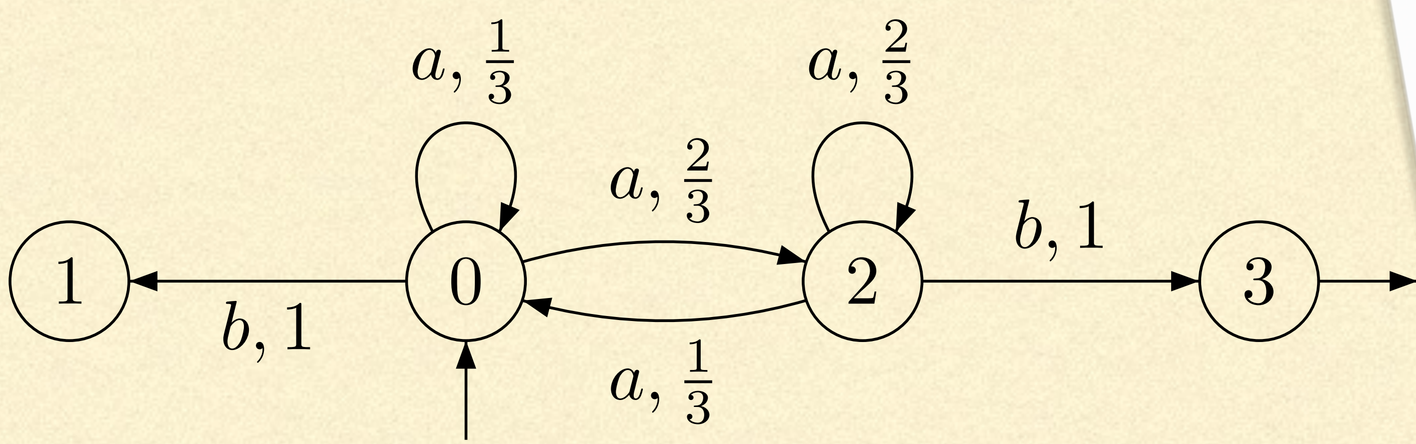


$$\mu(a) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad \mu(b) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

  - Qualitative: existence of an accepting path
  - Quantitative: number of accepting paths
  - Boolean semiring: $(\{0,1\}, \vee, \wedge, 0, 1)$ $\quad \mu(babaab) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
  - Natural semiring: $(\mathbb{N}, +, \times, 0, 1)$ $\quad \mu(babaab) = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

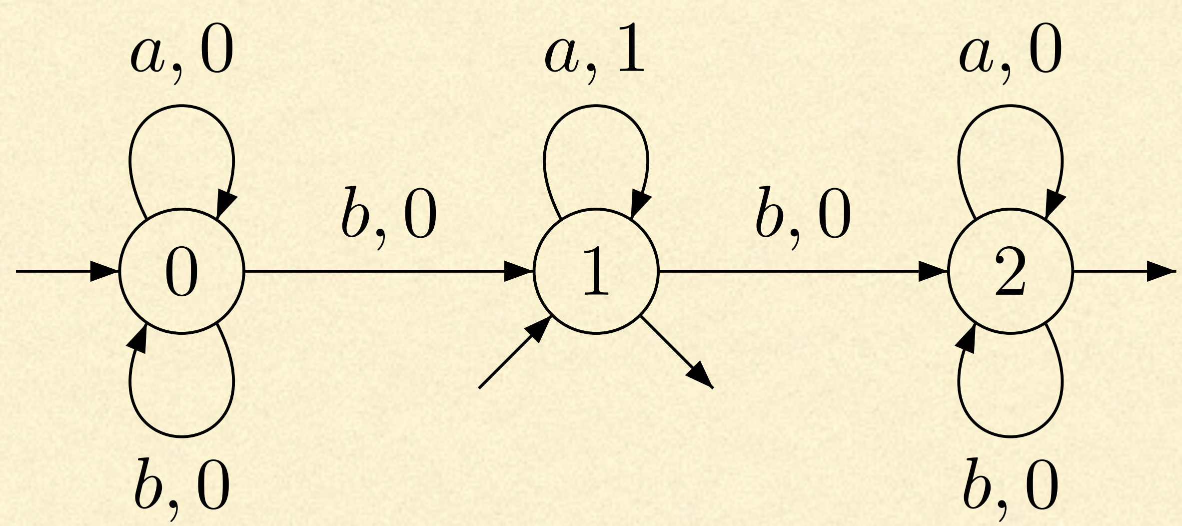
WEIGHTED AUTOMATA ON SEMIRINGS

- Probabilistic semiring: $(\mathbb{R}_{\geqslant 0}, +, \times, 0, 1)$



add weights to transitions

- (max,+) semiring: $(\mathbb{N} \cup \{-\infty\}, \max, +, -\infty, 0)$



max length of a's blocks

THE FIRST SOLUTION

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

Arbitrary constants from a semiring

Negation restricted to atomic formulae

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

- Semantics in a semiring $\mathbb{S} = (S, +, \times, 0, 1)$
  - Atomic formulae: 0,1
- Inspired from the boolean semiring $\mathbb{B} = (\{0,1\}, \vee, \wedge, 0, 1)$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

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$$\mid \boxed{\varphi \vee \varphi} \mid \varphi \wedge \varphi \mid \boxed{\exists x\, \varphi} \mid \forall x\, \varphi \mid \boxed{\exists X\, \varphi} \mid \forall X\, \varphi$$

- Semantics in a semiring $\mathbb{S} = (S, +, \times, 0, 1)$
  - Atomic formulae: 0, 1
  - disjunction, existential quantifications: sum
- Inspired from the boolean semiring $\mathbb{B} = (\{0,1\}, \vee, \wedge, 0, 1)$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

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$$\mid \varphi \vee \varphi \mid \boxed{\varphi \wedge \varphi} \mid \exists x\, \varphi \mid \boxed{\forall x\, \varphi} \mid \exists X\, \varphi \mid \boxed{\forall X\, \varphi}$$

- Semantics in a semiring $\mathbb{S} = (S, +, \times, 0, 1)$
  - Atomic formulae: 0,1
  - disjunction, existential quantifications: sum
  - conjunction, universal quantifications: product
- Inspired from the boolean semiring $\mathbb{B} = (\{0,1\}, \vee, \wedge, 0, 1)$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

- Examples

$$\varphi_1 = \exists x\, P_a(x) \qquad \varphi_2 = \forall x\, \exists y\, (y \leqslant x \wedge P_a(y))$$

$$[\![\varphi_1]\!](w) = |w|_a \qquad [\![\varphi_2]\!](abaab) = 1 \times 1 \times 2 \times 3 \times 3$$

$$[\![\varphi_2]\!](a^n) = n!$$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P ...) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid ...$$

We need to restrict weighted MSO

- Examples

$$\varphi_1 = \exists x \, P_a(x)$$

$$[\![\varphi_1]\!](w) = |w|_a$$

$$\varphi_2 = \forall x ...$$

Too big to be computed by a weighted automaton

$$... 1 \times 2 \times 3 \times 3$$

$$[\![\varphi_2]\!](a^n) = n!$$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \cancel{\forall X\, \varphi}$$

φ almost boolean
Defines
MSO-step functions

$$\cancel{\varphi_2 = \forall x\, \exists y\, (y \leqslant x \wedge P_a(y))}$$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall \cancel{X}\, \varphi$$

commutativity

φ almost boolean
Defines
MSO-step functions

Thm [Droste-Gastin] weighted automata = restricted wMSO

Google Scholar: +260 citations!

[Droste-Gastin, ICALP'05, TCS'[illegible], Handbook WA'09]

NTICS

Semantic Scholar

Hi Paul,

Your paper **"Weighted Automata and Weighted Logics"** is now on Semantic Scholar and has **287 citations** and a wealth of additional statistics and metadata produced by our software.

Learn more about the impact of your paper:

See Paper Details

$| \neg(x \in X)$

boolean
ines
functions

Thm [Droste-Gastin] weighted automata = restricted wMSO

Google Scholar: +260 citations!

[Droste-Gastin, ICALP'05, TCS' ... andbook WA'09]

WEIGHTED AUTOMATA TO RESTRICTED WMSO

$$\exists \overline{X} = (X_1, \ldots, X_n)\ \mathsf{run}(\overline{X}) \wedge \forall x\, \mathsf{weight}(\overline{X}, x)$$

sum over all runs

weight of the run

$$[\![\mathsf{run}(\overline{X})]\!](w, \sigma) = \begin{cases} 1 & \text{if } \overline{X} \text{ is a run} \\ 0 & \text{otherwise.} \end{cases}$$

weight of the transition taken at x by run X

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

WEIGHTED AUTOMATA TO RESTRICTED WMSO

$$\exists \overline{X} = (X_1, \ldots, X_n)\ \mathsf{run}(\overline{X}) \wedge \forall x\ \mathsf{weight}(\overline{X}, x)$$

sum over all runs

weight of the run

$$[\![\mathsf{run}(\overline{X})]\!](w, \sigma) = \begin{cases} 1 & \text{if } \overline{X} \text{ is a run} \\ \ldots \end{cases}$$

We need UNAMBIGUOUS formulas in the QUANTITATIVE semantics

...nsition
...aken at x by run X

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

UNAMBIGUOUS FORMULAS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

$$\begin{aligned}\exists x\, \exists y\ x < y \wedge P_a(x) \wedge P_b(y) & \\ & \wedge \forall z\, (x \leqslant z \vee (z < x \wedge \neg P_a(z))) \\ & \wedge \forall z\, (z \leqslant y \vee (y < z \wedge \neg P_b(z)))\end{aligned}$$

$$\varphi \mapsto (\varphi^+, \varphi^-)$$

$$[\![\varphi^+]\!](w,\sigma) = \begin{cases} 1 & \text{if } w,\sigma \models \varphi \\ 0 & \text{otherwise.} \end{cases} \qquad [\![\varphi^-]\!](w,\sigma) = \begin{cases} 0 & \text{if } w,\sigma \models \varphi \\ 1 & \text{otherwise.} \end{cases}$$

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

RESTRICTED WMSO TO WEIGHTED AUTOMATA

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \cancel{\forall X\, \varphi}$$

- disjunction : closure under sum
- existential quantification : closure under projections/renamings

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

RESTRICTED WMSO TO WEIGHTED AUTOMATA

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
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φ almost boolean
Defines
MSO-step functions

- disjunction : closure under sum
- existential quantification : closure under projections/renamings
- universal quantification: main difficulty

[Droste-Gastin, ICALP'05, TCS'07, Handbook WA'09]

MANY EXTENSIONS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

Thm: weighted automata = restricted wMSO

- Finite words: [Droste-Gastin, ICALP'05]
- Finite ranked trees: [Droste-Vogler, Theor. Comp. Sci.'06]
- Pictures: [Fichtner, STACS'06]
- Infinite words: [Droste-Rahonis, DLT'06]
- Finite unranked trees: [Droste-Vogler, Theor. of Computing Systems'09]
- Traces: [Fichtner-Kuske-Meinecke, Handbook of weighted automata '09]
- Nested words: [Mathissen, LMCS'10]
- …

MANY EXTENSIONS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P \ldots \in X)$$

$$\mid \varphi \vee \ldots$$

20+ papers by Manfred on weighted logics!

Thr... automata = restricted wMSO

- Finite words: [Droste-Gastin, ICALP'05]
- Finite ranked trees: [Droste-Vogler, Theor. Comp. Sci.'06]
- Pictures: [Fichtner, STACS'06]
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- Nested words: [Mathissen, LMCS'10]
- …

CHANGING THE SYNTAX TOWARDS EASIER FORMULAE

CHANGING THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- No need to write unambiguous formulae

$$\exists x\, \exists y\; x < y \wedge P_a(x) \wedge P_b(y)$$
$$\cancel{\wedge \forall z\, (x \leqslant z \vee (z < x \wedge \neg P_a(z)))}$$
$$\cancel{\wedge \forall z\, (z \leqslant y \vee (y < z \wedge \neg P_b(z)))}$$

[Bollig-Gastin, DLT'09] [Droste-Meinecke'12] [Fülöp-Stüber-Vogler'12]

CHANGING THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- No need to write unambiguous formulae

- Quantitative fragment

$$\Phi ::= s \mid \varphi \mid \Phi + \Phi \mid \Phi \times \Phi \mid \textstyle\sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Phi$$

- Formulae are more readable

$$\textstyle\sum_x P_a(x) \qquad\qquad \prod_x \sum_y (y \leqslant x \wedge P_a(y))$$

$$\exists x\, P_a(x) \qquad\qquad \forall x\, \exists y\, (y \leqslant x \wedge P_a(y))$$

[Bollig-Gastin, DLT'09] [Droste-Meinecke'12] [Fülöp-Stüber-Vogler'12]

CHANGING THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- No ne… s formulae

commutative

Φ has no Σ or Π

- Quantitative fragment

$$\Phi ::= s \mid \varphi \mid \Phi + \Phi \mid \Phi \times \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Phi$$

- Formulae are more readable

$\sum_x P_a(x)$ … $\leqslant x \wedge P_a(y))$

$\exists x\, P_a(x)$ … $y\, (y \leqslant x \wedge P_a(y))$

We still need restrictions

[Bollig-Gastin, DLT'09] [Droste-Meinecke'12] [Fülöp-Stüber-Vogler'12]

CHANGING THE SYNTAX

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

$$\Phi ::= s \mid \varphi \mid \Phi + \Phi \mid \Phi \times \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Phi$$

Thm: weighted automata = restricted wMSO

BEYOND SEMIRINGS
AVERAGE, DISCOUNTED SUMS, …

FROM PRODUCTS TO VALUATIONS

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- We compute the weight of the run $\mathsf{Val}(s_1 s_2 \cdots s_n)$
- product $\mathsf{Val}(s_1 s_2 \cdots s_n) = s_1 \times s_2 \times \cdots \times s_n$
- average: $\mathsf{Val}(s_1 s_2 \cdots s_n) = \frac{s_1 + s_2 + \cdots + s_n}{n}$
- discounted: $\mathsf{Val}(s_1 s_2 \cdots s_n) = s_1 + \lambda s_2 + \cdots + \lambda^{n-1} s_n$

Val: possibly non-associative, non distributive

[Droste-Meinecke, CIAA'10, IC'12]

FROM PRODUCTS TO VALUATIONS

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- We compute the weight of the run $\mathsf{Val}(s_1 s_2 \cdots s_n)$
- Final semantics $$[\![\mathcal{A}]\!](w) = \sum_{\rho \text{ run on } w} \mathsf{Val}(\mathsf{wgt}(\rho))$$
- Valuation monoid $(S, +, 0, \mathsf{Val})$ $\quad \mathsf{Val} \colon S^+ \to S$

[Droste-Meinecke, CIAA'10, IC'12]

FROM PRODUCTS TO VALUATIONS

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

$$\Phi ::= s \mid \varphi \mid \boxed{\Phi + \Phi} \mid \Phi \times \Phi \mid \boxed{\sum_x \Phi} \mid \boxed{\sum_X \Phi} \mid \prod_x \Phi$$

- Semantics in a valuation monoid $(S, +, 0, \mathsf{Val})$
  - $+, \Sigma$: ok

[Droste-Meinecke, CIAA'10, IC'12]

FROM PRODUCTS TO VALUATIONS

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

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- Semantics in a valuation monoid $(S, +, 0, \mathsf{Val})$
  - +, Σ : ok
  - Π : Val

$$\boxed{[\![\prod_x \Phi]\!](w,\sigma) = \mathsf{Val}(([\![\Phi]\!](w,\sigma[x \mapsto i]))_i)}$$

[Droste-Meinecke, CIAA'10, IC'12]

FROM PRODUCTS TO VALUATIONS

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

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  - $+, \Sigma$: ok
  - Π : Val

Thm [Droste-Meinecke]
weighted automata = restricted wMSO

$$\boxed{[\![\textstyle\prod_x \Phi]\!](w, \sigma) = \mathsf{Val}(([\![\Phi]\!](w, \sigma[x \mapsto i]))_i)}$$

[Droste-Meinecke, CIAA'10, IC'12]

EXTENDING THE SUM

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- We compute the weight of the run $\mathsf{Val}(s_1 s_2 \cdots s_n)$
- Final semantics

$$[\![\mathcal{A}]\!](w) = \mathsf{Average}\{\{\mathsf{Val}(\mathsf{wgt}(\rho)) \mid \rho \text{ run on } w\}\}$$

not associative

- Valuation structure $(S, F, \mathsf{Val}) \quad F : \mathbb{N}\langle S \rangle \to S$

[Droste-Perevoshchikov, CSR'13]

ABSTRACT SEMANTICS
MULTISETS OF WEIGHT-STRUCTURES

Joint work with Benjamin Monmege

MULTISETS OF WEIGHT STRUCTURES

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- Abstract semantics $\{\!|\mathcal{A}|\!\}(w) = \{\!\{\mathsf{wgt}(\rho) \mid \rho \text{ run on } w\}\!\}$

$$\{\!|\mathcal{A}|\!\} \colon \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$$

multiset

R: weights of A

- Aggregation $\mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

MULTISETS OF WEIGHT STRUCTURES

Semiring: sum-product

$$\mathsf{aggr}_{\mathtt{sp}}(A) = \sum \prod A = \sum_{r_1 \cdots r_n \in A} r_1 \times \cdots \times r_n$$

Valuation monoid: sum-valuation

$$\mathsf{aggr}_{\mathtt{sv}}(A) = \sum \mathsf{Val}(A) = \sum_{r_1 \cdots r_n \in A} \mathsf{Val}(r_1 \cdots r_n)$$

Valuation structure: F-valuation

$$\mathsf{aggr}_{\mathtt{fv}}(A) = F \circ \mathsf{Val}(A) = F(\{\{\mathsf{Val}(r_1 \cdots r_n) \mid r_1 \cdots r_n \in A\}\})$$

- Aggregation $\quad \mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

MULTISETS OF WEIGHT STRUCTURES

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- Abstract semantics $\{\!|\mathcal{A}|\!\}(w) = \{\!\{\mathsf{wgt}(\rho) \mid \rho \text{ run on } w\}\!\}$

  $\{\!|\mathcal{A}|\!\} \colon \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$

  finite multiset

  weights of A
- Aggregation $\mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$
- Concrete semantics $[\![\mathcal{A}]\!] = \mathsf{aggr} \circ \{\!|\mathcal{A}|\!\} \colon \Sigma^\star \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

THE FINAL TOUCH
CHANGING AGAIN THE SYNTAX

Joint work with Benjamin Monmege

CHANGING AGAIN THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- Step formulae $\quad \Psi ::= s \mid \varphi \,?\, \Psi : \Psi$

if … then … else …
no sum, no product

$$P_a(x) \,?\, 1 : 0$$

$$P_a(x) \,?\, 1 : (P_b(x) \,?\, -1 : 0)$$

$$\varphi_1(x) \,?\, s_1 : (\varphi_2(x) \,?\, s_2 : \cdots (\varphi_{n-1}(x) \,?\, s_{n-1} : s_n) \cdots)$$

$$[\![\Psi]\!](w, \sigma) = s$$

some value occurring in Ψ

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- Step formulae $\quad \Psi ::= s \mid \varphi\,?\,\Psi : \Psi$

$P_a(x)\,?\,1 : 0$

se …
duct

$P_a($

A step formula takes finitely many values
For each value, the pre-image is MSO-definable

$\varphi_1($ … $(\varphi_2(x)\,?\,s_2 : \cdots (\varphi_{n-1}(x)\,?\,s_{n-1} : s_n) \cdots)$

$[\![\Psi]\!](w, \sigma) = s$

some value occurring in Ψ

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- Step formulae $\Psi ::= s \mid \varphi \,?\, \Psi : \Psi$

- core wMSO $\Phi ::= \mathbf{0} \mid \varphi \,?\, \Phi : \Phi \mid \Phi + \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Psi$

no constants

if … then … else …
No binary products - No φ

Assigns a value from Ψ
to each position

$$[\![\textstyle\prod_x \Psi]\!](w,\sigma) = ([\![\Psi]\!](w,\sigma[x \mapsto i]))_{i \in \mathsf{pos}(w)}$$

$$\{\!|\textstyle\prod_x \Psi|\!\}(w,\sigma) = \{\!\{([\![\Psi]\!](w,\sigma[x \mapsto i]))_{i \in \mathsf{pos}(w)}\}\!\} \in \mathbb{N}\langle R^\star\rangle$$

singleton multiset

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

- Boolean fragment

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- Step formulae $\qquad \Psi ::= s \mid \varphi\,?\,\Psi : \Psi$

- core wMSO $\qquad \Phi ::= \mathbf{0} \mid \varphi\,?\,\Phi : \Phi \mid \Phi + \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Psi$

- Abstract semantics

empty multiset

sums over multisets

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$

$\Psi ::= s \mid \varphi ?\, \Psi : \Psi$

$\Phi ::= \mathbf{0} \mid \varphi ?\, \Phi : \Phi \mid \Phi + \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Psi$

Thm: weighted automata = core wMSO

- Abstract semantics $\{\!|\, - \,|\!\}\colon \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$
- Concrete semantics $[\![-]\!] = \text{aggr} \circ \{\!|\, - \,|\!\}\colon \Sigma^\star \to S$

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

$$\Psi ::= s \mid \varphi ?\, \Psi : \Psi$$

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Thm: weighted automata = core wMSO

- Abstract semantics $\{\!| - |\!\}: \Sigma^+$
- Concrete $\to S$

Easy constructive proofs
preservation of the constants
no restriction on core wMSO
no hypotheses on weights

- [Gastin-Monmege, 14]

CONCLUDING REMARKS

- Similar approaches
  - M-expressions for multi-operator monoids [Fülöp-Stüber-Vogler'12]
  - Assignment logic [Perevoshchikov PhD'15]
- Other structures: ranked trees, unranked trees, …
  - multisets of weight-structures
  - core-wMSO: only the boolean fragment changes
- More operators in core-wMSO
  - any binary operation $\Diamond : S \times S \to S$ can be lifted to multisets of weight-structures and added to core-wMSO