

INFORMEL

Indo-French Formal Methods Lab



CHENNAI  
MATHEMATICAL  
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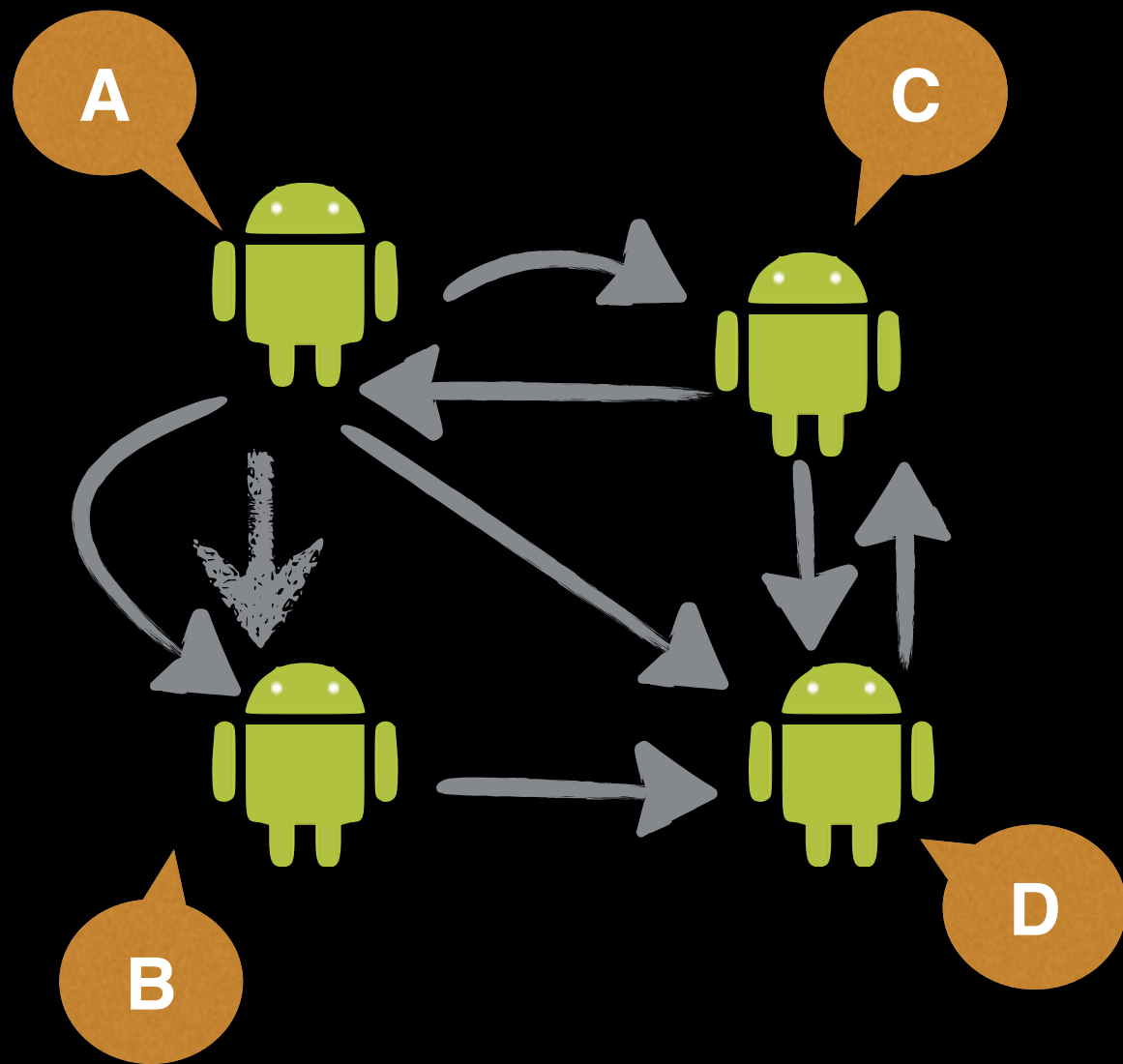
C. Aiswarya, Paul Gustin, K. Narayan Kumar

# Gossip

## Maintaining Latest Information Beyond Channel Bounds

ALFA, June 16th, 2015

# Distributed Systems

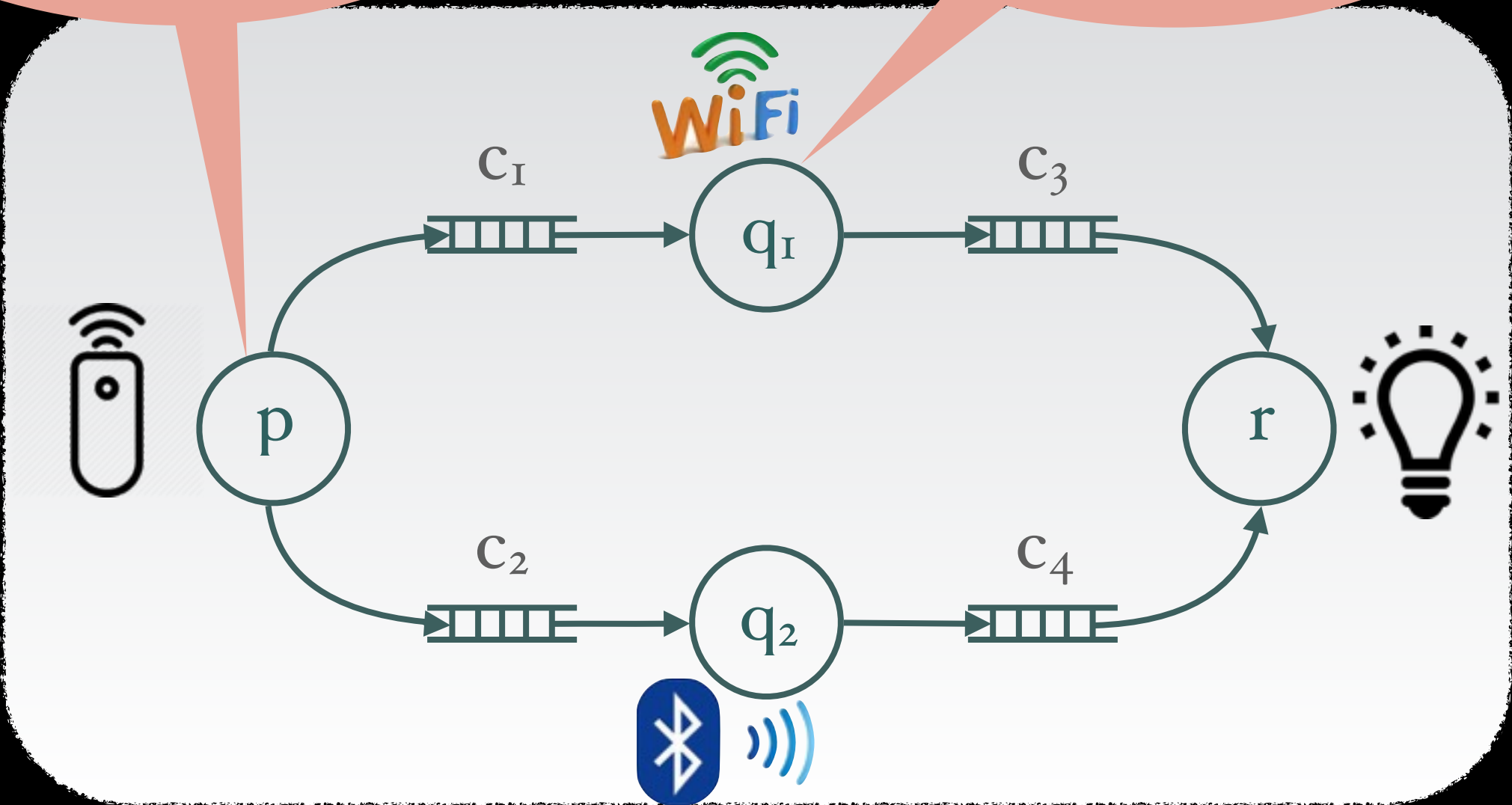
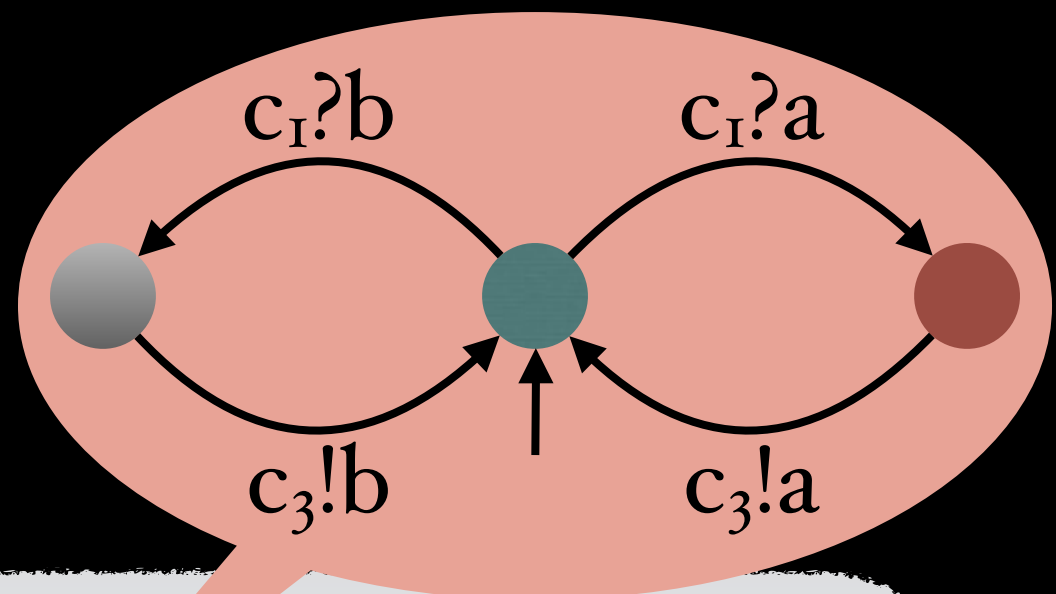
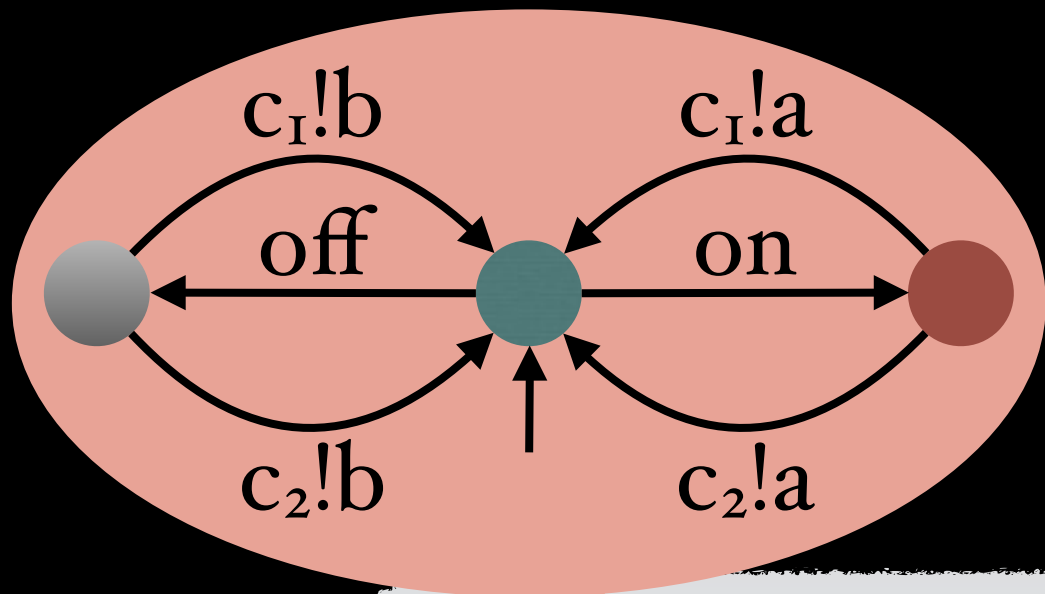


**Finite set of processes**

**Communicating via reliable  
FIFO message passing**

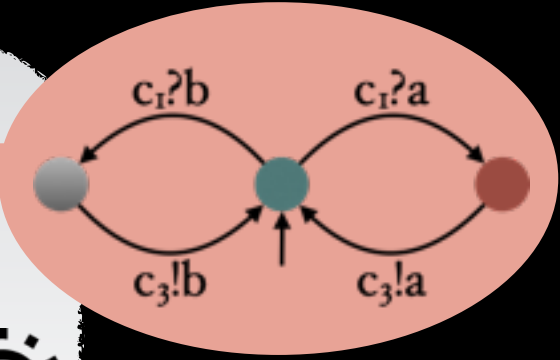
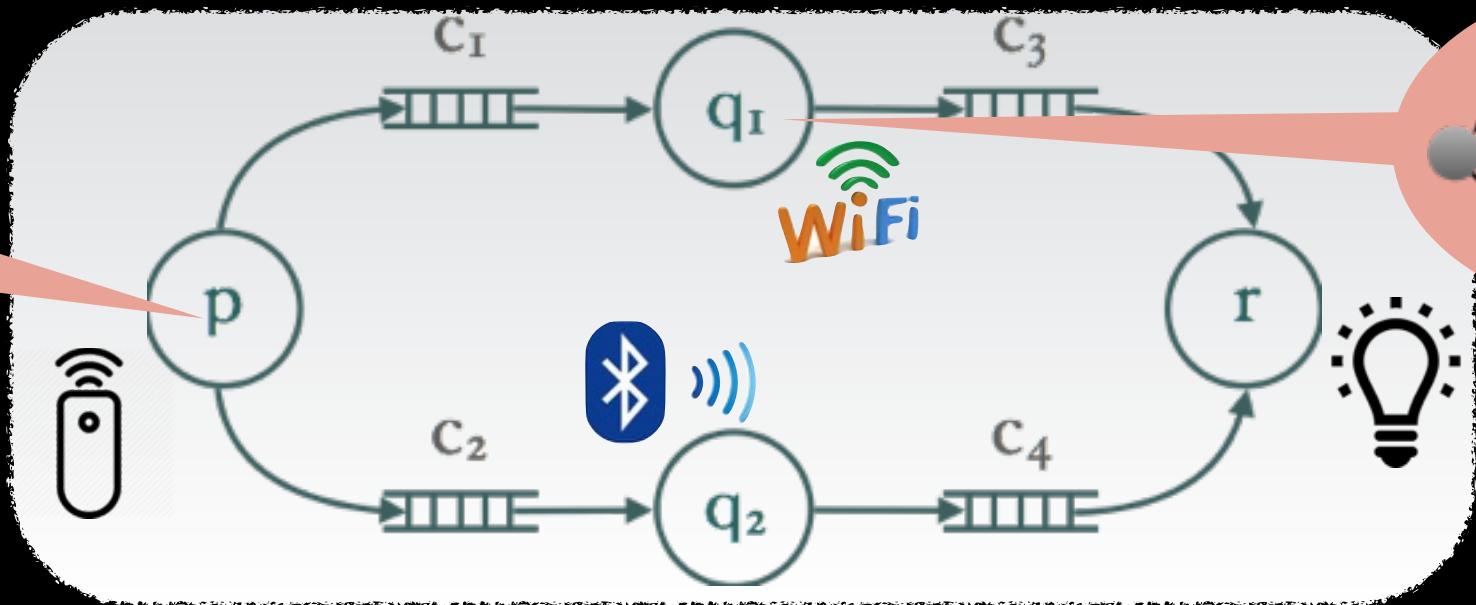
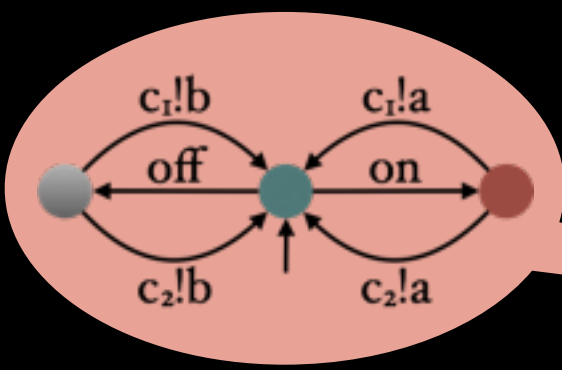
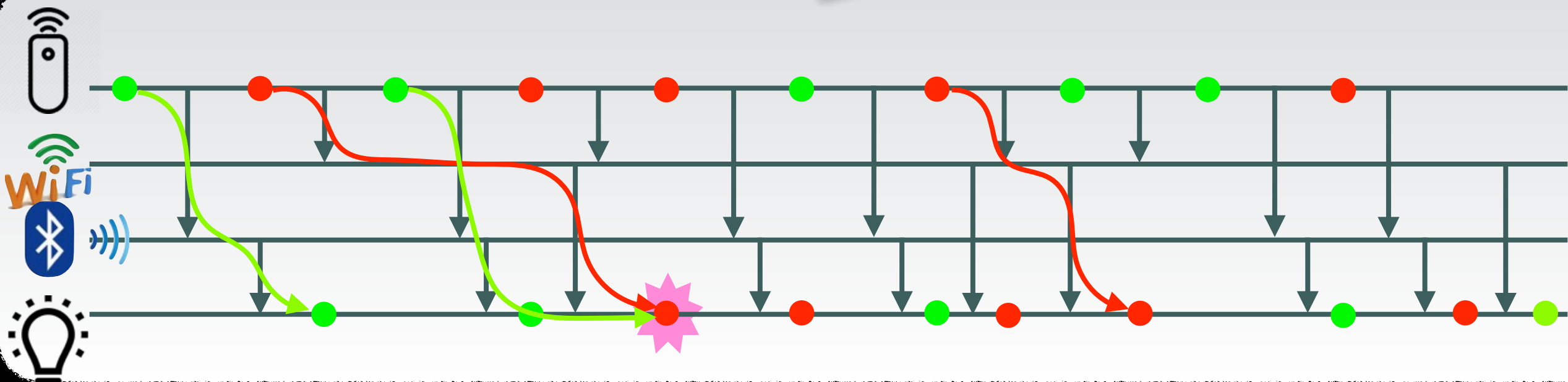
**multiple channels between  
processes**

# Remote control light



Obey the latest order

Message Sequence Charts  
ITU Standard



# Gossip

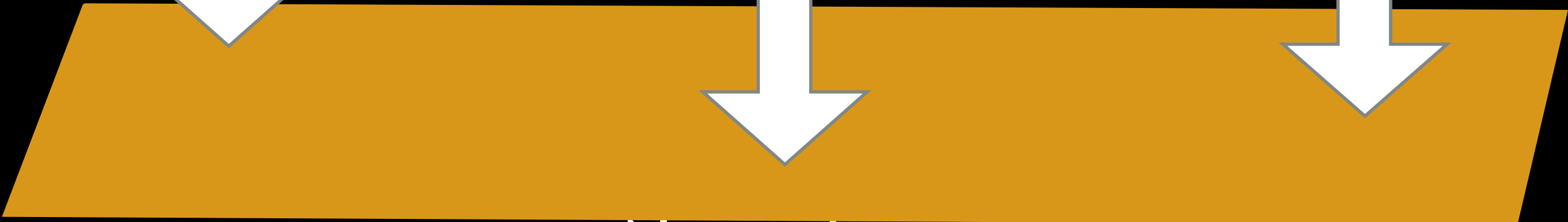
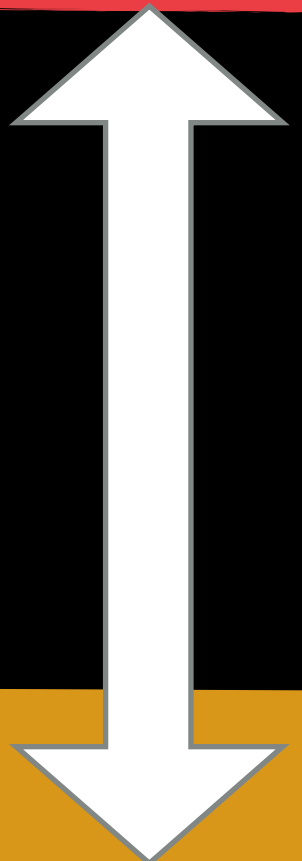
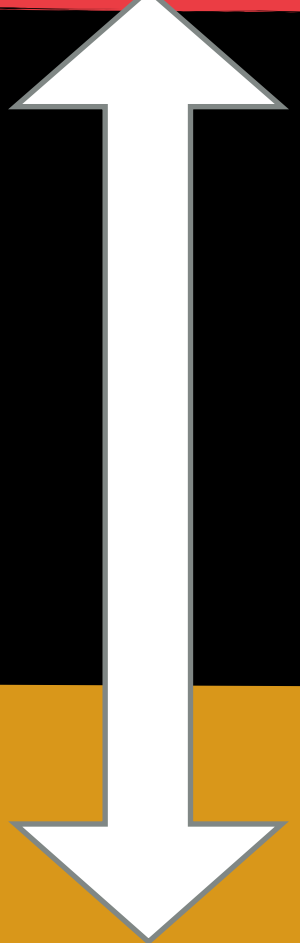
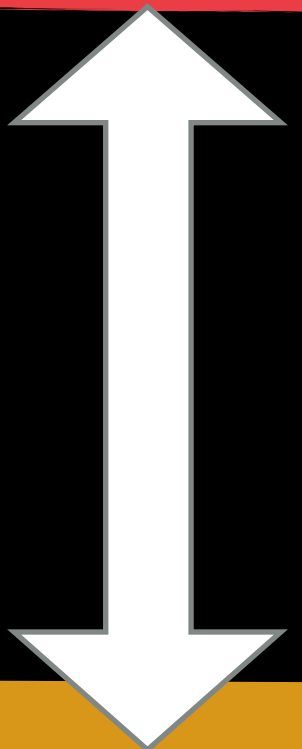
- Cooperate so that every process maintains latest information about every other process
- When receiving a message, a process needs to identify which is more recent:
  - the information it has,
  - the information transmitted by the sender



Process 1

Process 2

Process 3



Network



Process 1

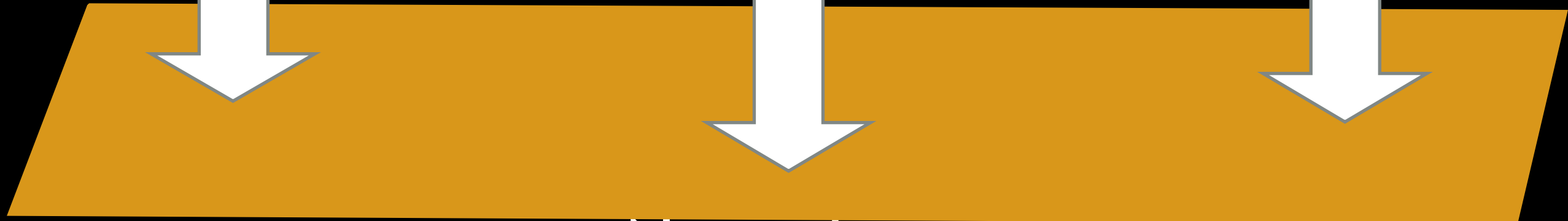
Process 2

Process 3

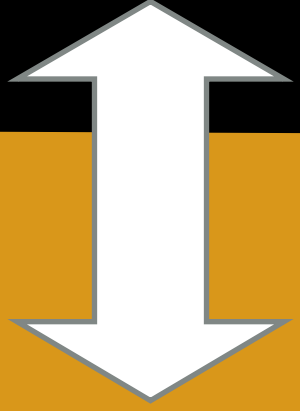
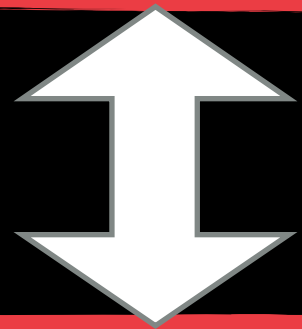
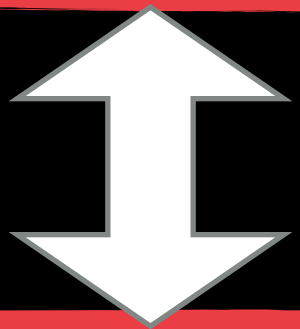
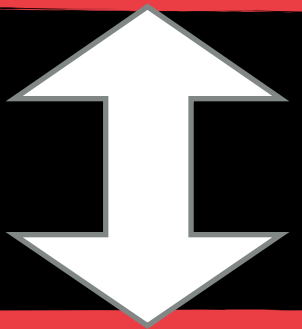
Gossip 1

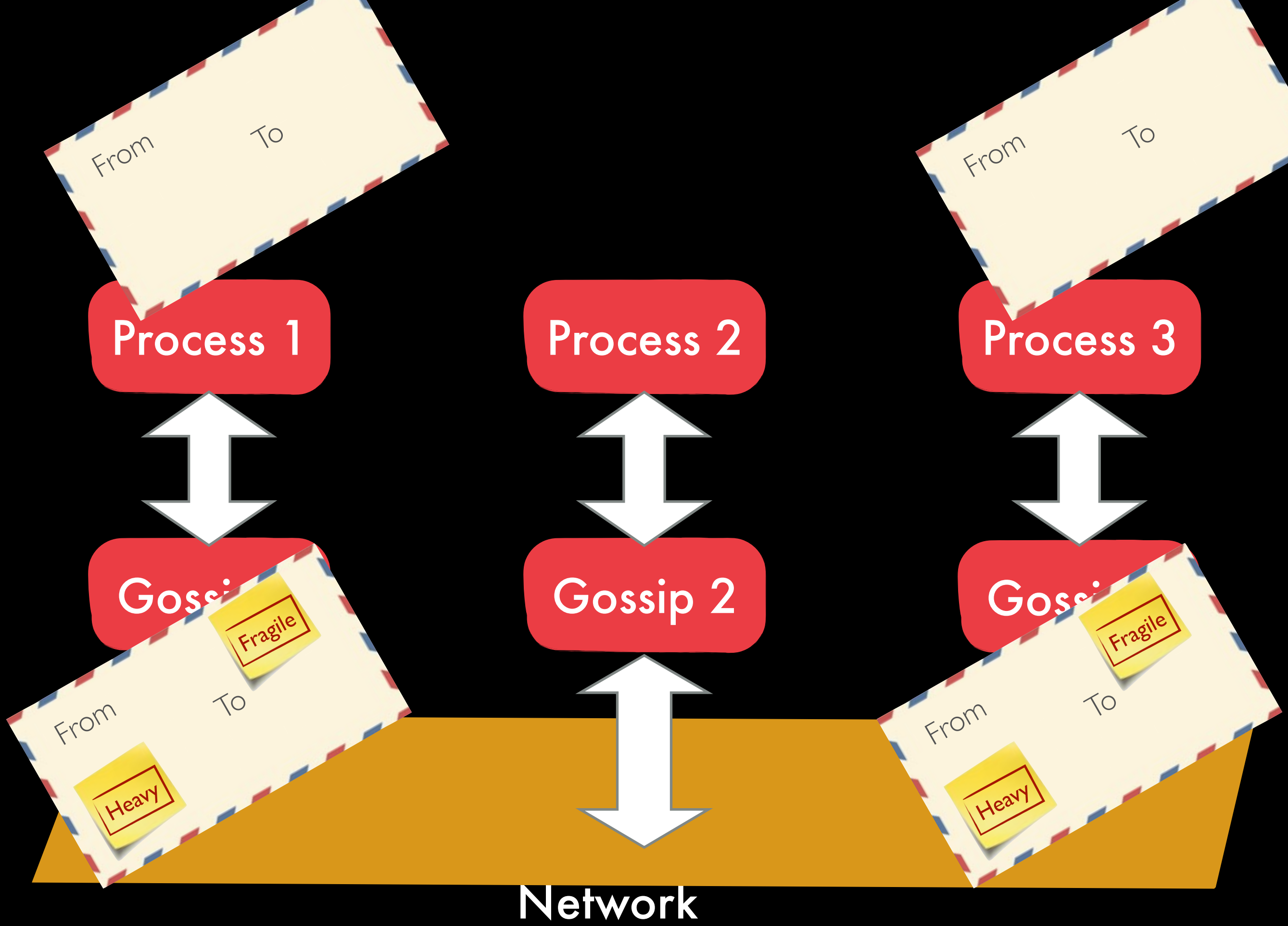
Gossip 2

Gossip 3



Network





From

To

Process 1

Process 2

Process 3

Gossip 1

Gossip 2

Gossip 3

From

To

Heavy

Fragile

From

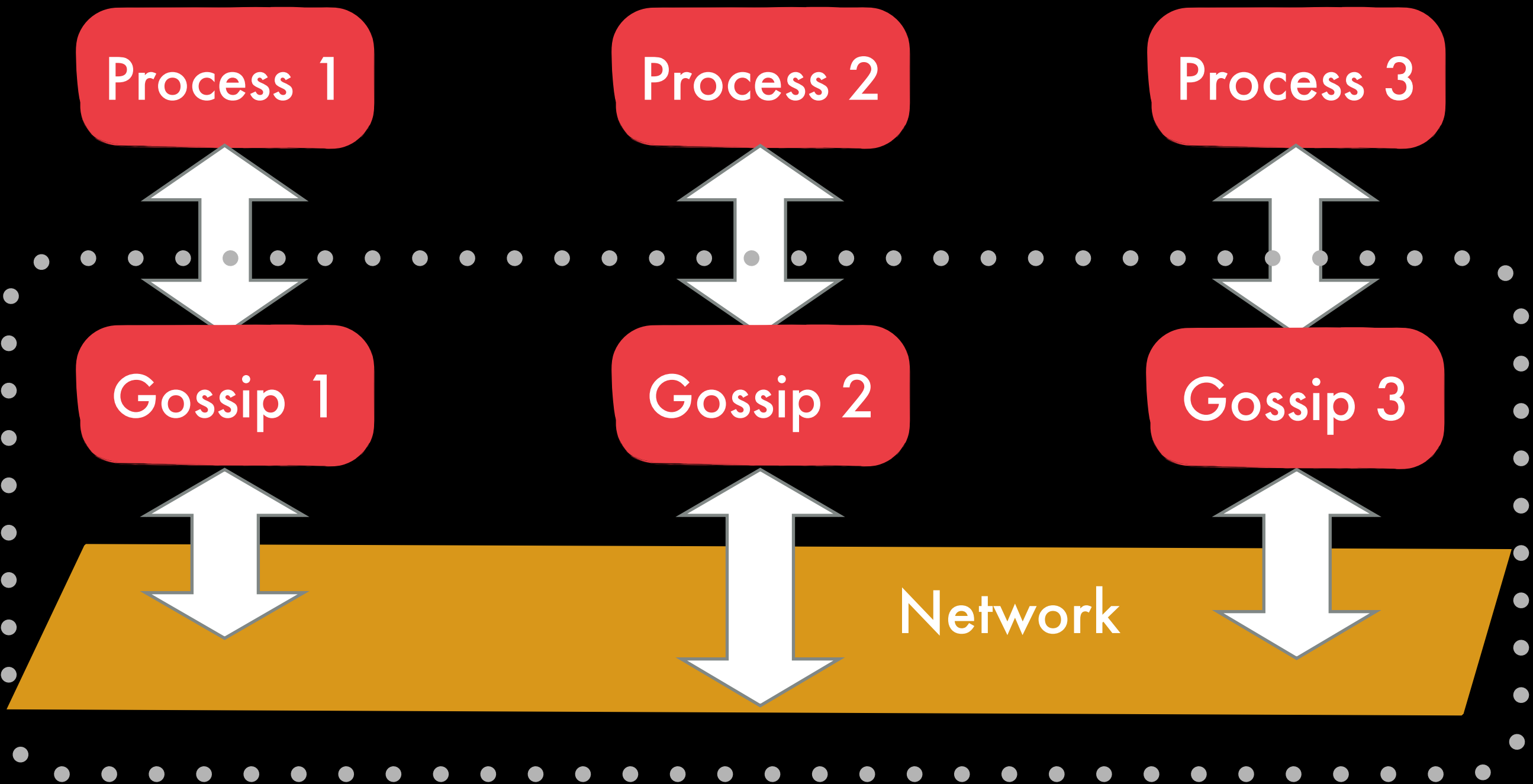
To

Heavy

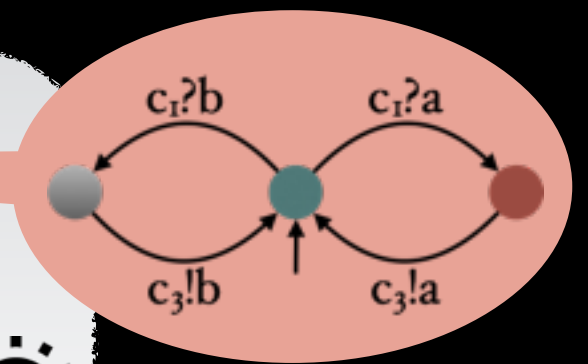
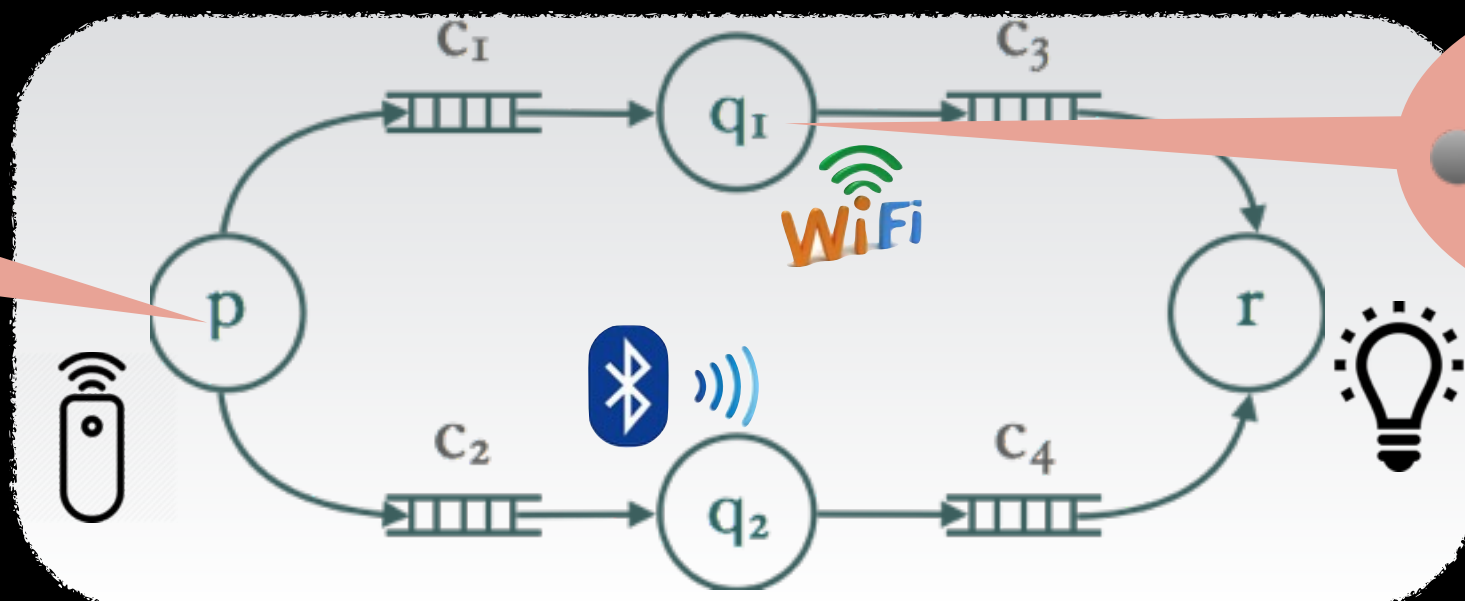
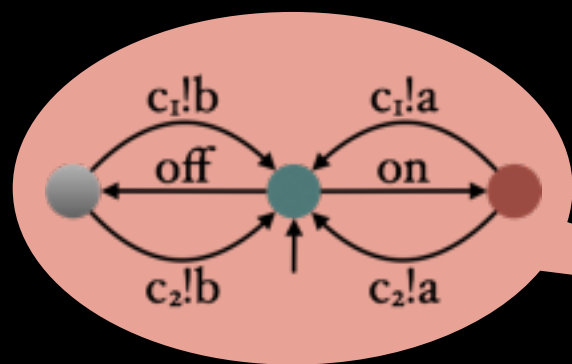
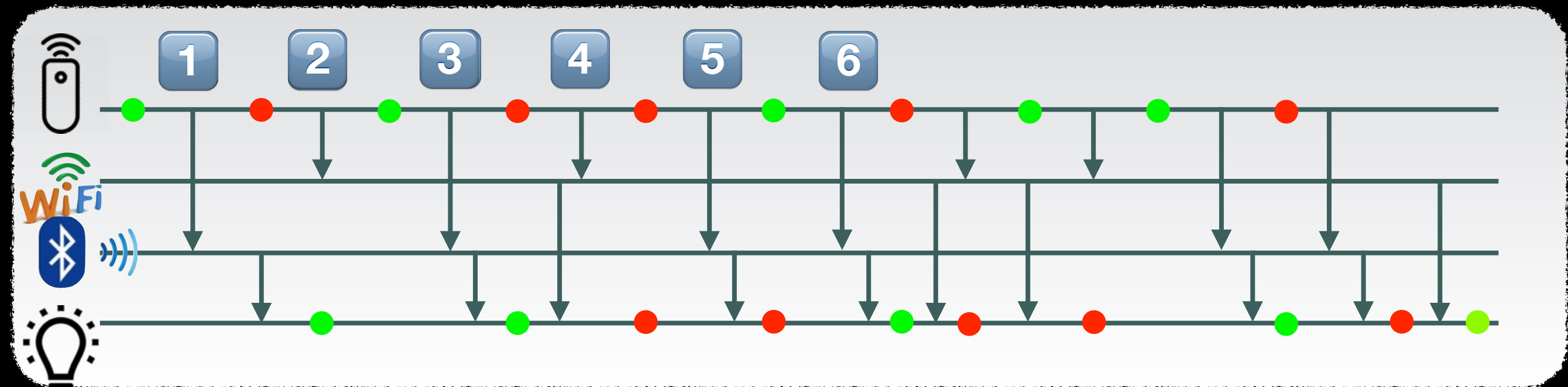
Fragile

Network

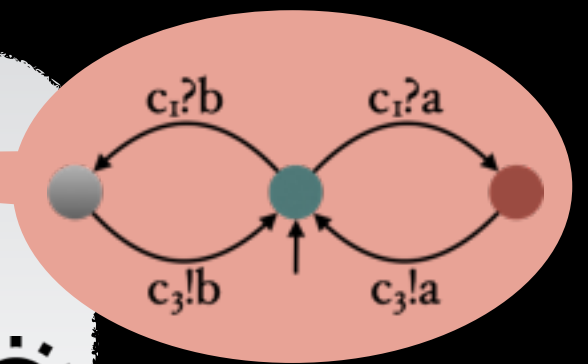
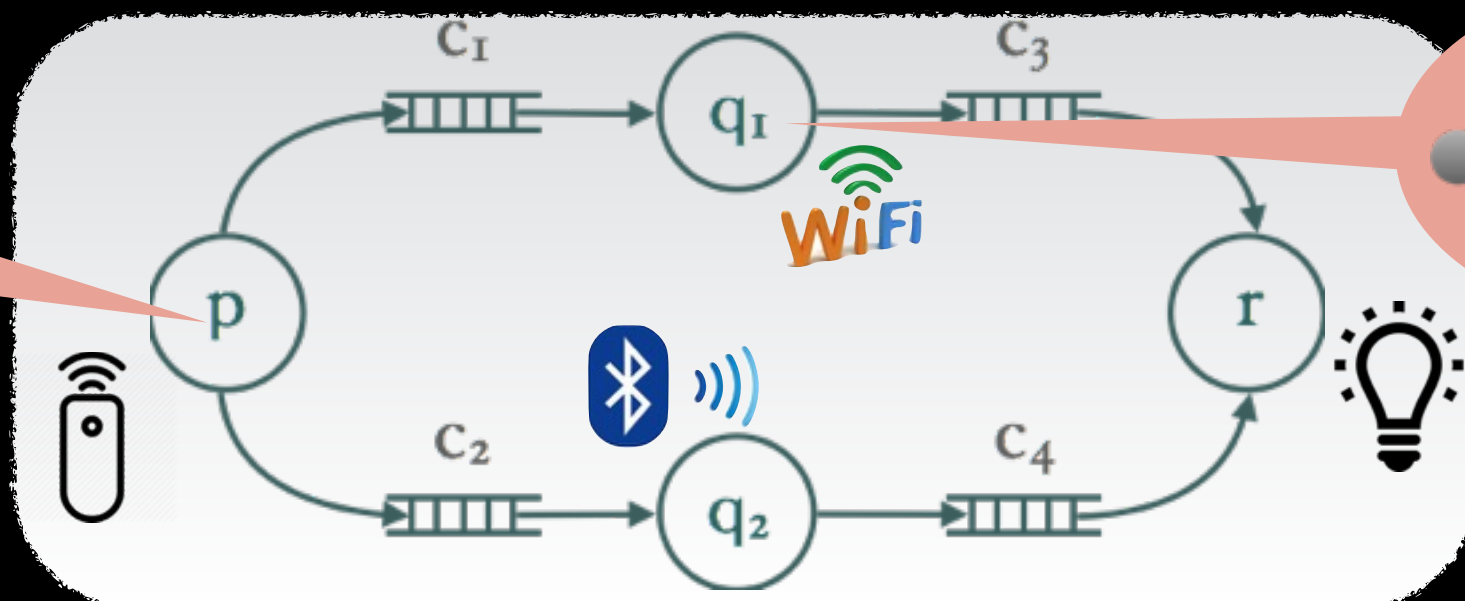
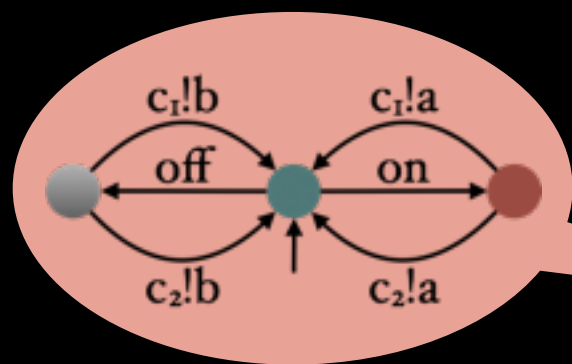
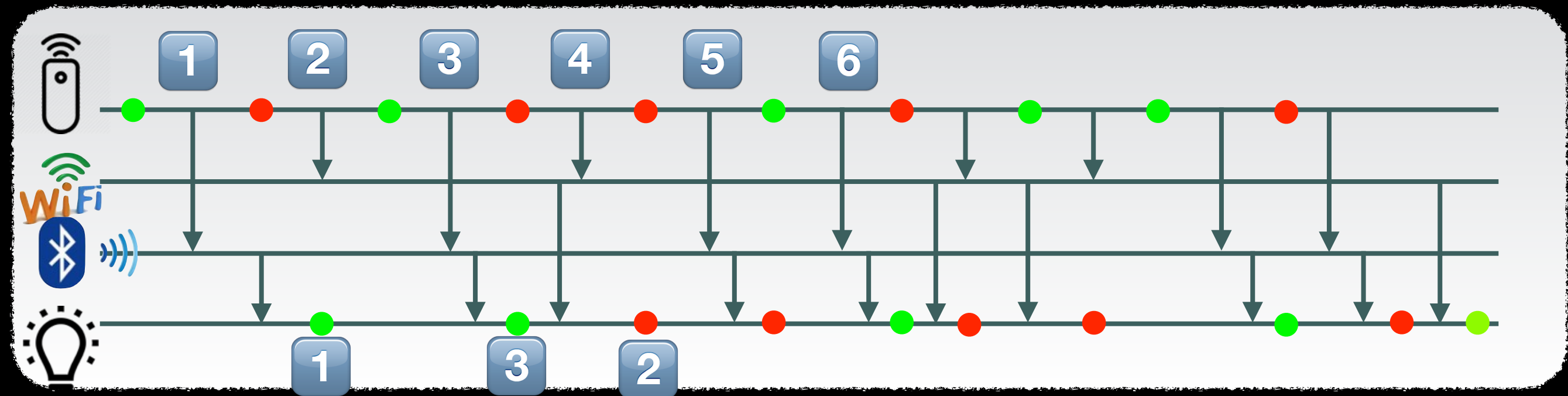




# How to maintain the latest information?



# How to maintain the latest information?



# Why ~~How~~ to maintain the latest information using only finite set of messages?

Finite  
communication  
complexity

Formal Methods  
Model checking

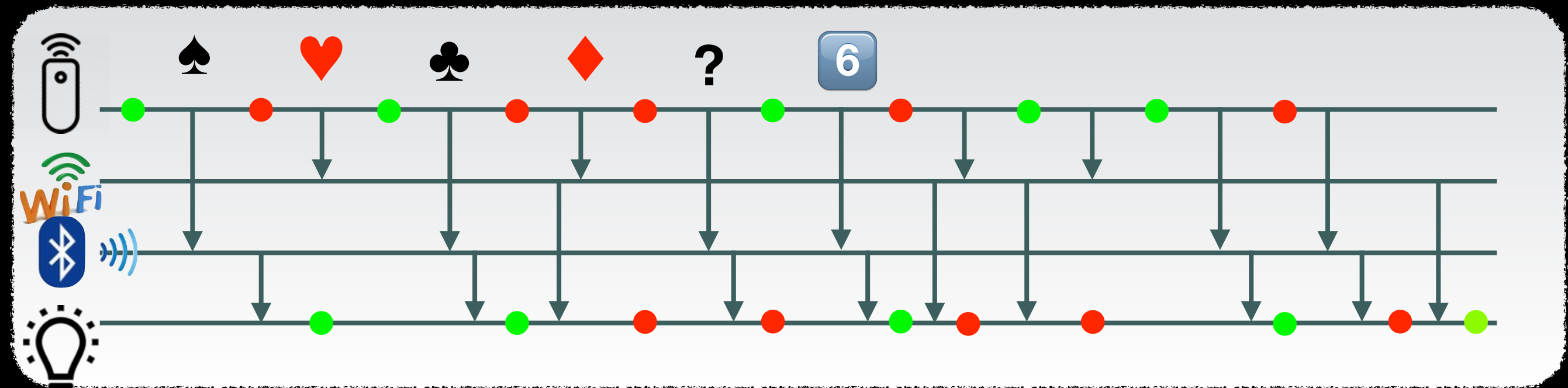
Distributed  
Synthesis

Global Snapshots  
Causal Ordering

Bounded  
implementations of  
replicated data-types

Local testing

# How to maintain the latest information using only finite set of messages?



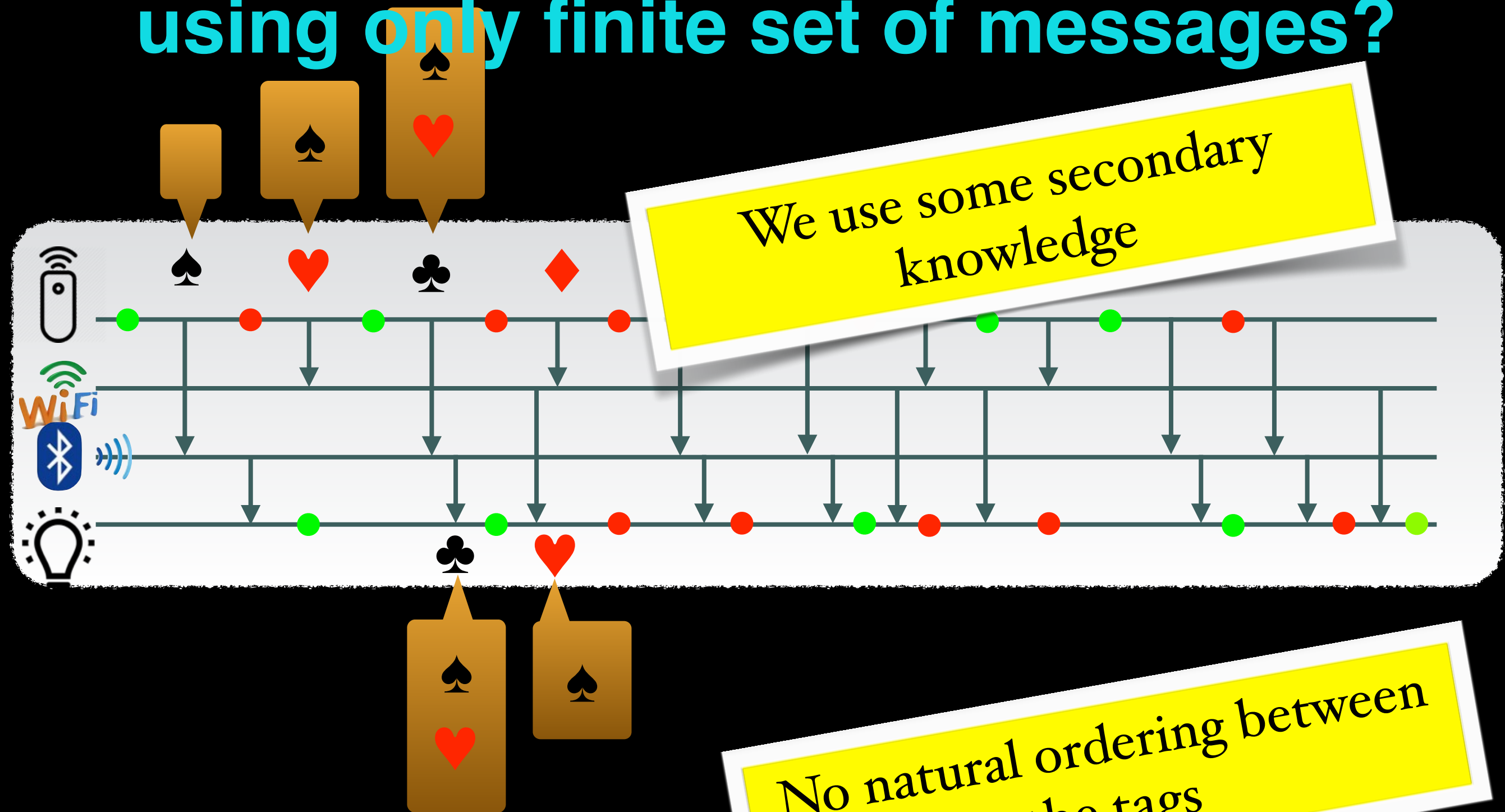
need to reuse tags

No natural ordering between  
the tags

# How to maintain the latest information using only finite set of messages?

We use some secondary knowledge

No natural ordering between the tags



# How to maintain the latest information **using only finite set of messages?**

Is it even possible?

At least in some cases?

Synchronous communication  
[Zielonka87]

Bounded channels  
[Mukund et al.03]

Beyond Bounded channels?

# Challenges

## How to maintain the latest information using only finite set of messages?

Lets analyse for k-Bounded channels

When is a color not needed any more?

requires k colors

I can reuse a color when I know that the tagged message has been received

necessary, but not sufficient

and I know that everyone knows that the tagged message has been received

Secondary knowledge

requires  $k^2$  colors

colors are not freed in the order they were used

showing a bound, and using a round-robin does not work

k-Bounded channels permit finite time-stamping

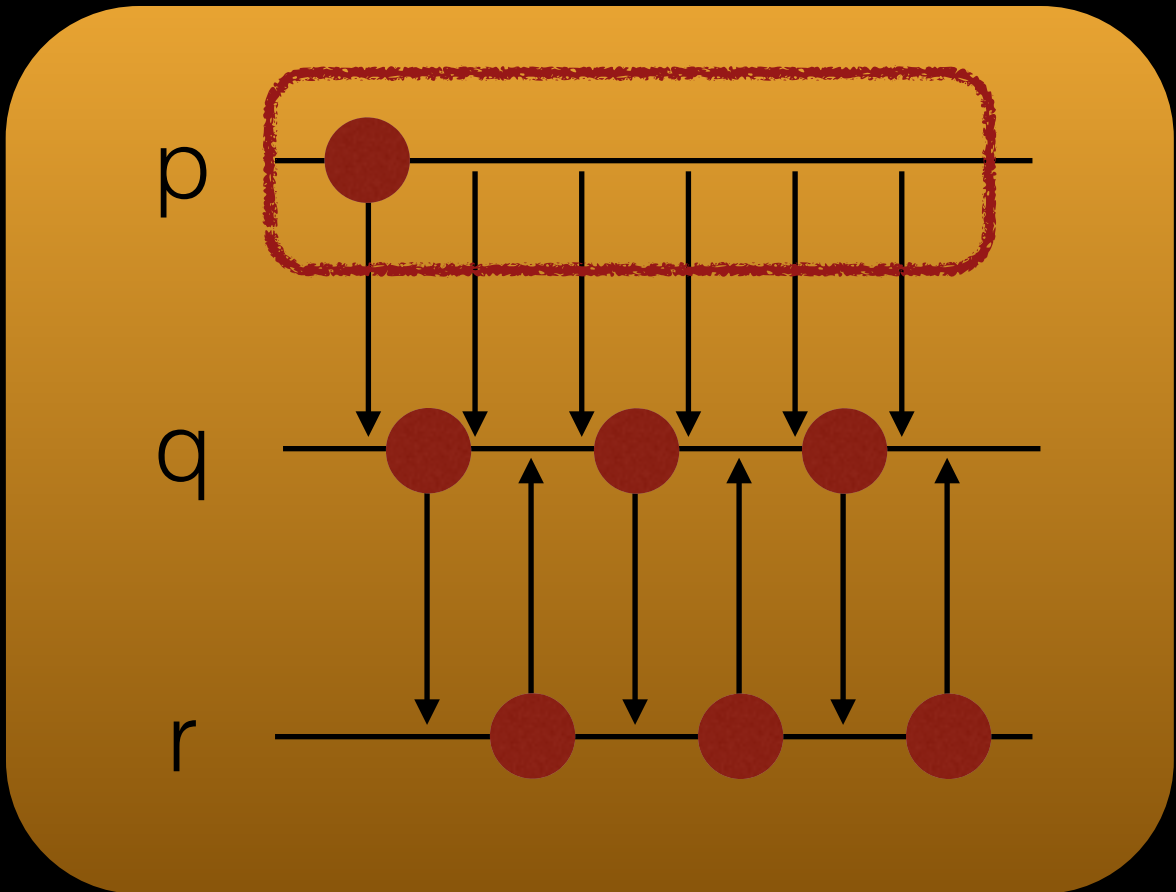


**k-Bounded channels**

# How to maintain the latest information using only finite set of messages?

k-Bounded channels permit finite time-stamping

Are channel bounds necessary for finite time-stamping?



Equivalent writes

Not simply stuttering

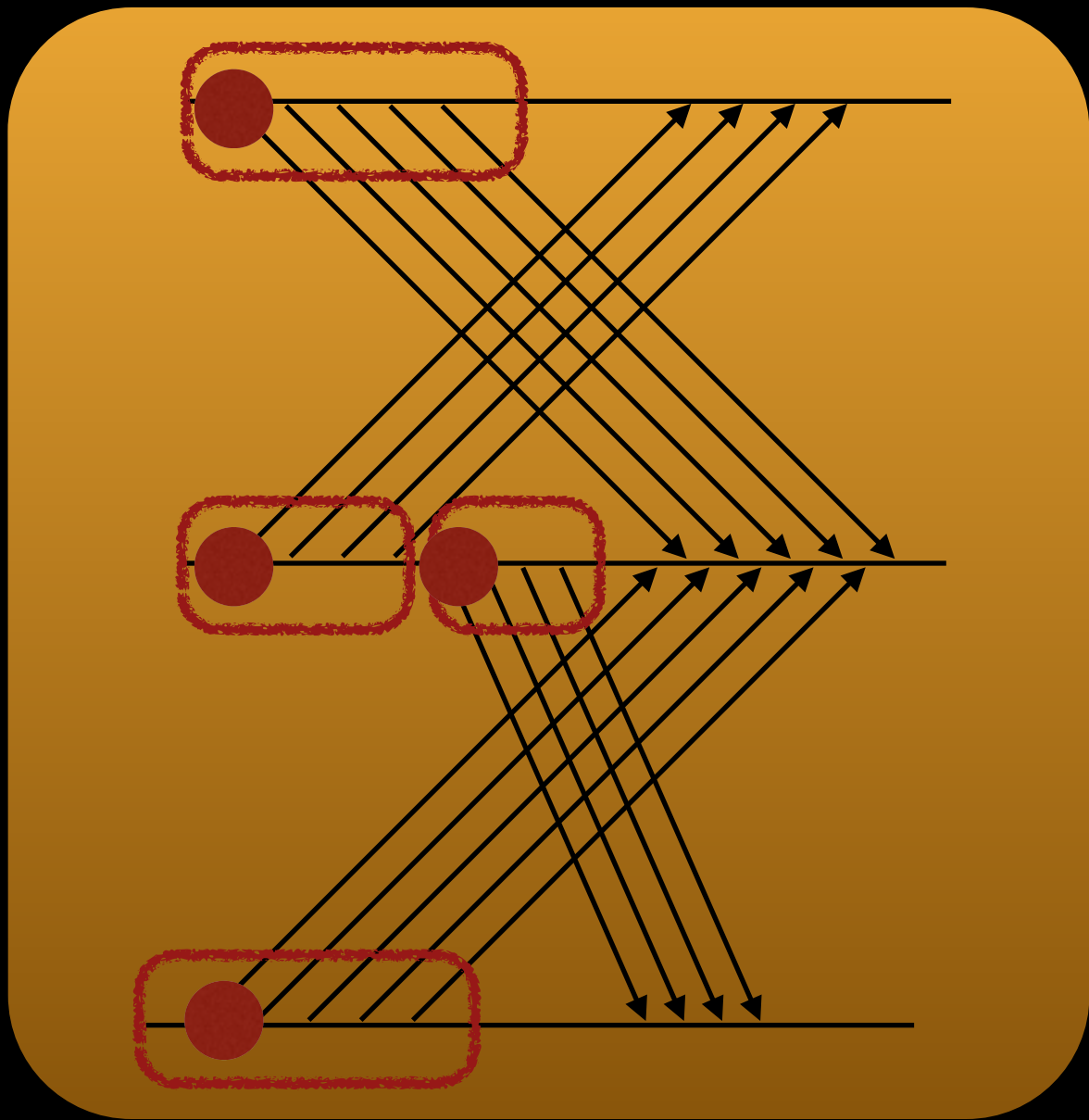
Important writes

Are existential channel bounds necessary?

Important writes

# How to maintain the latest information using only finite set of messages?

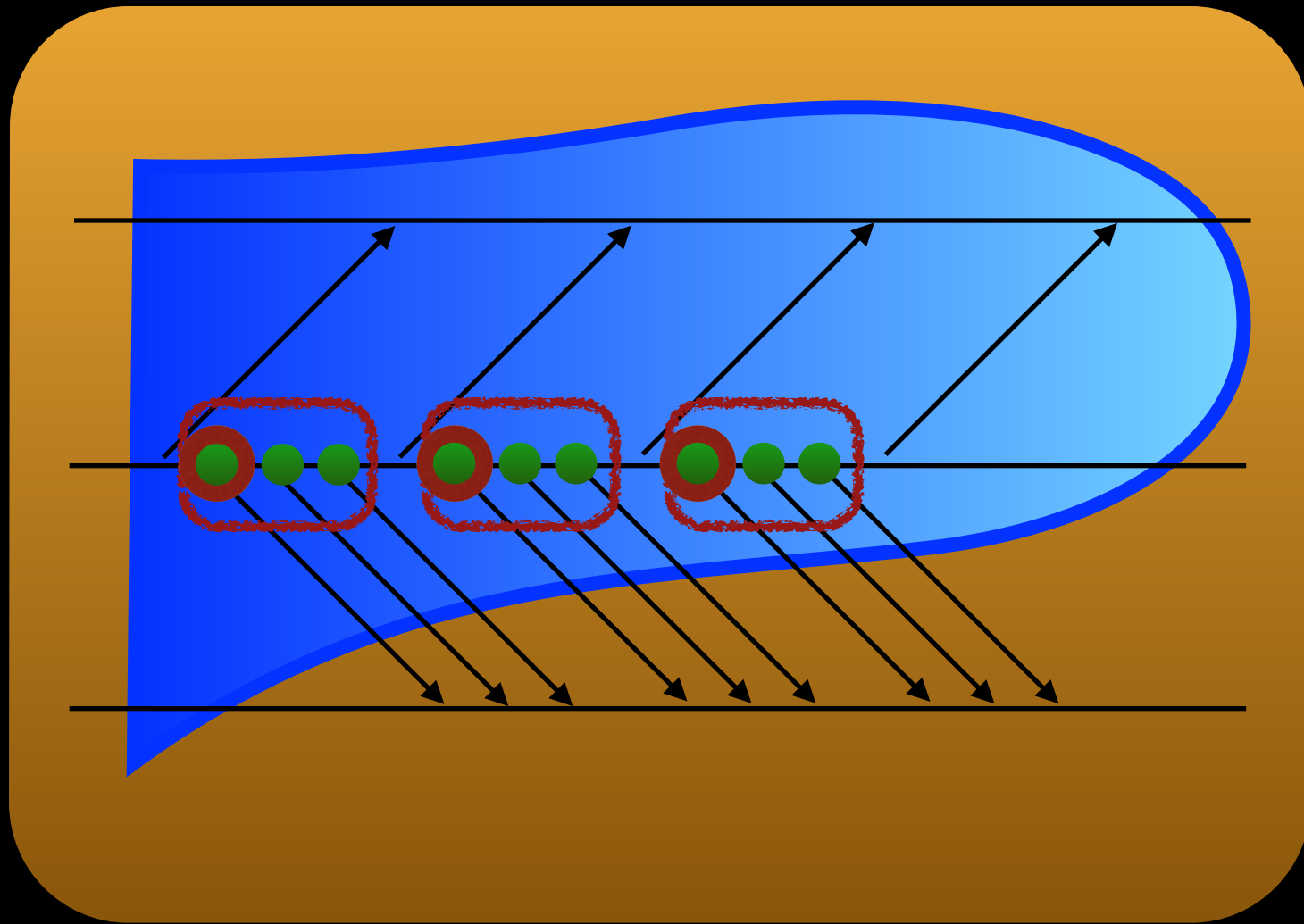
Are existential channel bounds necessary?



Equivalent writes

Important writes

# We need some bound: Primary information

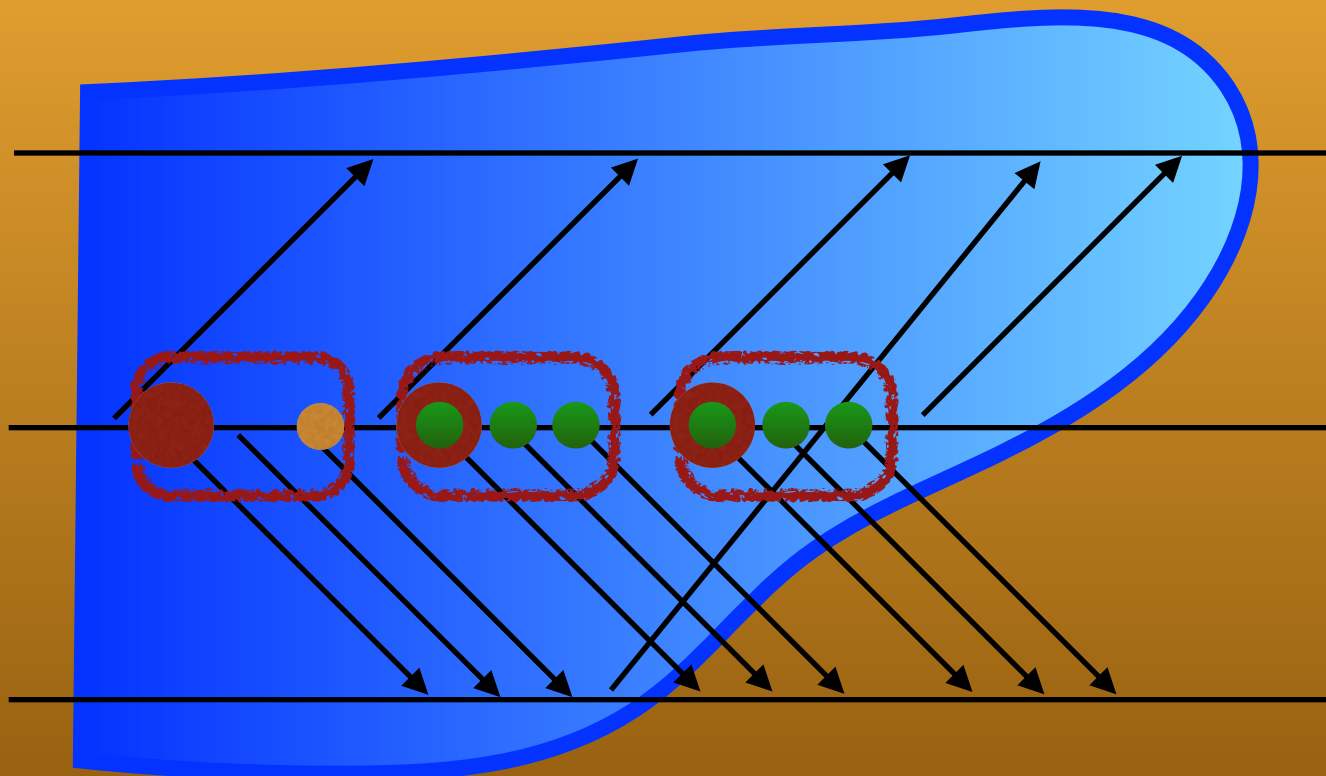


Pending writes

Equivalent writes

Primary writes

# We need some bound: Primary information



Pending writes

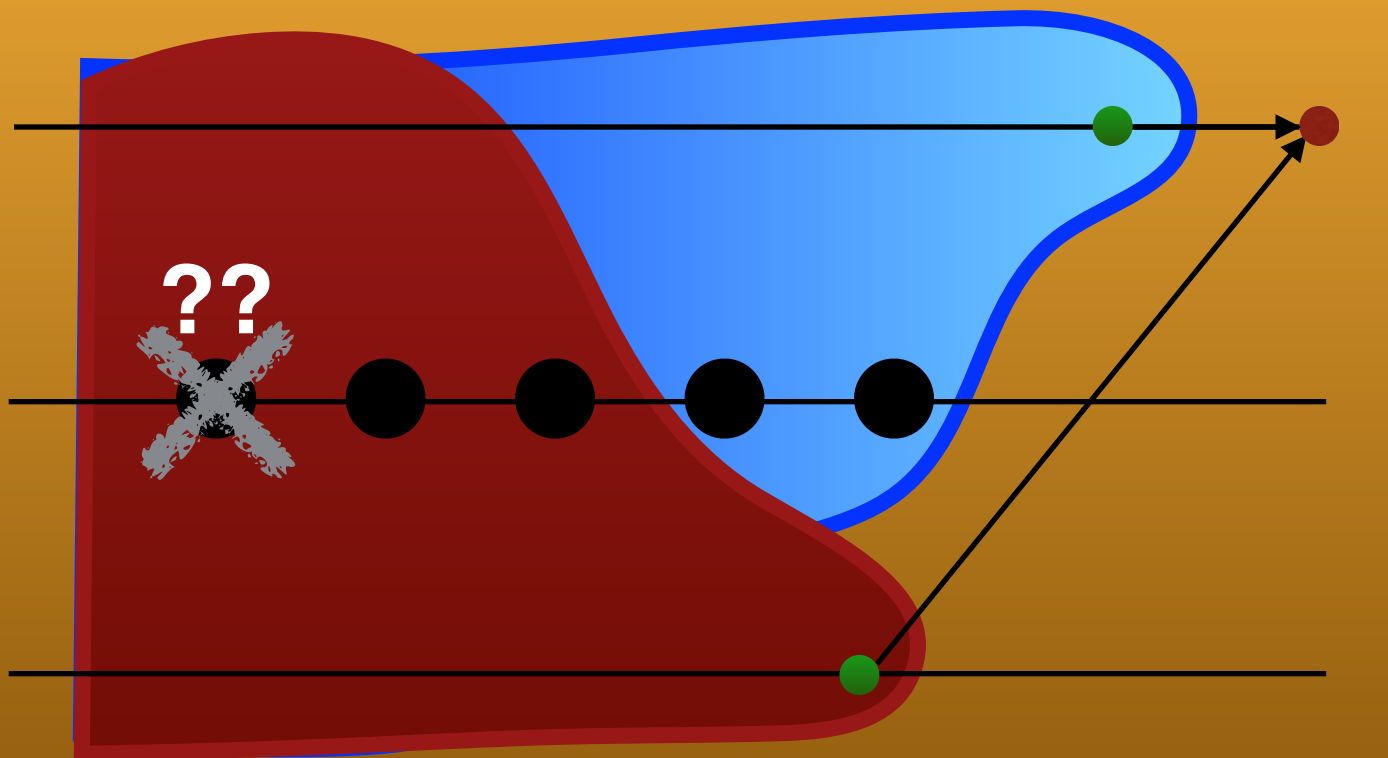
Latest received writes

Equivalent writes

Primary writes

**We solve the gossip problem for primary bounded**

# How do we maintain the primary?

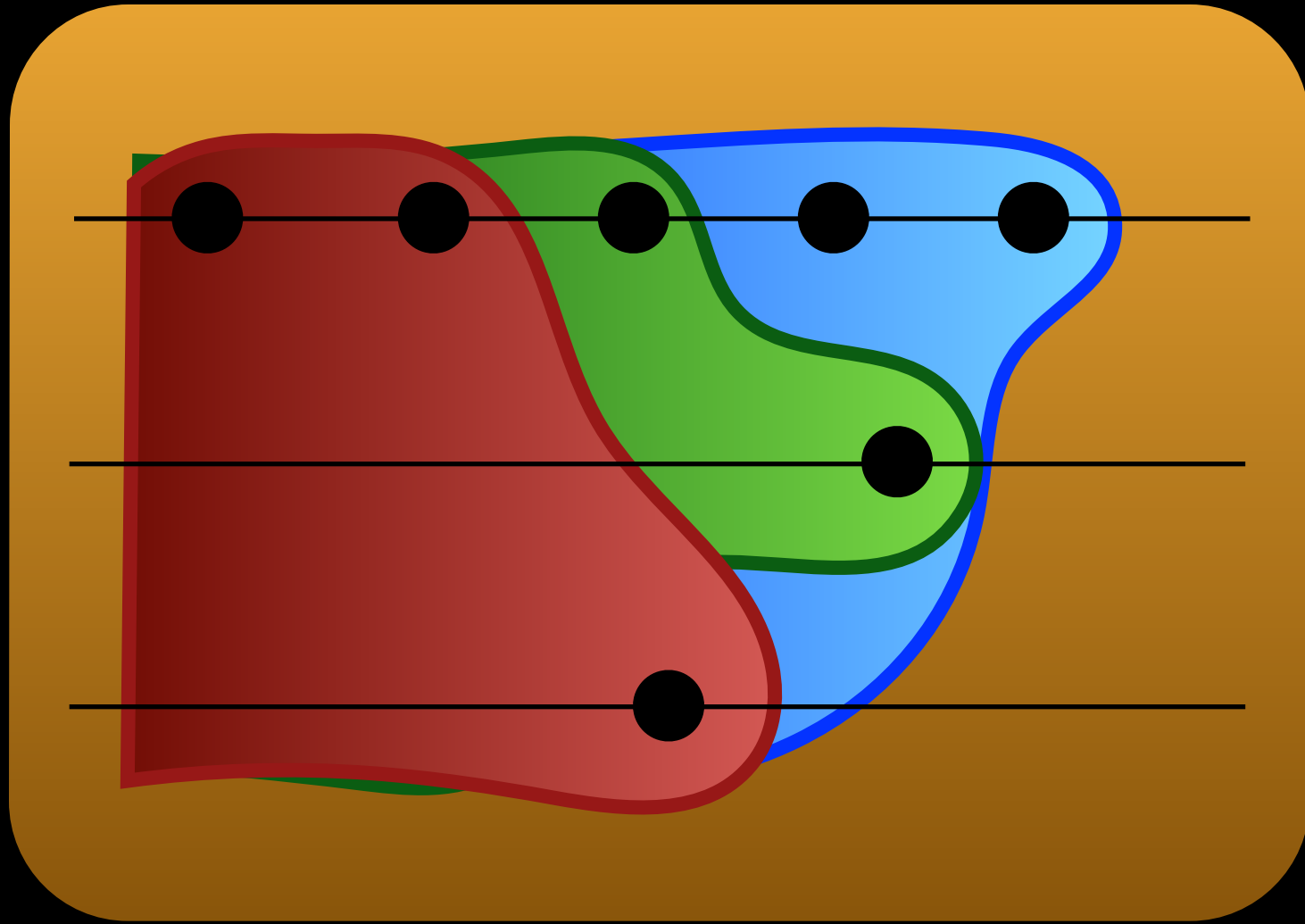


Keeping primary alone is not enough

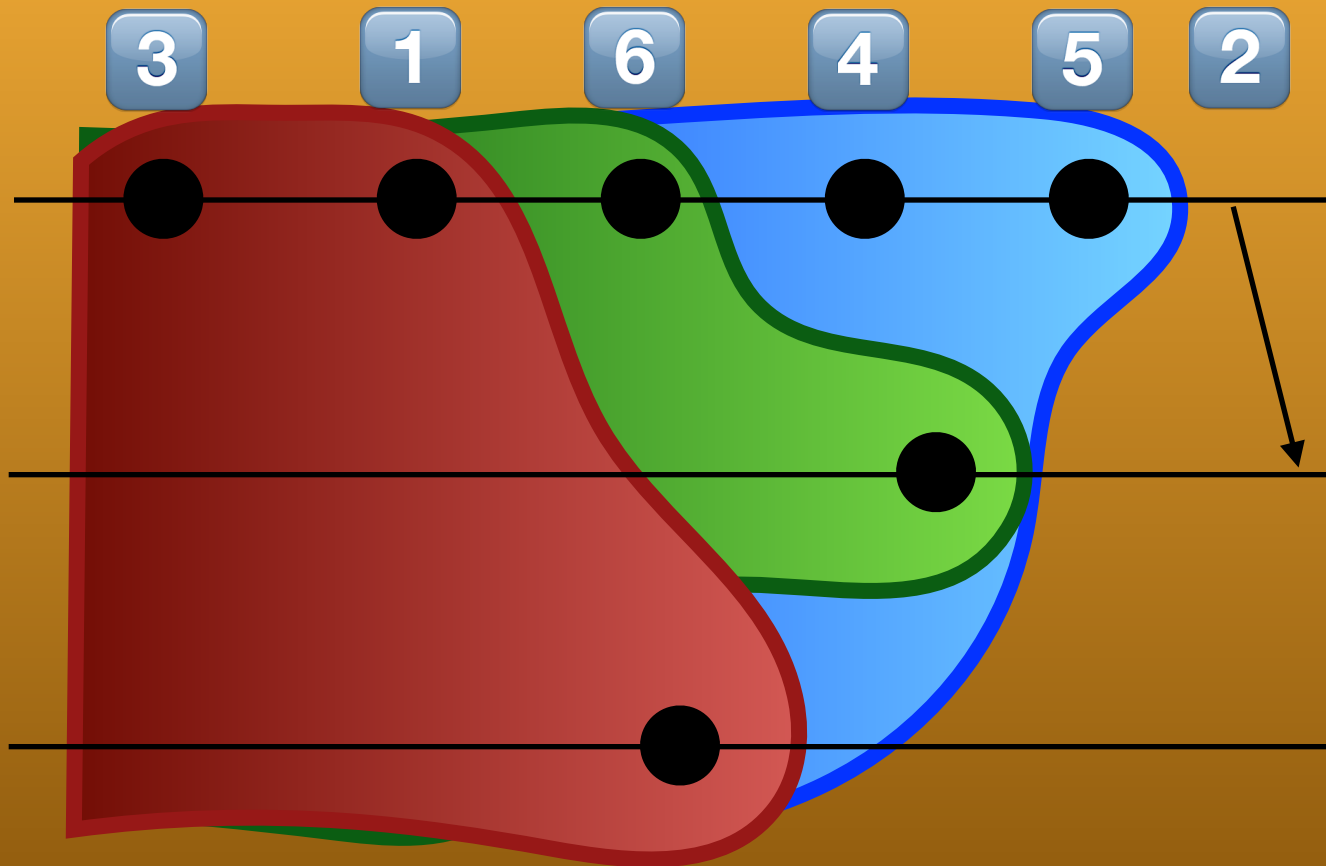
Need secondary knowledge

What is secondary knowledge?

Secondary = Primary of Primary



# Secondary = Primary of Primary



When can I reuse a color?

When it is not in the secondary

$\sim k^2$  colors  
 $\sim$  size of secondary

Can we maintain secondary?

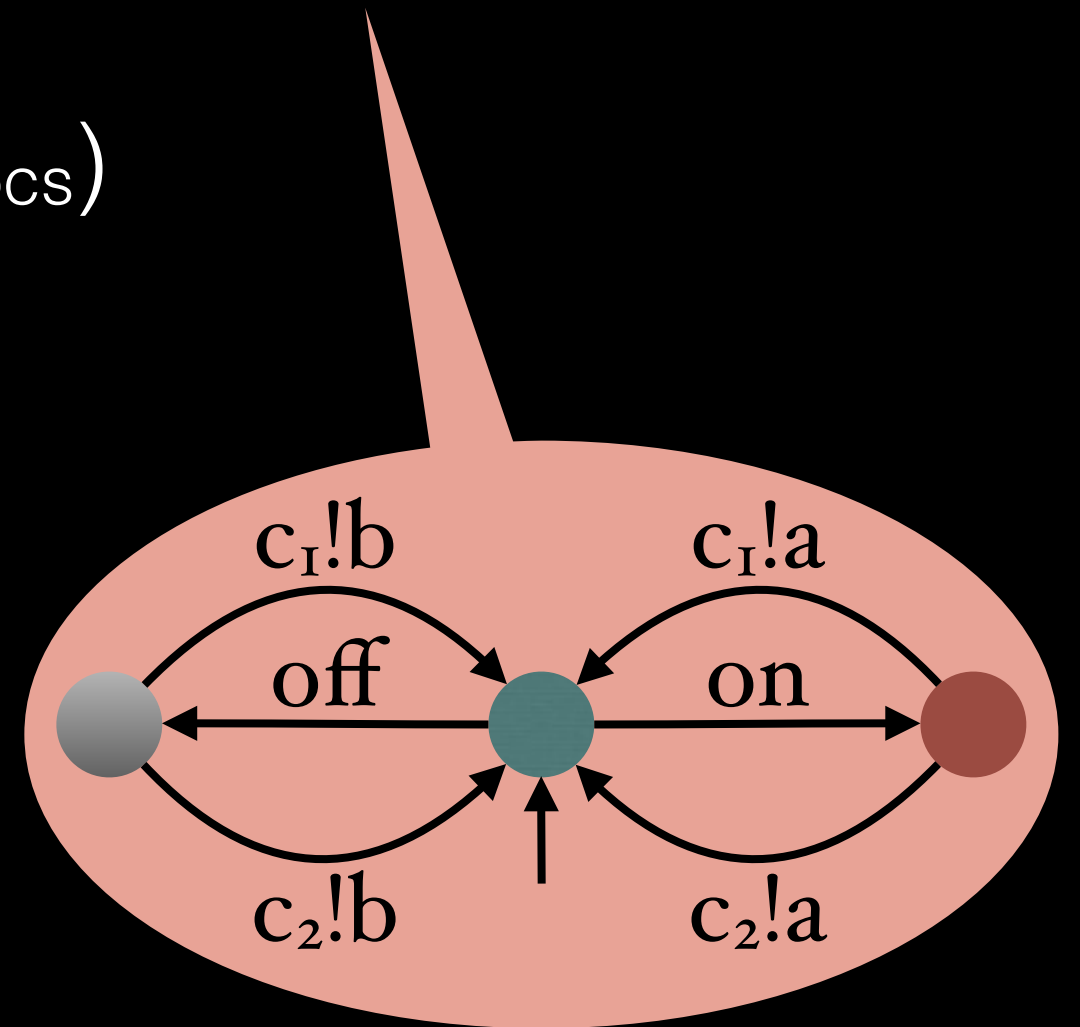
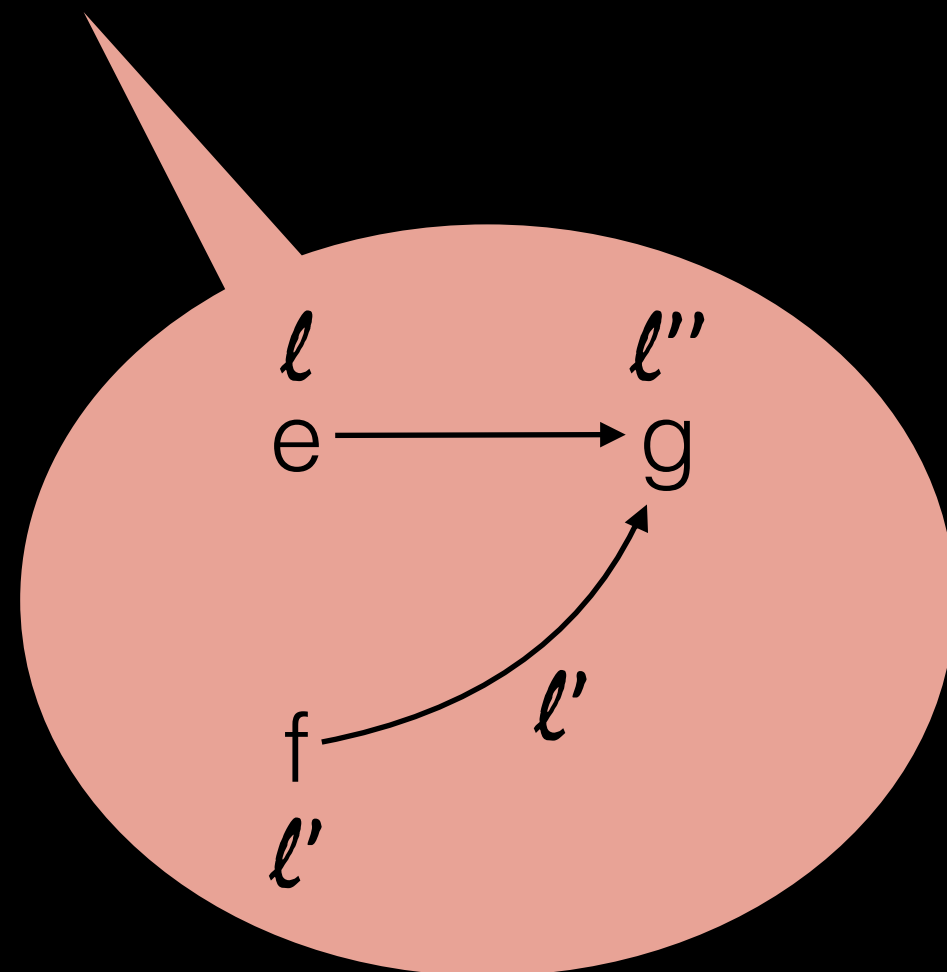
YES, WE CAN!

# Gossip: more precisely

- Message passing automaton (MPA or CFM)

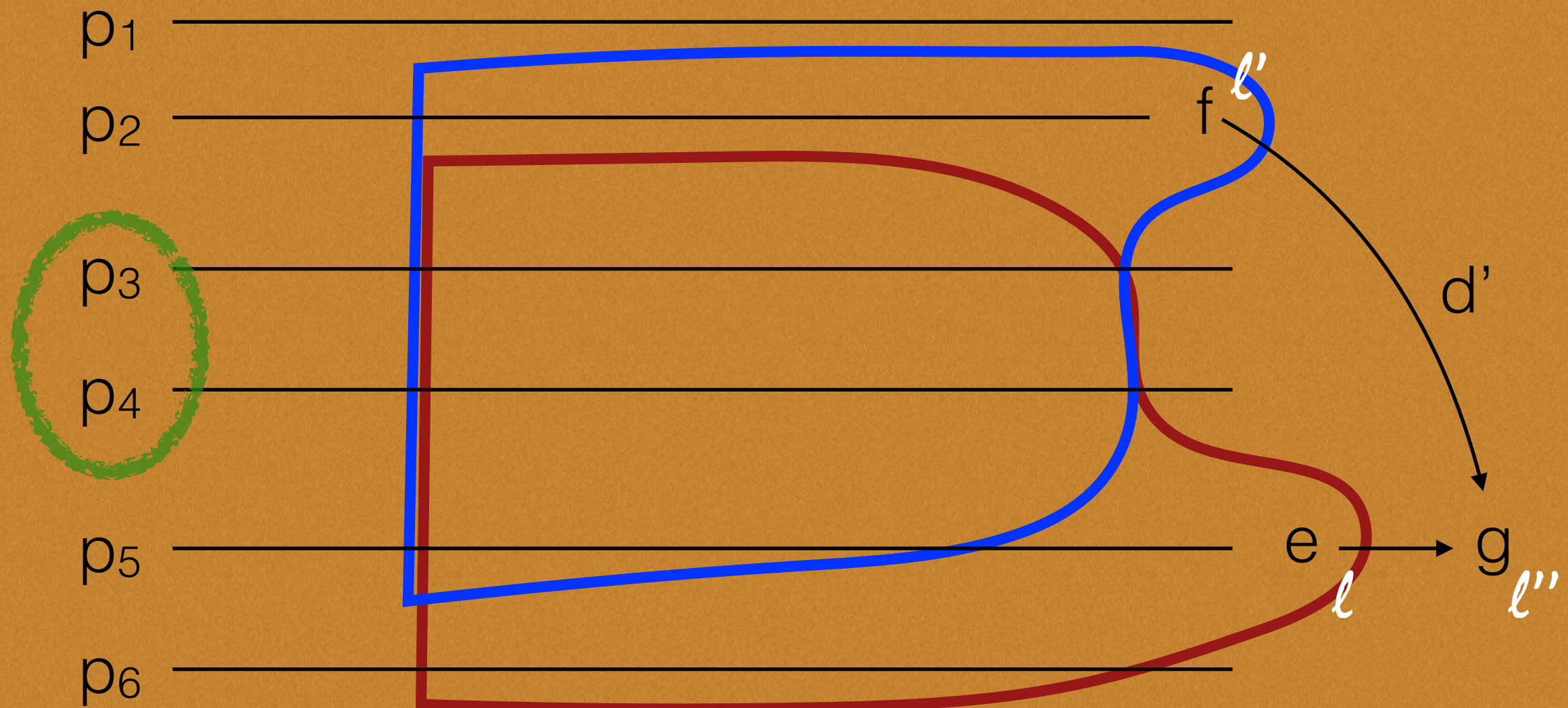
Gossip = (Locs, (Trans<sub>p</sub>)<sub>p ∈ Procs</sub>)

Run:  $\rho : \text{Events} \rightarrow \text{Locs}$





# Known and Latest



Known: Locs  $\rightarrow 2^{\text{Procs}}$

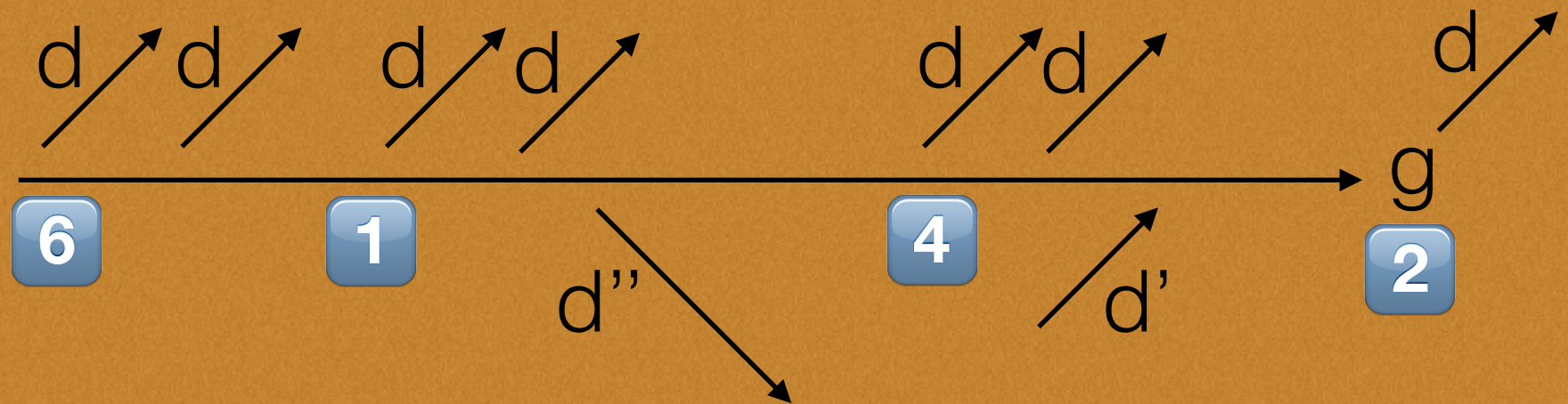
$\text{Known}(l'') = \{p_2, p_3, \dots, p_6\}$

Latest: Locs<sup>2</sup>  $\rightarrow 2^{\text{Procs}}$

$\text{Latest}(l, l') = \{p_3, p_5, p_6\}$

# Colors and time-stamps

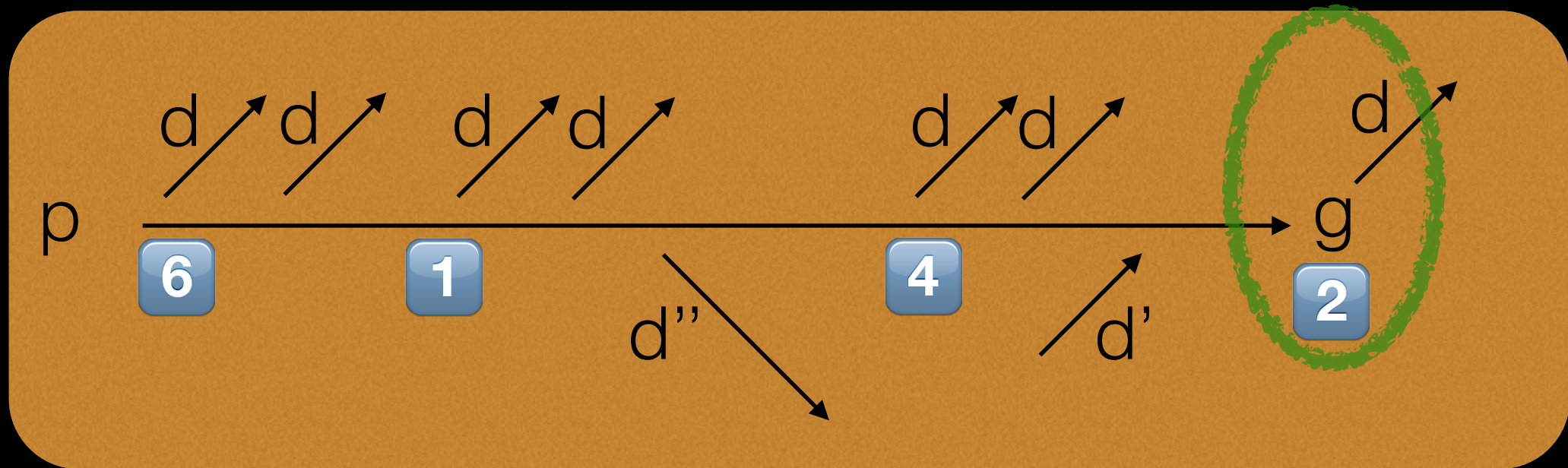
$$\chi(g) = \min(\mathbb{N} \setminus \chi(\text{Sec}(\downarrow g) \cap \text{Send}(d)))$$



$$h(g) = (d, \chi(g))$$

$$K^1(g) = \{h(e) \mid e \in \text{Prim}(\downarrow g)\}$$

# Locations of Gossip

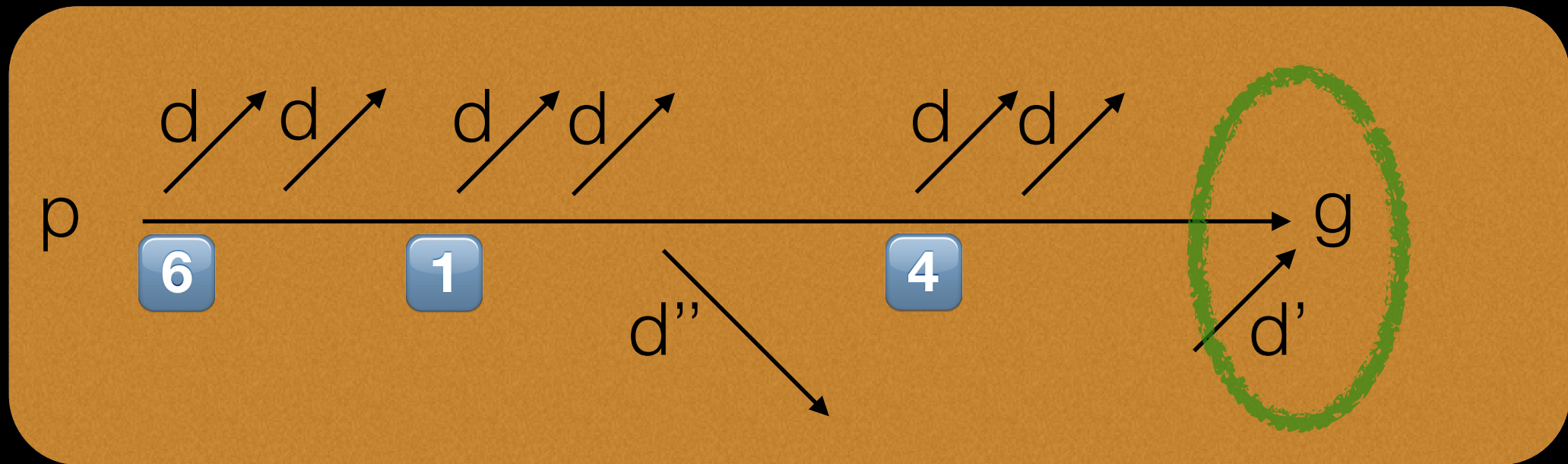


$$K^2(g) = (\text{pid}(g), d, c, K^1(g), (K^1(e))_{e \in \text{Prim}(g)})$$

$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$



# Locations of Gossip

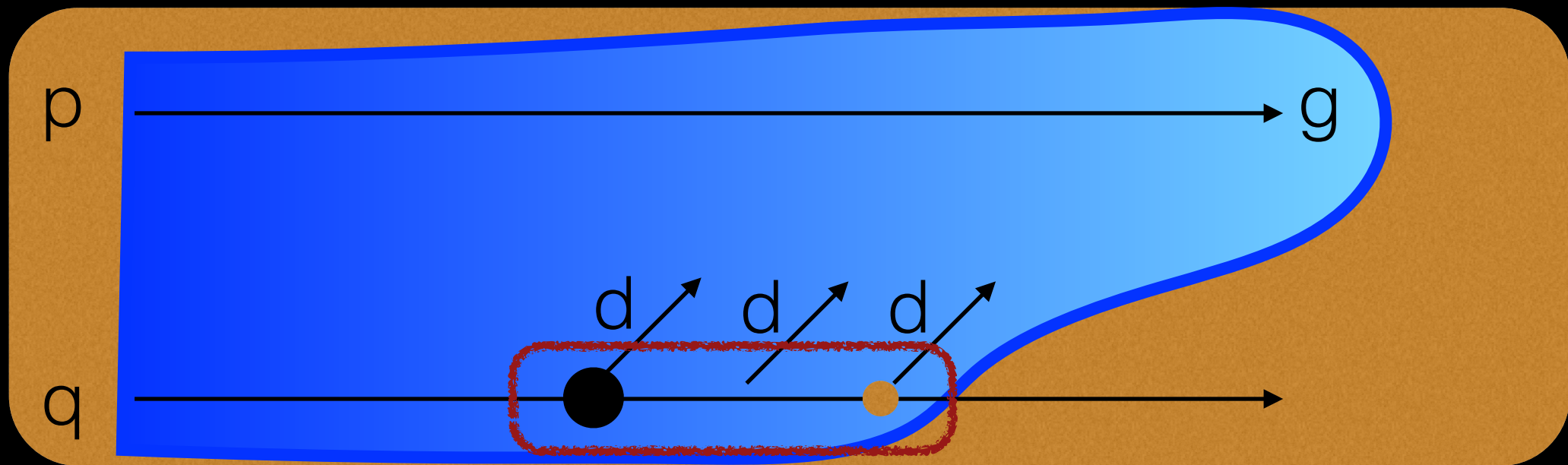


$\perp, \perp$

$$K^2(g) = (\text{pid}(g), d, c, K^1(g), (K^1(e))_{e \in K^1(g)})$$

$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

# Known



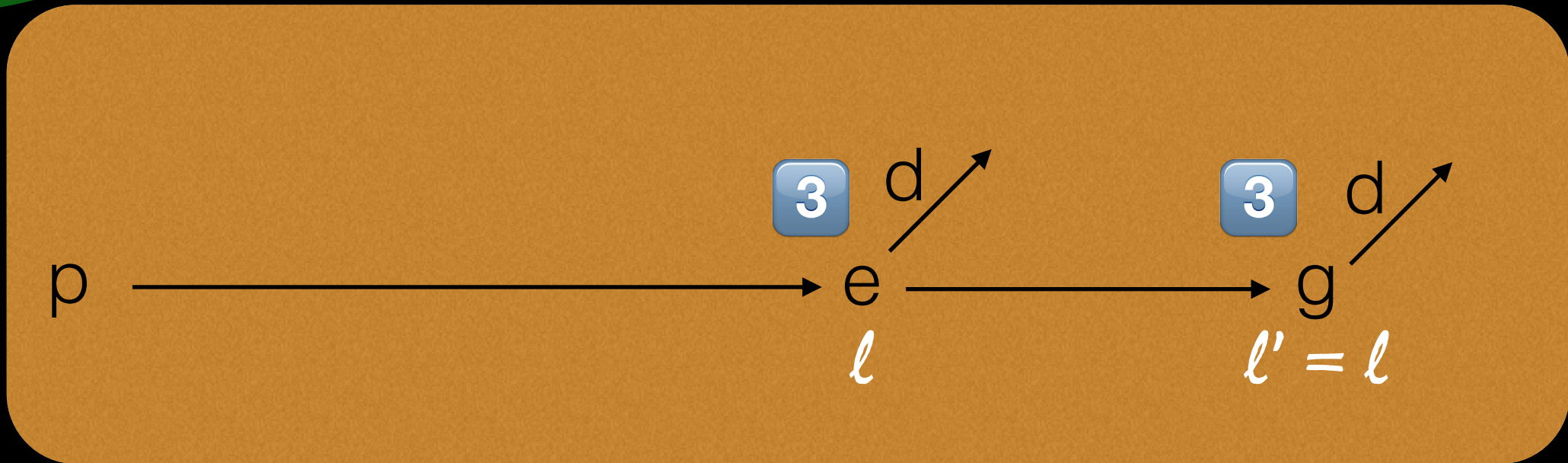
$$\text{pid}(\downarrow g) = \{\text{pid}(g)\} \cup \text{pid}(\text{Prim}(\downarrow g))$$

$$\text{Known}(\ell) = \{p\} \cup \text{pid}(P)$$

$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

write event  
case 1

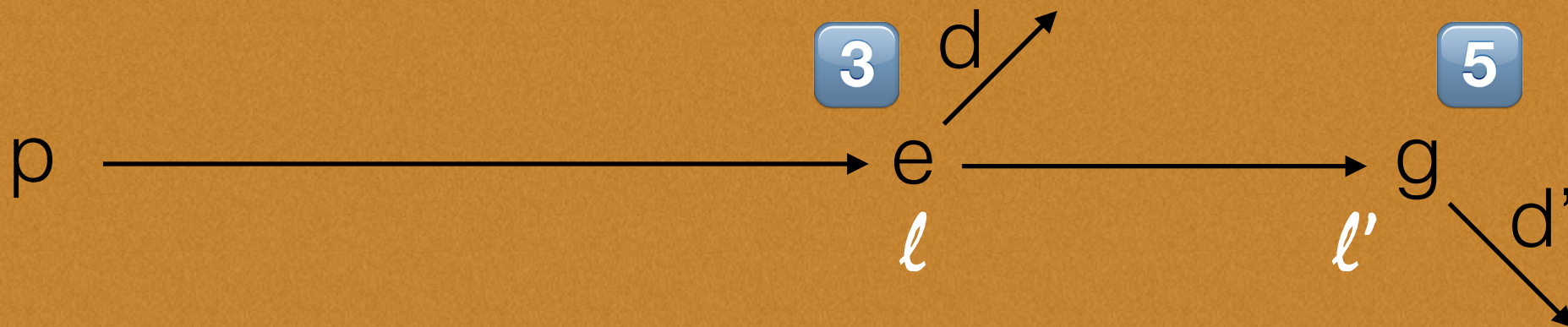
# Maintaining $K^2$



Equivalent writes : no changes

write event  
case 2

# Maintaining $K^2$



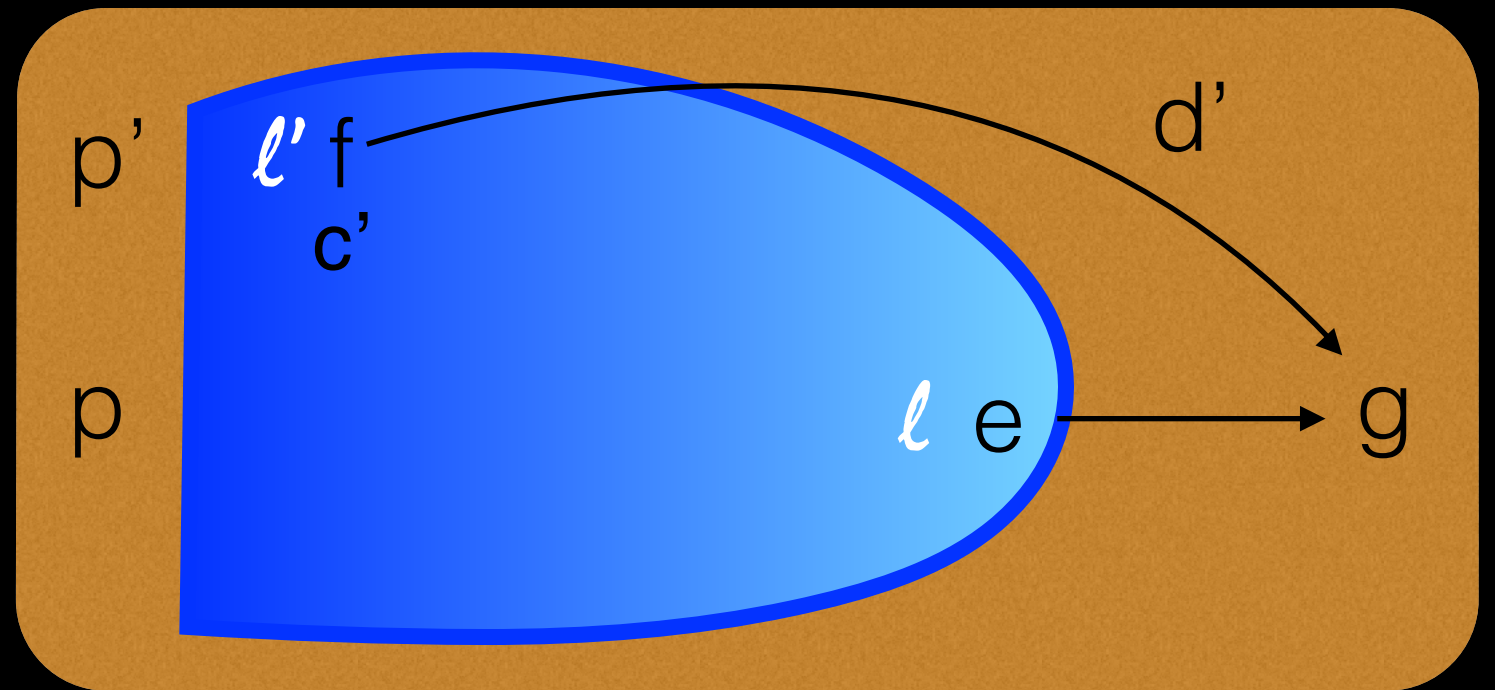
New channel: requires an available color

$$l = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$l' = (p, d', c', P' = P \cup \{(d', c')\}, (S'_\gamma)_{\gamma \in P'})$$

read event  
case 1

# Maintaining $K^2$



$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$\ell' = (p', d', c', P', (S'_\gamma)_{\gamma \in P'})$$

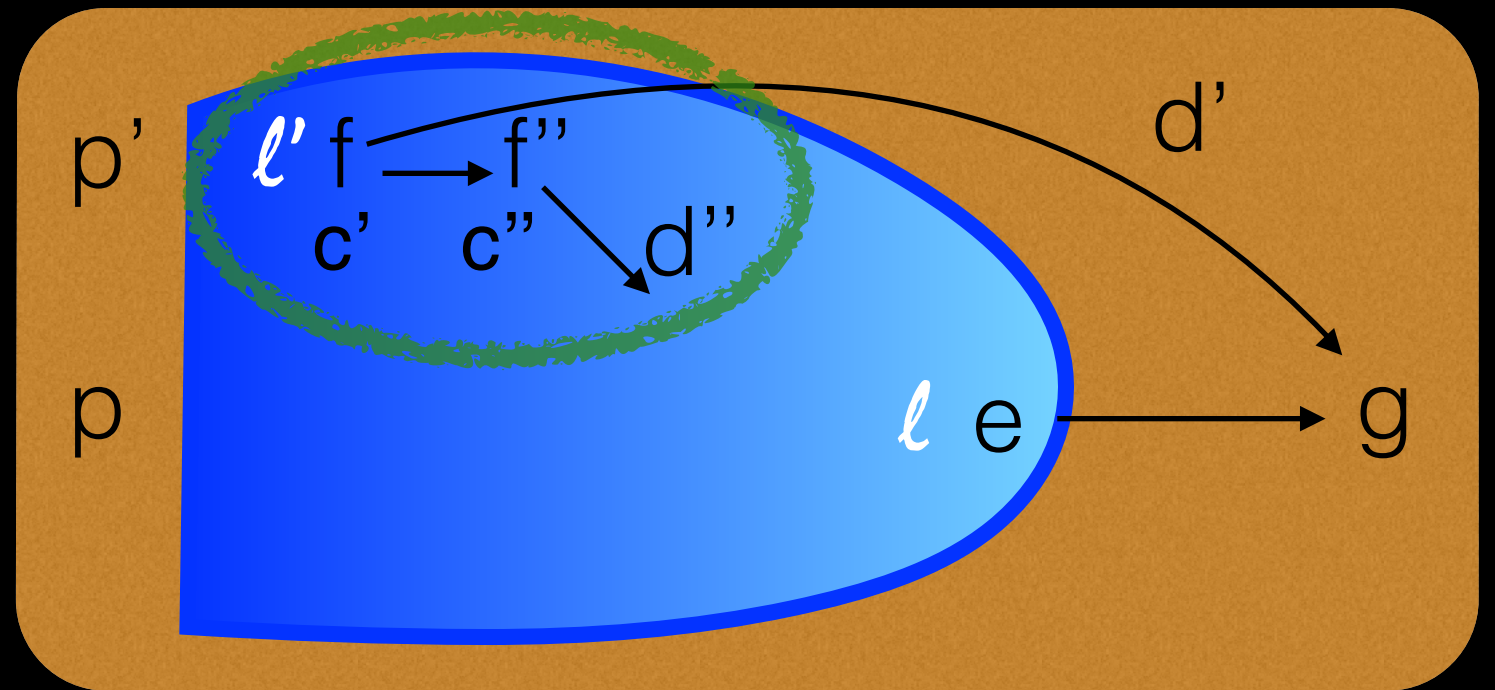
$f < e$  iff

$$(d', c') \in P \wedge \exists (d'', c'') \in P \setminus P' (d'' \neq d' \wedge W(d'') = W(d'))$$



read event  
case 1

# Maintaining $K^2$



$$l = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$l' = (p', d', c', P', (S'_\gamma)_{\gamma \in P'})$$

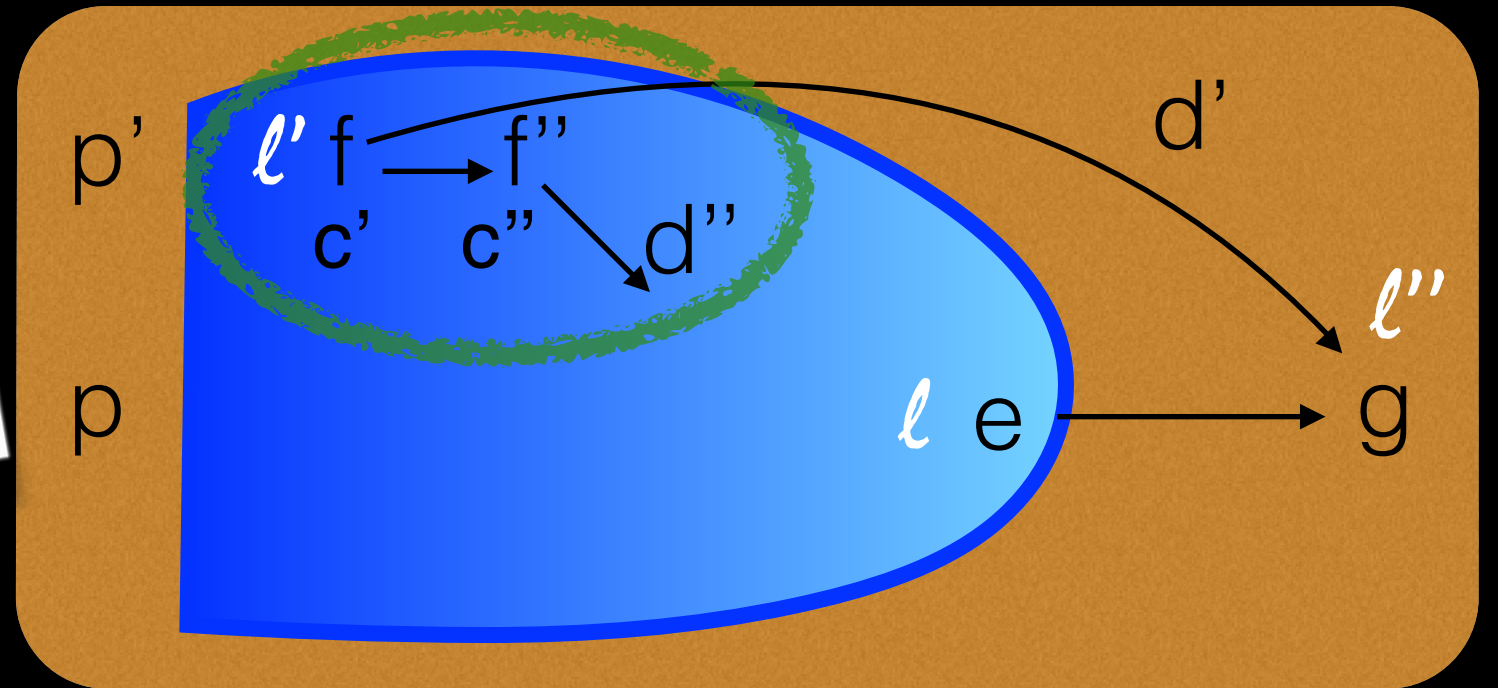
$f < e$  iff

$$(d', c') \in P \wedge \exists (d'', c'') \in P \setminus P' (d'' \neq d' \wedge W(d'') = W(d'))$$

read event  
case 1

# Maintaining $K^2$

$$\text{Latest}(\ell, \ell') = \text{Known}(\ell)$$



$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$\ell' = (p', d', c', P', (S'_\gamma)_{\gamma \in P'})$$

$$\ell'' = (p, \perp, \perp, P'', (S''_\gamma)_{\gamma \in P''})$$

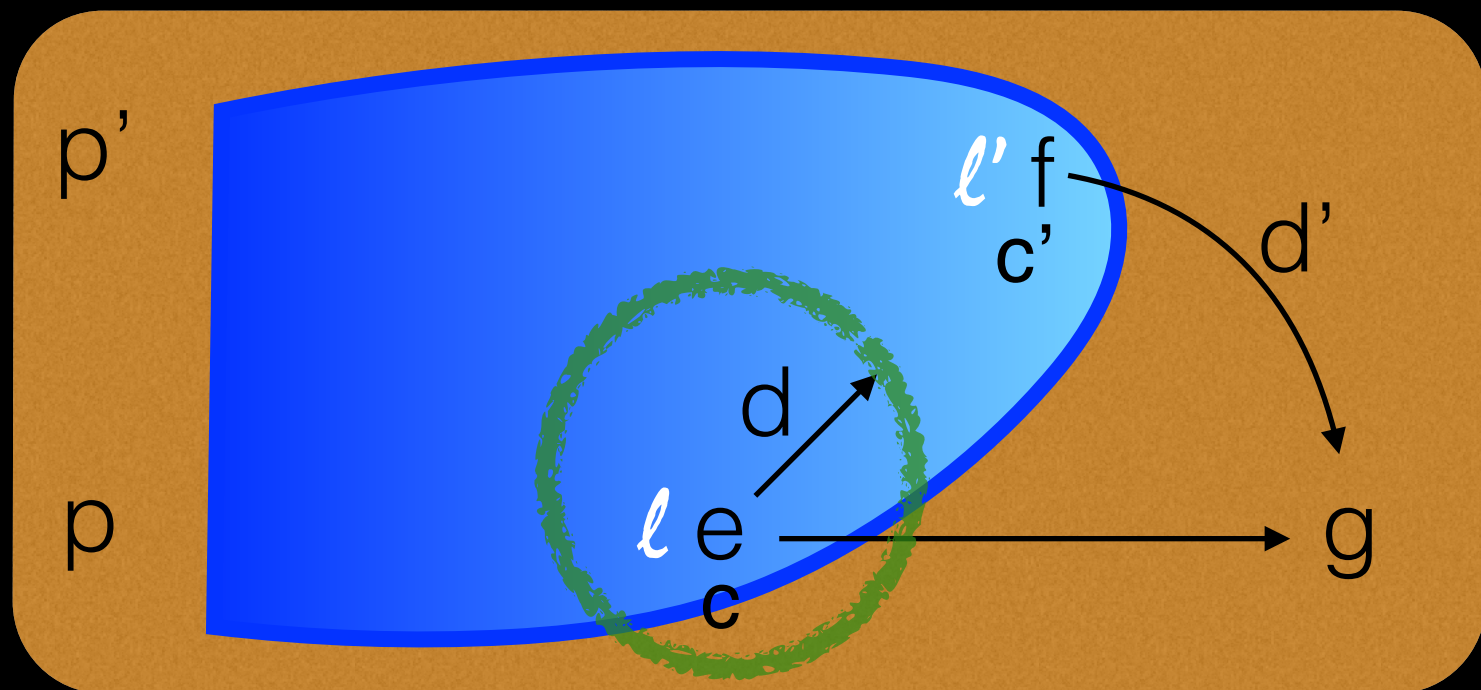
$$P'' = (P \setminus (P' \cap d')) \cup \{(d', c')\}$$

$f < e$  iff

$$(d', c') \in P \wedge \exists (d'', c'') \in P \setminus P' (d'' \neq d' \wedge W(d'') = W(d'))$$

read event  
case 2

# Maintaining $K^2$



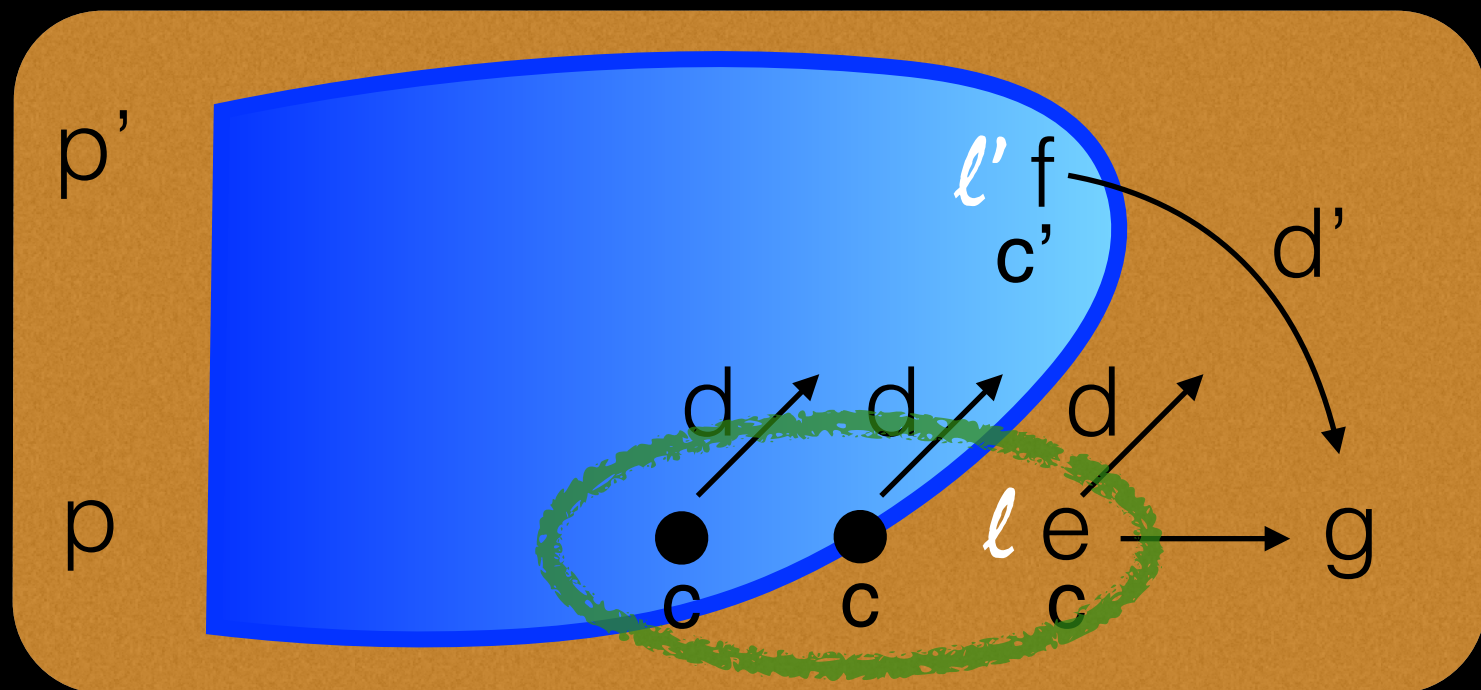
$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$$

$e < f$  implies  $(d, c) \in P'$

read event  
case 2

# Maintaining $K^2$



$$l = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$l' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$$

$e < f$  implies  $(d, c) \in P'$

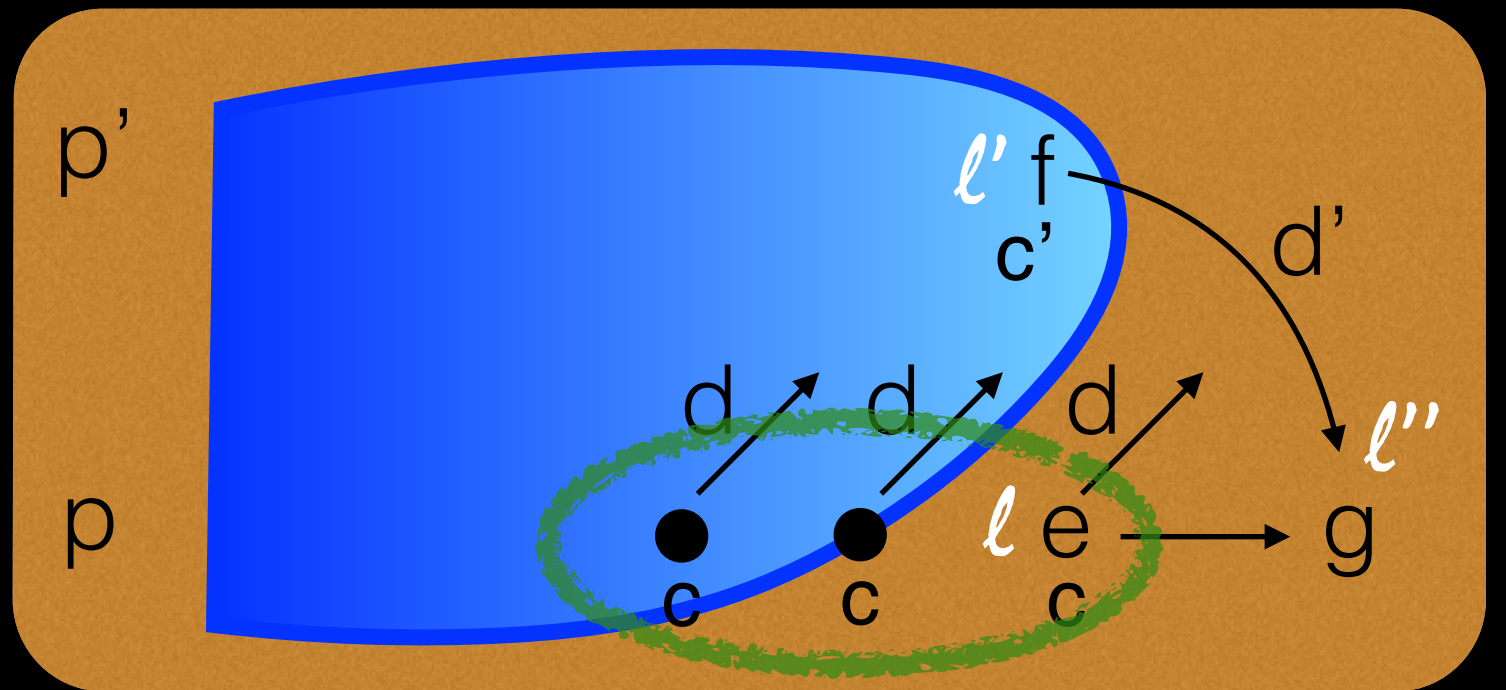
not iff



read event  
case 2

# Maintaining $K^2$

$$\text{Latest}(\ell, \ell') = \{p\}$$



$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$$

$$\ell'' = (p, \perp, \perp, P'', (S''_\gamma)_{\gamma \in P''})$$

$$P'' = (P' \setminus (P' \cap d')) \cup \{(d', c')\}$$

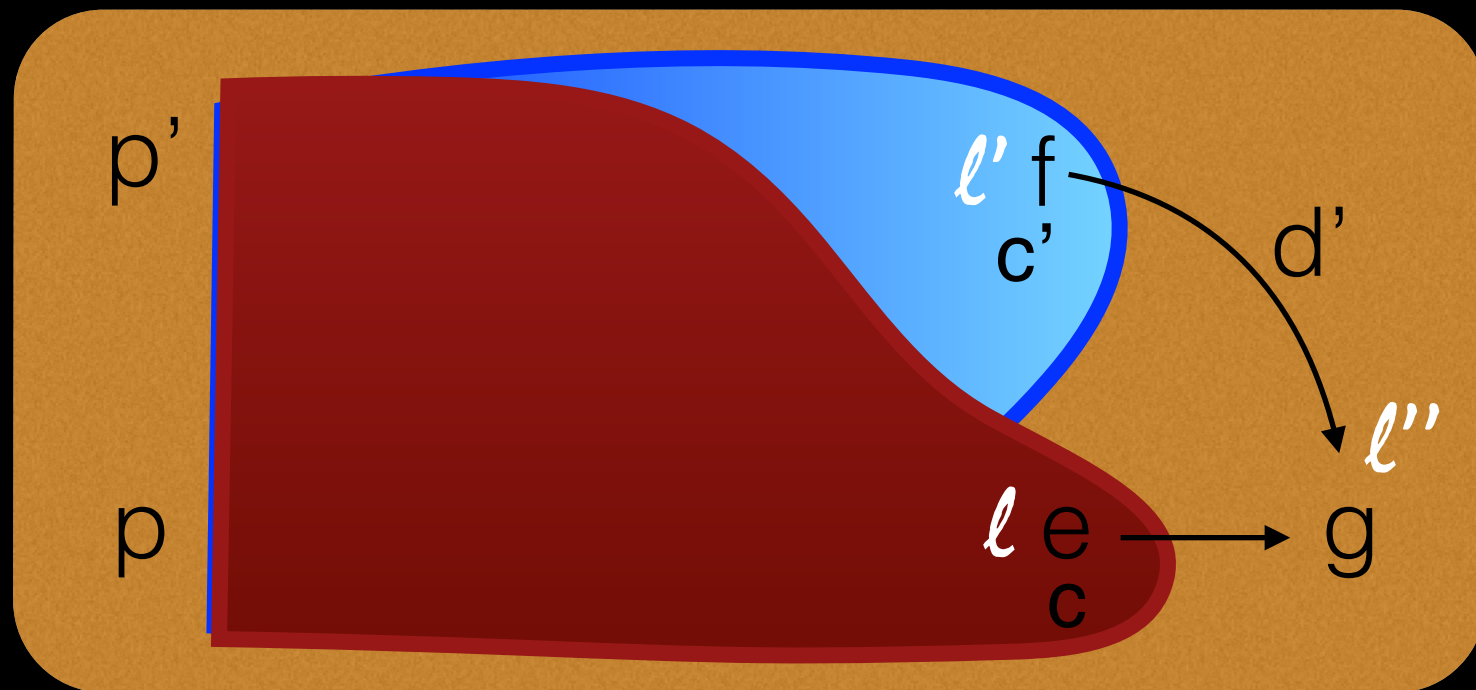
$e < f$  implies  $(d, c) \in P'$

not case 1 and  $(d, c) \in P'$  implies ...

not iff

read event  
case 3

# Maintaining $K^2$



not (case 1 or case 2) implies  $e \parallel f$

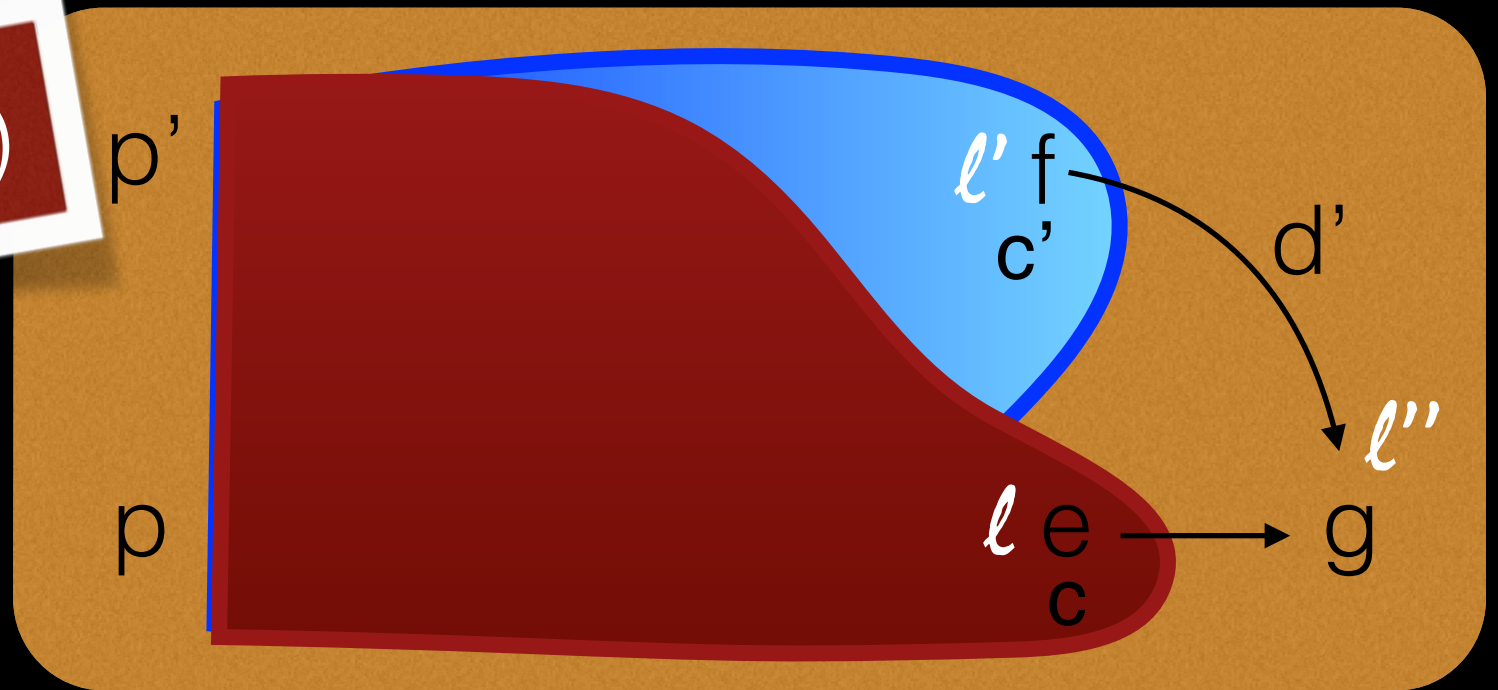
$$\text{Prim}(\downarrow g) = (\text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)) \cup (\text{Prim}(\downarrow e) \setminus \downarrow f) \cup (\text{Prim}(\downarrow f) \setminus \downarrow e)$$

$$\text{Prim}(\downarrow e) \setminus \bigcup_{e' \in \text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)} \text{Prim}(\downarrow e')$$

read event  
case 3

# Maintaining $K^2$

$h$  injective on  $\text{Sec}(\downarrow e \cup \downarrow f)$



not (case 1 or case 2) implies  $e \parallel f$

$$\text{Prim}(\downarrow g) = (\text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)) \cup (\text{Prim}(\downarrow e) \setminus \downarrow f) \cup (\text{Prim}(\downarrow f) \setminus \downarrow e)$$

$$P''' = (P \cap P') \cup (P \setminus \bigcup_{\gamma \in P \cap P'} S_\gamma) \cup (P' \setminus \bigcup_{\gamma \in P \cap P'} S'_\gamma)$$

$$\text{Prim}(\downarrow e) \setminus \bigcup_{e' \in P \cap P'} \text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f) \quad \text{Prim}(\downarrow e')$$

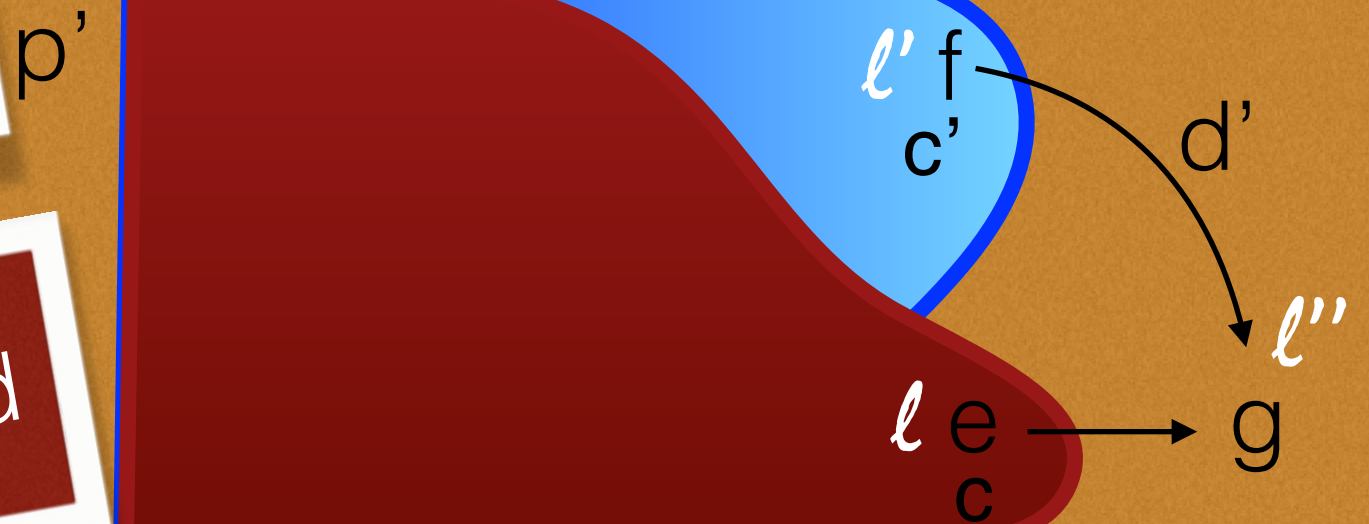


read event  
case 3

# Maintaining $K^2$

$h$  injective on  $\text{Sec}(\downarrow e \cup \downarrow f)$

$\text{Latest}(\ell, \ell')$  can be computed



$$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$$

$$\ell'' = (p, \perp, \perp, P'', (S''_\gamma)_{\gamma \in P''})$$

$$\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$$

$$P'' = (P''' \setminus (P' \cap d')) \cup \{(d', c')\}$$

not (case 1 or case 2) implies  $e \parallel f$

$$P''' = (P \cap P') \cup \left( P \setminus \bigcup_{\gamma \in P \cap P'} S_\gamma \right) \cup \left( P' \setminus \bigcup_{\gamma \in P \cap P'} S'_\gamma \right)$$



# How to maintain the latest information using only finite set of messages?

Is it even possible?

At least in some cases?

Synchronous communication  
[Zielonka87]

Bounded channels  
[Mukund et al.03]

**Primary bounded**