Distributed Timed Automata with Independently Evolving Clocks

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Motivations

Aim

Study the expressive power of local clocks as a synchronization mechanism in a distributed system.

- Distributed systems with no explicit communication or synchronization.
- Clocks as a synchronization mechanism.
- Clocks on different processes evolve independently according to local times.

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Plan

- Distributed Timed Automata
 - **Region abstraction and existential semantics**
 - Universal semantics and undecidability
- **Reactive (Game) Semantics**

Timed automata (Alur & Dill)



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Distributed Timed automata

Definition: DTA

- $\mathcal{D} = ((\mathcal{A}_p)_{p \in Proc}, \pi)$ where
 - each A_p is a classical timed automaton
 - $\pi: \mathcal{Z} \to Proc$ assigns processes to clocks. If $\pi(x) = p$ then
 - \blacktriangleright clock x evolves according to local time on process p
 - \blacktriangleright only process p may reset clock x
 - > all processes may read clock x (i.e., use x in guards or invariants)

Example: DTA with
$$\pi(x) = p$$
 and $\pi(y) = q$
 $\mathcal{A}_p: \quad \checkmark \underbrace{s_0} \qquad y \leq 1, a \qquad & \underbrace{s_1} \qquad a, \{x\} \qquad & \underbrace{s_2} \qquad & \underbrace{s_2} \qquad & \underbrace{s_3} \qquad & \underbrace{s_4} \qquad & \underbrace{s_5} \qquad & \underbrace{s_5} \qquad & \underbrace{s_6} \qquad$

Local Times

Local Times

- Processes do not have access to the absolute (global) time.
- Each process has its own local time: $\tau_p: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

 $\tau_p(t)$: local time on process p at absolute time t

continuous, strictly increasing, diverging, $\tau_p(0) = 0$.



Runs of DTA's & Untimed Behaviours

Example: DTA with $\pi(x) = p$ and $\pi(y) = q$



$$\mathcal{A}_q: \quad \mathbf{A}_q: \quad \mathbf{A}$$

If $\tau_p >$	$ au_q$ ther	n abal	$b \in \mathcal{L}($	\mathcal{D}, au) (e.g.	$ au_p(t)$ =	= 2t a	and $ au_q(t) = t$)	
s_0	a	s_1	b	s_1	a	s_2	b	s_2		
r_0	0.2	r_0	0.6	r_1	0.7	r_1	0.8	r_2		
x = 0		0.4		1.2		0		0.2		
y = 0		0.2		0.6		0.7		0.8		

Runs of DTA's & Untimed Behaviours

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$$\mathcal{A}_q: \quad \mathbf{A}_q: \quad \mathbf{A}$$



If
$$\tau_p = \tau_q$$
 then $abab \notin \mathcal{L}(\mathcal{D}, \tau)$ (e.g. $\tau_p(t) = \tau_q(t) = 2t$)
 $s_0 \xrightarrow{a} 0.2 \xrightarrow{s_1} 0 \xrightarrow{b} s_1 \xrightarrow{s_1} 0 \xrightarrow{a} 0.5 \xrightarrow{s_2} \tau_1$
 $x = 0 \qquad 0.4 \qquad 1 \qquad 0$
 $y = 0 \qquad 0.4 \qquad 1 \qquad 1$

Let $\mathcal{D} = ((\mathcal{A}_p)_{p \in Proc}, \pi)$ be an DTA with local times $\tau = (\tau_p)_{p \in Proc}$.

Definition: (Infinite) Transition System $TS(\mathcal{D}, \tau)$

- Configurations are tuples (s, t, v) where
 - $s = (s_p)_{p \in Proc}$ where s_p is a state of \mathcal{A}_p for each $p \in Proc$
 - $t \in \mathbb{R}_{\geq 0}$ is the absolute time
 - $v: \mathcal{Z} \to \mathbb{R}_{\geq 0}$ is the valuation of clocks.

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- For t < t' we define $v_{t,t'}(x) = v(x) + \tau_{\pi(x)}(t') \tau_{\pi(x)}(t)$.

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- For t < t' we define $v_{t,t'}(x) = v(x) + \tau_{\pi(x)}(t') \tau_{\pi(x)}(t)$.
- Transitions : $(s, t, v) \xrightarrow{g,a,R} (s', t', v')$ if
 - ▶ $s_p \xrightarrow{g,a,R} s'_p$ for some $p \in Proc$ and $s'_q = s_q$ for all $q \neq p$,
 - $\blacktriangleright v_{t,t''} \models \bigwedge_{q \in Proc} I_q(s_q) \text{ for all } t \leq t'' \leq t',$

$$v_{t,t'} \models g$$

•
$$v' = v_{t,t'}[R]$$
 (clocks in R are reset)

•
$$v' \models \bigwedge_{q \in Proc} I_q(s'_q).$$

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with s_0 initial, $t_0 = 0$, $v_0(x) = 0$ for all $x \in \mathcal{Z}$ and s_n final.

Semantics of DTA's

Consider the following DTA ${\cal D}$



with $\pi(x) = p$ and $\pi(y) = q$ and the local times on the right.



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a occurs every local time unit of p.



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with $\pi(x) = p$ and $\pi(y) = q$ and the local times on the right.

 \boldsymbol{a} occurs every local time unit of $\boldsymbol{p}.$

b occurs every local time unit of q.

 $\mathcal{L}(\mathcal{D}, \tau)$ are the finite prefixes of $bab^2ab^4ab^8a\cdots$



Existential & Universal Semantics

Definition: Existential & Universal Semantics

Let \mathcal{D} be a DTA. • $\mathcal{L}_{\exists}(\mathcal{D}) = \bigcup_{\tau} \mathcal{L}(\mathcal{D}, \tau)$ • $\mathcal{L}_{\forall}(\mathcal{D}) = \bigcap_{\tau} \mathcal{L}(\mathcal{D}, \tau)$

Example: $\mathcal{L}_{\exists}(\mathcal{D}) = \{aa, abab, baab\}$ $\mathcal{L}_{\forall}(\mathcal{D}) = \{aa\}$ $\mathcal{A}_{p}: \rightarrow \underbrace{s_{0}}_{a, y \leq 1} \xrightarrow{a, \{x\}} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \leq 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \xrightarrow{b, 0 < x < 1} \underbrace{s_{2}}_{a, y \in 1} \xrightarrow{b, 0 < x < 1} \xrightarrow{$

Negative & Positive Specifications

Aim: robustness of a DTA $\ensuremath{\mathcal{D}}$ against relative local times

Definition: Negative Specifications (Safety)

Given a set Bad of undesired behaviours,

Does a DTA ${\mathcal D}$ robustly avoid Bad

 $\mathcal{L}_\exists(\mathcal{D})\cap \underline{\mathrm{Bad}}=\emptyset$

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Definition: Positive Specifications (Liveness)

Given a set Good of desired behaviours,

Does a DTA ${\mathcal D}$ robustly exhibit Good

 $\mathbf{Good} \subseteq \mathcal{L}_\forall(\mathcal{D})$

Plan

Distributed Timed Automata

- 2 Region abstraction and existential semantics
 - Universal semantics and undecidability
 - **Reactive (Game) Semantics**











Regions when $\pi(x) = \pi(y)$





Regions when $\pi(x) = \pi(y)$



Regions when $\pi(x) \neq \pi(y)$



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Regions when $\pi(x) = \pi(y)$





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Regions when $\pi(x) \neq \pi(y)$



Theorem: Region abstraction

Let $\mathcal D$ be a DTA. Let $\mathcal R_{\mathcal D}$ be its region abstraction.

$$\mathcal{L}_{\exists}(\mathcal{D}) = \mathcal{L}(\mathcal{R}_{\mathcal{D}})$$

and

$$|\mathcal{R}_{\mathcal{D}}| \le |\mathcal{D}| \cdot (2C+2)^{|\mathcal{Z}|} \cdot |\mathcal{Z}|!$$

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Corollary: Negative specifications

Model checking regular negative specifications for DTA's is decidable.

 $\mathcal{L}_\exists(\mathcal{D})\cap \underline{Bad}=\emptyset$

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Plan

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Undecidability of the universal semantics

Theorem: Undecidability

Skip proof.

Let \mathcal{D} be a DTA.

emptiness: $\mathcal{L}_{\forall}(\mathcal{D}) = \emptyset$ is undecidable.

universality: $\mathcal{L}_{\forall}(\mathcal{D}) = \Sigma^*$ is undecidable.

Even for 2 processes, 1 clock each and bounded drifts: $\exists \alpha > 0, \forall t > 0$,

 $1 - \alpha \leq \frac{\tau_q(t)}{\tau_p(t)} < 1 + \alpha \quad \text{or} \quad |\tau_q(t) - \tau_p(t)| \leq \alpha$

Corollary: Positive specifications

$\operatorname{Good} \subseteq \mathcal{L}_\forall(\mathcal{D})$

Model checking regular positive specifications for DTA's is undecidable.

Proof: Reduction from Post Correspondance Problem

- Given two morphisms $f, g: A^+ \to \{0, 1\}^+$ with $A = \{a_1, \ldots, a_k\}$.
- Does there exist $w \in A^+$ such that f(w) = g(w)?

Proof: Reduction from Post Correspondance Problem

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Definition: Directions defined by local times

Each pair of local times $\tau = (\tau_p, \tau_q)$ is mapped to a word $\operatorname{dir}(\tau) \in \{0, 1, 2\}^{\omega}$.



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Next direction of $dir(\tau)$ is 1

$$guard(0) := x = 2 \land 1 < y < 2$$

 $guard(1) := 1 < x < 2 \land y = 2$

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Definition: Directions defined by local times

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Next direction of $dir(\tau)$ is 2

$$guard(0) := x = 2 \land 1 < y < 2$$

guard(1) := 1 < x < 2 \lapha y = 2
guard(2) := (x = 2 \lapha (y < 1 \lapha y = 2)) \lapha (x < 1 \lapha y = 2)

Directions defined by local times

Clocks x, y are reset when reaching the 2×2 square boundary



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 $\operatorname{dir}(\tau) = 0100\cdots$

Definition: Macro transition

For $a \in A$ and $\sigma = d_1 d_2 \dots d_n \in \{0, 1, 2\}^+$ we define



From u with x = y = 0, we reach

- ▶ s with x = y = 0 if local times $\tau = (\tau_p, \tau_q)$ evolve according to σ
- r otherwise

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Proposition:

$$\mathcal{L}(\mathcal{D}_g, \tau) = \{ wb \in A^+b \mid g(w)2 \le \operatorname{dir}(\tau) \}$$



Proposition: $\mathcal{L}_{\forall}(\mathcal{D}) = \{wb \in A^+b \mid f(w) = g(w)\}$

- $s_0 \xrightarrow{w} s_1$ iff $f(w) \leq \operatorname{dir}(\tau)$
- $s_0 \xrightarrow{w} r_1$ iff $f(w) \not\leq \operatorname{dir}(\tau)$
- $s_0 \xrightarrow{w} s_2$ iff $g(w) \leq \operatorname{dir}(\tau)$

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Remark: Positive Specifications and universal semantics

Good $\subseteq \mathcal{L}_{\forall}(\mathcal{D})$ does not imply that the system can be controlled in order to exhibit all Good behaviours, whatever local times are.





Definition: Reactive (Game) Semantics

Environment controls how local times evolve (time-elapse transitions)



- System observes current region and controls discrete transitions
- Not turn-based: system may execute several discrete transitions

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 $\mathcal{L}_{\text{react}}(\mathcal{D}) = \{ w \in \Sigma^* \mid \text{System has a winning strategy} \}$

Decidability of the reactive semantics

Theorem: Regularity

Let \mathcal{D} be a DTA. $\mathcal{L}_{react}(\mathcal{D})$ is regular.

Proof: construct an alternating automaton with ε -transitions accepting $\mathcal{L}_{react}(\mathcal{D})$.

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Corollary: Positive specifications

Model checking regular poitive specifications is decidable for the reactive semantics.

 $\underline{Good} \subseteq \mathcal{L}_{\mathrm{react}}(\mathcal{D})$

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Let \mathcal{D} be a DTA. $\mathcal{L}_{react}(\mathcal{D})$ is regular.

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$$\underline{Good} \subseteq \mathcal{L}_{\mathrm{react}}(\mathcal{D})$$

Proposition: Reactive vs. Universal

- $\mathcal{L}_{react}(\mathcal{D}) \subseteq \mathcal{L}_{\forall}(\mathcal{D})$ for all DTA's \mathcal{D} .
- In general, L_{react}(D) ⊊ L_∀(D).
 Even for DTA's over 2 processes having 1 clock each.

Conclusion

Summary

- Distributed system using clocks with local times to synchronize.
- Regular existential semantics suited for negative specifications
- Regular reactive semantics suited for positive specification
- Undecidable universal semantics

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Further work: Synthesis Problem

Given a regular specification $\operatorname{Spec} \subseteq \Sigma^*$ and an architecture A, Construct a DTA \mathcal{D} over A such that $\mathcal{L}_{\operatorname{react}}(\mathcal{D}) = \operatorname{Spec}$

