Local testing of MSCs

Paul Gastin
LSV, ENS Cachan

Joint work with
Puneet Bhateja, Madhavan Mukund, K Narayan Kumar
CMI, Chennai

ANR DOTS, Bordeaux
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1. MSC

HMSC

Local testing of HMSC

Undecidability of 1-testability for 2 processes

Undecidability of $k$-testability

Decidability of 1-testability without message contents
Scenarios

- A scenario describes a pattern of interaction
- Attractive visual formalism
- Telecommunications
  - Message sequence charts (MSC)
  - Messages sent between communicating agents
- UML
  - Sequence diagrams
  - Interaction between objects
    e.g., method invocations etc
An ATM

Customer

ATM

Bank

passwd

authen

correct

OK

funds?

no

amount

sorry

OK

correct
MSCs as labelled partial orders

- Linearizations give a word language
  \[ p!q(m_1) p!r(m_3) q?p(m_1) q!r(m_2) r?q(m_2) r?q(m_3), \]
  \[ p!q(m_1) q?p(m_1) q!r(m_2) p!r(m_3) r?q(m_2) r?q(m_3), \ldots \]

- A single linearization is sufficient to reconstruct MSC
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Collections of MSCs

- Often need to specify a collection of scenarios
- Finite collection can be exhaustively enumerated
- Infinite collection needs a generating mechanism
High level MSCs (HMSCs)

- A finite state automaton
- Each state is labelled by an MSC
- Each (legal) path in the automaton generates an MSC
All processes must traverse the same path in an HMSC

... but processes move asynchronously

Some processes may be (unboundedly) far ahead of others

After $k$ iterations, we could have $r$ and $s$ in the final copy of $M_2$ while $p$ and $q$ are in the first copy of $M_1$
Regular MSC languages

- An MSC is (uniquely) determined by its linearizations
  - Set of strings over send actions $p!q(m)$ and receive actions $p?q(m)$
- Collection of MSCs $\Leftrightarrow$ word language over send/receive actions
- Regular collection of MSCs $\triangleq$ linearizations form a regular language
**HMSCs and regularity**

- HMSC specifications may not be regular
- **Problem 1** Unbounded buffers

- **Problem 2** Global synchronization yields context-free behaviours
HMSCs and regularity

- HMSC specifications may not be regular
- **Problem 1** Unbounded buffers
- **Problem 2** Global synchronization yields context-free behaviours
- Sufficient structural conditions on HMSCs to guarantee regularity...
  - Locally synchronized
- ... but checking if an HMSC specification is regular is undecidable
  - [AY99,MP99]
- Every regular MSC language can be implemented as network of communicating finite-state automata with bounded channels
  - [HMNST05]
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3 Local testing of HMSC

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(Local) testing using scenarios

- Does an implementation conform to an HMSC specification?
- Local testing
  - Inject messages from some process(es) and observe the response
  - For each process $p$, local observer records sequence of events at $p$
  - If each local observation is consistent with some MSC defined by the HMSC, the implementation passes the test
- Does local testing suffice to check conformance of (regular) HMSC languages?
Implied scenarios [AEY00]

- **p** and **q** believe **M** is **M_1**
- **r** and **s** believe **M** is **M_2**
- MSC **M** is implied by **L** if for each process **p**, the **p**-projection of **M** matches the **p**-projection of some MSC in **L**
- An MSC language is **locally testable** if it is closed with respect to implied MSCs
- Originally studied in context of **realizability**
Implied scenarios . . .

- Even for regular MSC languages, checking local testability is undecidable! [AEY01]
- Even if the original language has bounded channels, its implied scenarios may not
Implied scenarios . . .

- Even for regular MSC languages, checking local testability is undecidable! [AEY01]
- Even if the original language has bounded channels, its implied scenarios may not

Confusing $M^{2k} M'^k$ and $M'^k M^{2k}$ generates upto $k$ messages in $p \rightarrow s$ channel
Joint observations

- What if we have observers who can record the behaviours of sets of processes?

- $M$ is detected as an illegal MSC by $\{p, s\}$.

- Joint observers have more discriminating power.
Joint observations . . .

- Fix some observers $P_1, P_2, \ldots, P_r$
- Each observer records the events on the processes in the set $P_i$

*Given a HMSC $G$, is its language testable with observers $P_1, P_2, \ldots, P_r$?*
**P-Observations**

\( M \) an MSC, \( P \) a set of processes

**P-observation** of \( M \) \( \triangleq \) tuple of projections of \( M \) on each process in \( P \)

\( M\mid_P : P \)-observation of \( M \).

\( L\mid_P = \{ M\mid_P \mid M \in L \} : P \)-observation of a language \( L \)

\[
M\mid_{p,s} = \langle p!q(m)p!s(m), s?r(m)s?p(m) \rangle.
\]
\textbf{$k$-testability}

- Record $P$-observations for every set $P$ of processes of size $k$.
- $k$-closure of a language $L \triangleq \{ M \mid \forall P \text{ s.t. } |P| = k, M|_P \in L|_P \}$
- Scenario $k$-implied by $L \triangleq$ MSC in the $k$-closure of $L$ but not in $L$
- A language is $k$-testable if it equals its $k$-closure
- Local testability is 1-testability
The set \( \{M_1, M_2\} \) is 2-testable but not 1-testable.
$k$-testability . . .

- 1-testability is undecidable for 4 or more processes. [AEY 01]
- $n$-testability is trivial
- What about $k$-testability for $1 < k < n$?
- What is the smallest $k \leq n$ such that $k$-testability is decidable?
Our results

- For all $n$ and $k < n$ there are regular HMSC languages over $n$ processes that are not $k$-testable
- $k$-testability is undecidable for $n \geq 3$ processes and $1 < k < n$
- $1$-testability is undecidable for $2$ processes
  - Improves result from $4$ processes in [AEY01]
- $k$-testability remains undecidable for $n \geq 3$ processes and $1 < k < n$ even without message contents
- $1$-testability is decidable without message contents
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4 Undecidability of 1-testability for 2 processes

Undecidability of $k$-testability

Decidability of 1-testability without message contents
Theorem: 2 processes

For $n \geq 2$, 1-testability is undecidable for regular 4-bounded MSG-definable languages over $n$ processes.

Proof:

Reduction to Modified Post Correspondence Problem (MPCP).
Modified Post Correspondence Problem

Definition: MPCP

Instance: sequence \( (v_1, w_1), (v_2, w_2), \ldots, (v_r, w_r) \) of pairs of words such that

\[ 1 \leq |v_i| \leq 4 \text{ and } 1 \leq |w_i| \leq 4 \text{ for } 1 \leq i \leq r, \]

\[ w_1 < v_1 \text{ and is shorter by at least 2 letters.} \]

Solution: sequence \( 1 = i_1, i_2, i_3, \ldots, i_m \) of indices from \( \{1, 2, \ldots, r\} \) such that

\[ w_{i_1} w_{i_2} \cdots w_{i_m} = v_{i_1} v_{i_2} \cdots v_{i_m} \]

and for \( k < m \),

\[ w_{i_1} w_{i_2} \cdots w_{i_k} < v_{i_1} v_{i_2} \cdots v_{i_k} \]

Theorem: MPCP

The Modified Post Correspondence Problem is undecidable.
Undecidability: Reduction

Proof:

Let $\Delta = (v_1, w_1), (v_2, w_2), \ldots, (v_t, w_t)$ be an instance of the MPCP.

$M_{v_\ell}$ with $v_\ell = a_1a_2\ldots a_r$

$M_{w_\ell}$ with $w_\ell = b_1b_2\ldots b_s$

$L_\Delta = M_{v_1}\cdot\{M_{v_1}, \ldots, M_{v_t}\}^* + M_{w_1}\cdot\{M_{w_1}, \ldots, M_{w_t}\}^*$

Lemma:

The MPCP $\Delta$ has a solution iff $L_\Delta$ has some 1-implied scenario.
Undecidability: Reduction

Proof: Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1} v_{i_2} \ldots v_{i_m} = a_1 a_2 \ldots a_\ell = w_{i_1} w_{i_2} \ldots w_{i_m}$$

$$L = M_{v_1} \cdot \{ M_{v_1}, \ldots, M_{v_t} \}^* + M_{w_1} \cdot \{ M_{w_1}, \ldots, M_{w_t} \}^*$$

- $M \notin L$
- $M \upharpoonright_1 = (M_{v_{i_1}} M_{v_{i_2}} \ldots M_{v_{i_m}}) \upharpoonright_1 \in L \upharpoonright_1$
- $M \upharpoonright_2 = (M_{w_{i_1}} M_{w_{i_2}} \ldots M_{w_{i_m}}) \upharpoonright_2 \in L \upharpoonright_2$
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Undecidability of $k$-testability

Decidability of 1-testability without message contents
Theorem: $k$-testability

For $1 < k < n$, $k$-testability is undecidable for regular 1-bounded MSG-definable languages over $n$ processes.

Proof:

Reduction to Modified Post Correspondence Problem (MPCP).
Undecidability: Reduction

A basic MSC

\[
\begin{align*}
&i & i + 1 & i + 2 & j - 2 & j - 1 & j \\
&\quad & \quad & \quad & \quad & \quad & \\
\end{align*}
\]

\[
\begin{align*}
&i & i + 1 & i + 2 & j - 2 & j - 1 & j \\
\quad & \quad & \quad & \quad & \quad & \quad & \\
\quad & \quad & \quad & \quad & \quad & \quad & \\
\end{align*}
\]
Let $\Delta = (v_1, w_1), (v_2, w_2), \ldots, (v_t, w_t)$ be an instance of the MPCP.

$M_{v_\ell}$ with $v_\ell = a_1 a_2 \ldots a_r$

$M_{w_\ell}$ with $w_\ell = b_1 b_2 \ldots b_s$
Let $\triangle = (v_1, w_1), (v_2, w_2), \ldots, (v_t, w_t)$ be an instance of the MPCKP.

$L_\triangle = M_{v_1} \cdot \{M_{v_1}, \ldots, M_{v_t}\}^* \cup M_{w_1} \cdot \{M_{w_1}, \ldots, M_{w_t}\}^*$

\[ \cup \bigcup_{j} M_{v_1, w_1}^j \cdot \{M_{v_1, w_1}^j, \ldots, M_{v_t, w_t}^j\}^* \]
Lemma:
The MPCP $\Delta$ has a solution iff $L_\Delta$ has some $(k - 1)$-implied scenario.

Proof: Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1}v_{i_2} \cdots v_{i_m} = a_1a_2 \cdots a_\ell = w_{i_1}w_{i_2} \cdots w_{i_m}$$

We build an MSC $M \notin L_\Delta$ which is $(k - 1)$-implied by $L_\Delta$. 
Proof: Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1}v_{i_2}\ldots v_{i_m} = a_1a_2\ldots a_\ell = w_{i_1}w_{i_2}\ldots w_{i_m}$$

$$M \notin L_\Delta$$
Proof: Let $1 = \hat{i}_1, \hat{i}_2, \ldots, \hat{i}_m$ be a solution of MPCP

\[
v_{i_1} v_{i_2} \ldots v_{i_m} = a_1 a_2 \ldots a_\ell = w_{i_1} w_{i_2} \ldots w_{i_m}
\]

\[
M|_{1,\ldots,k-1} = (M_{v_{i_1}} M_{v_{i_2}} \ldots M_{v_{i_m}})|_{1,\ldots,k-1}
\]
Proof: Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

\[ v_{i_1} v_{i_2} \cdots v_{i_m} = a_1 a_2 \cdots a_\ell = w_{i_1} w_{i_2} \cdots w_{i_m} \]

\[ M \mid_{2,\ldots,k} = (M_{w_{i_1}} M_{w_{i_2}} \cdots M_{w_{i_m}}) \mid_{2,\ldots,k} \]
Proof: Let $1 = i_1, i_2, \ldots, i_m$ be a solution of MPCP

$$v_{i_1} v_{i_2} \ldots v_{i_m} = a_1 a_2 \ldots a_\ell = w_{i_1} w_{i_2} \ldots w_{i_m}$$

$$M|_{1, \ldots, j-1, j+1, \ldots, k} = (M_{{v_{i_1}, w_{i_1}}}^j M_{{v_{i_2}, w_{i_2}}}^j \ldots M_{{v_{i_m}, w_{i_m}}}^j)|_{1, \ldots, j-1, j+1, \ldots, k}$$
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Undecidability of 1-testability for 2 processes

Undecidability of $\kappa$-testability

Decidability of 1-testability without message contents
1-testability over the singleton alphabet

- Each channel behaves as counter
- Encode the behaviours of the HMSC as a Petri Net
- HMSC defines a regular language \( \Rightarrow \) channels are bounded by some constant \( B \)
  1. Check if net has \( B + 1 \) messages in a channel en route to final marking
  2. If yes, implied scenario exists
  3. Otherwise, language of net is regular
    - Check if net exhibits any behaviour not described by HMSC
- Special case of a result due to Morin [M02]
Future work

- Local testability is undecidable in most situations
- Look for sufficient conditions that indicate violation of testability
- Testability by piggybacking auxiliary information?

By tagging auxiliary information to $m$,
$p$ informs $s$ whether it has sent a message to $q$
This rules out the implied scenario $M$

- Can we piggyback a bounded amount of auxiliary information to ensure testability?
- Bounded auxiliary information suffices to check causal closure [AMNN05]