How to get decidability of distributed synthesis ?

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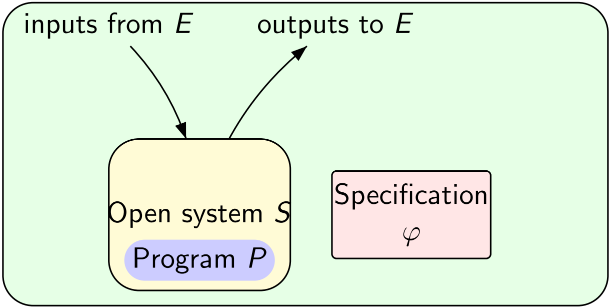
Outline

1. Introduction
2. Model
3. Specification
4. Decidability Results

Synthesis of a reactive system

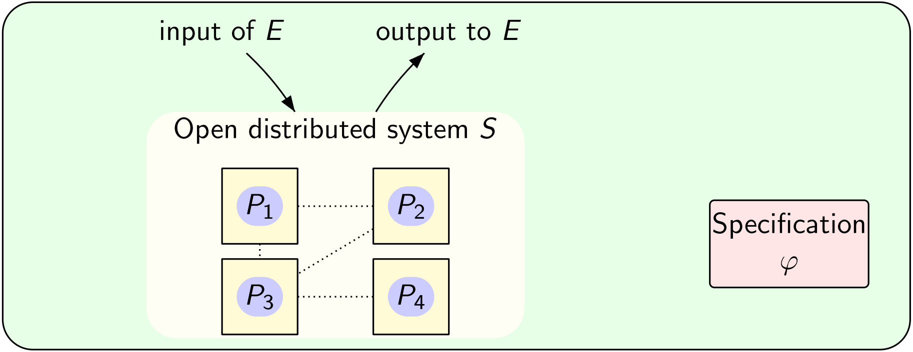


Two problems

- Decide whether there exists a program st. $P \| E \models \varphi, \quad \forall E$.
- Synthesis: If so, compute such a program.

For reasonable systems and specifications, the problems are decidable.

Distributed synthesis



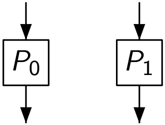
Two problems

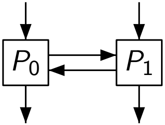
- Decide the existence of a distributed program such that their joint behavior $P_1||P_2||P_3||P_4||E$ satisfies φ, for all E.
- Synthesis : If it exists, compute such a distributed program.

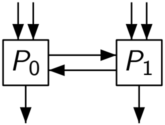
Distributed synthesis: Undecidable in general!?

Synchronous semantics: Introduced by Pnueli Rosner '90

- At each tick of a global clock, all processes and the environment output their new value
- Undecidable with global specifications.
- Undecidable with constraints on internal channels.
- Undecidable with bandwidth constraints.
- Decidable for some architectures, e.g., pipelines.







Asynchronous semantics

P.G., Benjamin Lerman, Marc Zeitoun

- Behaviors are Mazurkiewicz traces
- Players = controllable actions
- Causal memory
- Specification : regular over Mazurkiewicz traces

Theorem

Synthesis problem is decidable for co-graph dependence alphabets, i.e., for series-parallel systems.

Asynchronous semantics

Our model

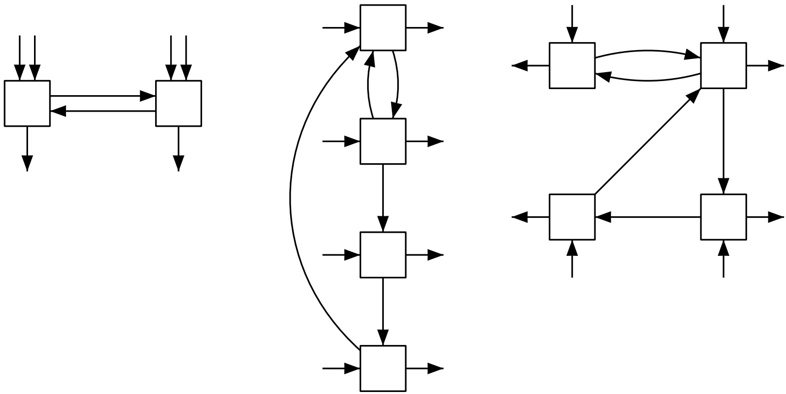
- Processes evolve asynchronously for local actions (i.e., communications with the environment)
- They can synchronize by signals = common actions initiated by only one process. A process cannot refuse reception of a signal.
- Specifications :
  - over partial orders
  - will not restrain communication abilities

Decidability Results

Theorem

Synthesis problem is decidable for

- strongly-connected architectures,
- disjoint unions of decidable architectures.

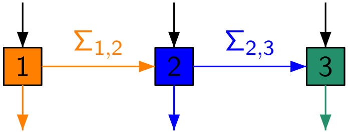


Outline

The model

Architectures

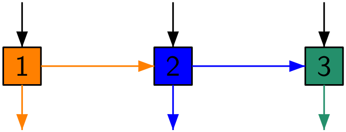
- Communication graph $(Proc, E)$
- For each process i, sets In_i and Out_i of input and output signals: $\Gamma = \bigcup_{i \in Proc} \mathrm{In}_i \cup \bigcup_{i \in Proc} \mathrm{Out}_i$
- Processes choose sets $\Sigma_{i,j}$ for $(i,j) \in E$
- $\Sigma = \Gamma \cup \bigcup_{(i,j) \in E} \Sigma_{i,j}$
- For each process i,
  $\Sigma_i^c = \mathrm{Out}_i \cup \bigcup_{j,(i,j) \in E} \Sigma_{i,j}$ is the set of signals it can send (control),
  $\Sigma_i = \mathrm{In}_i \cup \Sigma_i^c$ is its alphabet.

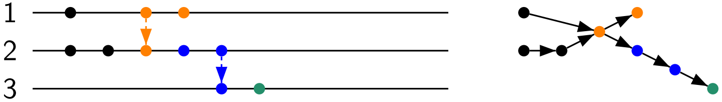


The model: runs

Runs

A run is a Mazurkiewicz trace $t = (V, \lambda, \leq)$ over (Σ, D)
where $a \; D \; b$ if there is a process that takes part both in a and b

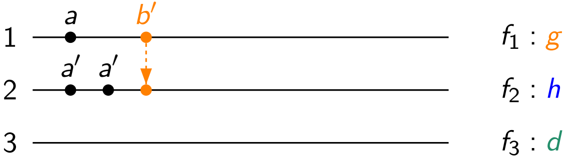




The model: strategies

Strategies

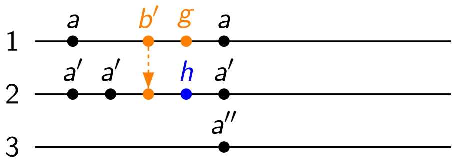
- Strategies are partial functions $f_i : \Sigma_i^* \to \Sigma_i^c$ with local memory.
- Signal semantics implies reactivity of processes to events.



The model: strategies

Strategies

- Strategies are partial functions $f_i : \Sigma_i^* \to \Sigma_i^c$ with local memory.
- Signal semantics implies reactivity of processes to events.
- A run respects a strategy $f = (f_i)_{i \in \mathrm{Proc}}$ (is an f-run) if each event of process i labelled with a controllable action respects the strategy f_i.
- A run $t = (V, \lambda, \leq)$ is f-maximal if for each process i either $V_i = \lambda^{-1}(\Sigma_i)$ is infinite, or f_i is undefined on the maximal event of V_i.



The model

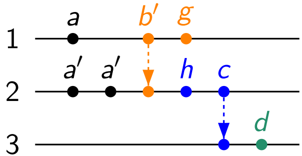
Observable runs

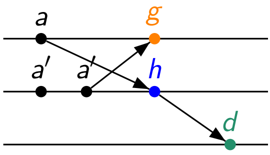
Given a run $t = (V, \lambda, \leq)$, we define the observable run by

$$\pi_\Gamma(t) = (\Gamma, \lambda_{|\Gamma}, \leq \cap (\Gamma \times \Gamma))$$

where

$$\Gamma = \bigcup_{i \in Proc} \mathrm{In}_i \cup \bigcup_{i \in Proc} \mathrm{Out}_i$$





The synthesis problem

Given

- $\mathcal{A} = (\mathrm{Proc}, E, \Gamma)$
- φ a specification over Γ-labelled partial orders (observable runs)

Do there exist

- sets $\Sigma_{i,j}$ for each $(i, j) \in E$
- and strategies $f_i : \Sigma_i^* \to \Sigma_i^c$ for each $i \in \mathrm{Proc}$

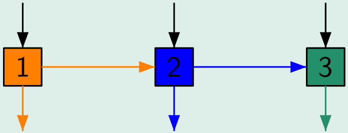
such that every f-maximal f-run t is such that $\pi_\Gamma(t) \models \varphi$?

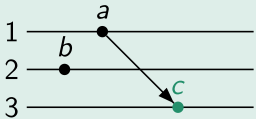
If so, compute them

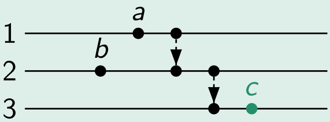
Outline

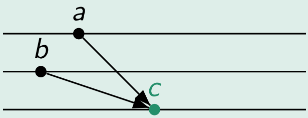
Specifications

Communication induces order relation









Specifications

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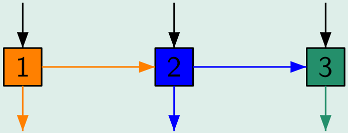
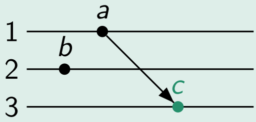
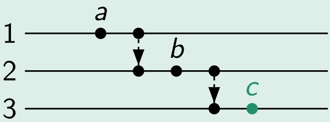
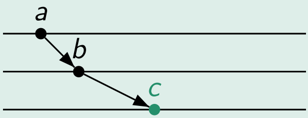


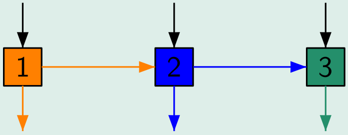


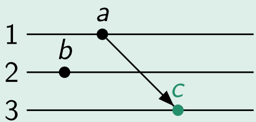


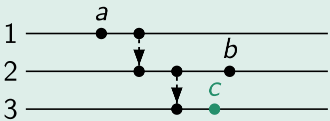


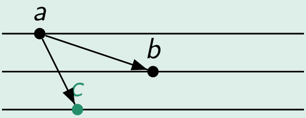
Specifications

Communication induces order relation





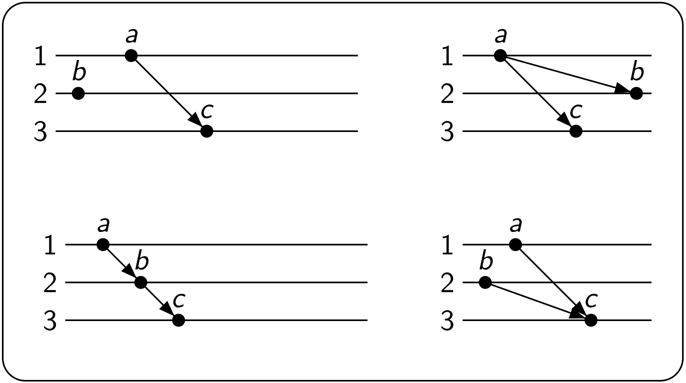




Specifications

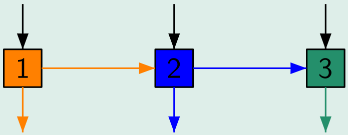
Restrictions on specifications

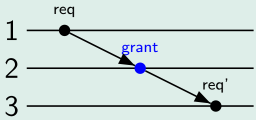
- Specifications should not discriminate between a partial order and its order extensions

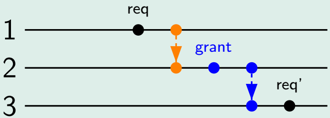


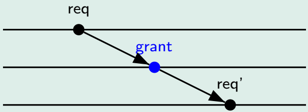
Specifications

Input events are not controllable by processes









Specifications

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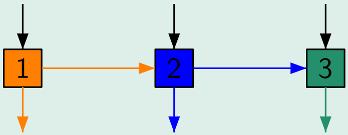
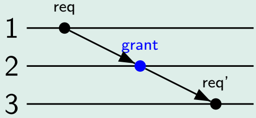
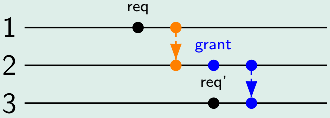
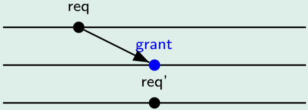




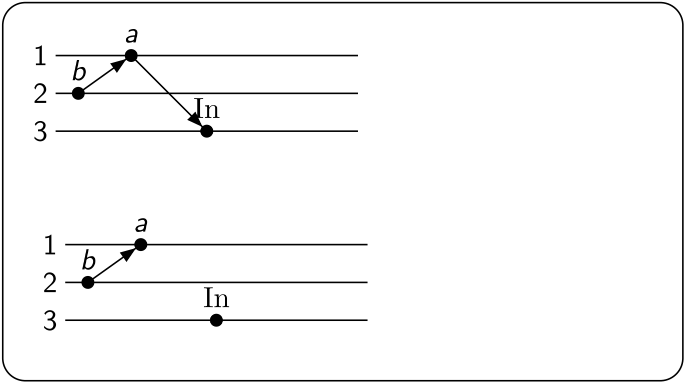




Specifications

Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions
- Specifications should not discriminate between a partial order and its "weakenings"

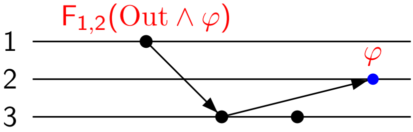


Example of a logic closed by extension and weakening

AlocTL

$$\begin{aligned}\varphi ::= a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \\ \mid \mathsf{X}_i\,\varphi \mid \varphi\,\mathsf{U}_i\,\varphi \mid \neg\,\mathsf{X}_i\,\top \mid \varphi\,\widetilde{\mathsf{U}}_i\,\varphi \\ \mid \mathsf{Y}_i\,\varphi \mid \varphi\,\mathsf{S}_i\,\varphi \mid \neg\,\mathsf{Y}_i\,\top \mid \varphi\,\widetilde{\mathsf{S}}_i\,\varphi \\ \mid \mathsf{F}_{i,j}(\mathrm{Out} \wedge \varphi) \mid \mathrm{Out} \wedge \mathsf{H}_{i,j}\,\varphi\end{aligned}$$

with $a \in \Gamma$ and $i, j \in \mathrm{Proc}$



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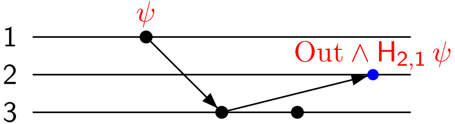

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with $a \in \Gamma$ and $i, j \in \mathrm{Proc}$

Formulae

- $\mathsf{G}_1(\texttt{request} \longrightarrow \mathsf{F}_{1,2}(\mathrm{Out} \wedge \texttt{grant}))$
- $\mathsf{G}_2(\texttt{grant} \longrightarrow (\mathrm{Out} \wedge \mathsf{H}_{2,1}\,\texttt{request}))$

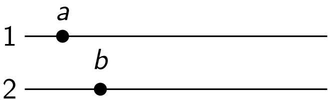
Theorem

AlocTL is closed under extension and weakening

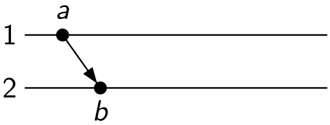
Closure by extension

- $\neg\,\mathsf{F}_{i,j}\,\varphi$ forbidden!

$$a \wedge \neg\,\mathsf{F}_{1,2}\,b$$



OK

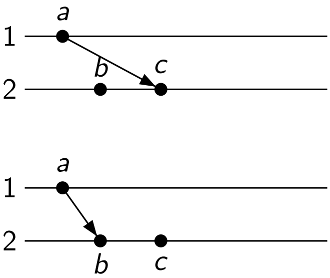


KO

Closure by extension

- $\neg\, \mathsf{F}_{i,j}\, \varphi$ forbidden!
- $X_{i,j}\varphi$ forbidden!

$a \wedge \mathsf{X}_{1,2}\, c$



OK

KO

Closure by extension

- $\neg\, \mathsf{F}_{i,j}\, \varphi$ forbidden!
- $X_{i,j}\varphi$ forbidden!

Specification is not allowed to require concurrency

Closure by weakening

Ensured by $\mathsf{F}_{i,j} \wedge \mathrm{Out}$ and $\mathrm{Out} \wedge \mathsf{H}_{i,j}\, \varphi$.

Outline

Decidability Results

Theorem

The synthesis problem over singleton architectures is decidable for regular specifications.

Theorem

The distributed synthesis problem over strongly connected architectures is decidable for AlocTL specifications.

Proof

By reduction to the singleton case.

Note that we do not need to change the specification since it is closed under extension

Strongly connected architectures (2)

Proposition

If there are communication sets $\Sigma_{i,j}$ for $(i, j) \in E$ and a winning distributed strategy on the strongly connected architecture, then there is a winning strategy on the singleton.

Proof

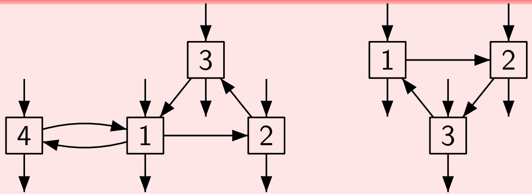
Easy.

Strongly connected architectures

Proposition

If there is a winning strategy f over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected architecture.

Proof



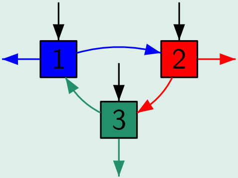
- We select a master process and a cycle.
- The master process will centralize information in order to simulate f and tell other processes which value to output
- Aim: create a run that will be a weakening of some f-run over the singleton

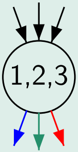
Centralize information

Example

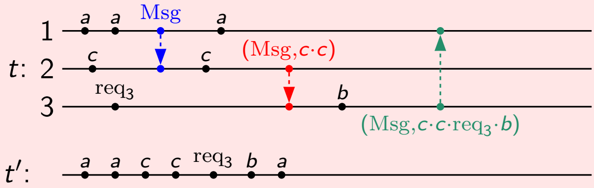
Specification: $\mathrm{req}_3 \rightarrow \mathsf{F}_{32}(\neg\, \mathsf{Y}_2\, \mathrm{alert} \leftrightarrow \mathrm{grant})$

Strategy for the singleton: $f(\sigma) = \mathrm{grant}$ iff σ contains req_3 but no alert





Master collect information by sending a signal Msg through the cycle



Tell processes what to output

Example

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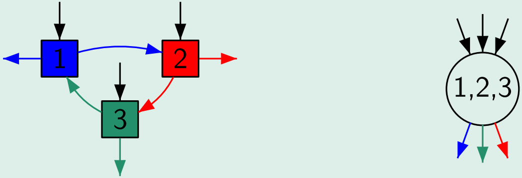
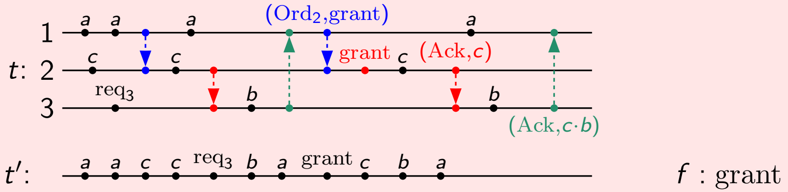


Master sends orders to other processes to simulate the strategy f



Tell processes what to ouptut (2)

Example

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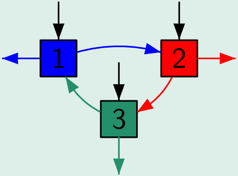

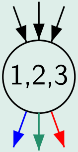

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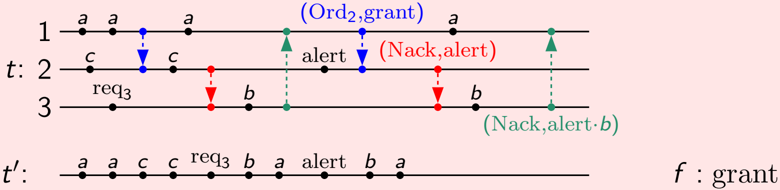

Proof - end

Lemma

t' is an extension of $\pi_\Gamma(t)$.

Lemma

If $x <' y$ in t' and $x \parallel y$ in $\pi_\Gamma(t)$ then $\lambda(y) \in \mathrm{In}$.

Corollary

$\pi_\Gamma(t)$ is a weakening of t'.

Lemma

t' is an f-maximal f-run.

Conclusion

Then $t' \models \varphi$ and, by closure property $\pi_\Gamma(t) \models \varphi$.

Memory

Proposition

If the strategy f over the singleton has finite memory, then we can distribute the strategy for the strongly connected architecture using

- finite alphabets $\Sigma_{i,j}$
- local strategies with finite memories

Conclusion

- Asynchrony removes undecidability causes
- We have defined a new model of communication
- We have identified a class of decidable architectures
- Hopefully, many more to come!