10 YEARS OF WEIGHTED LOGICS FOR WEIGHTED AUTOMATA

Paul Gastin,

LSV, ENS Cachan

joint work with Benjamin Monmege

ULB

GOAL

- Qualitative, Boolean: [Büchi’60], [Elgot’61], [Trakhtenbrot’61]

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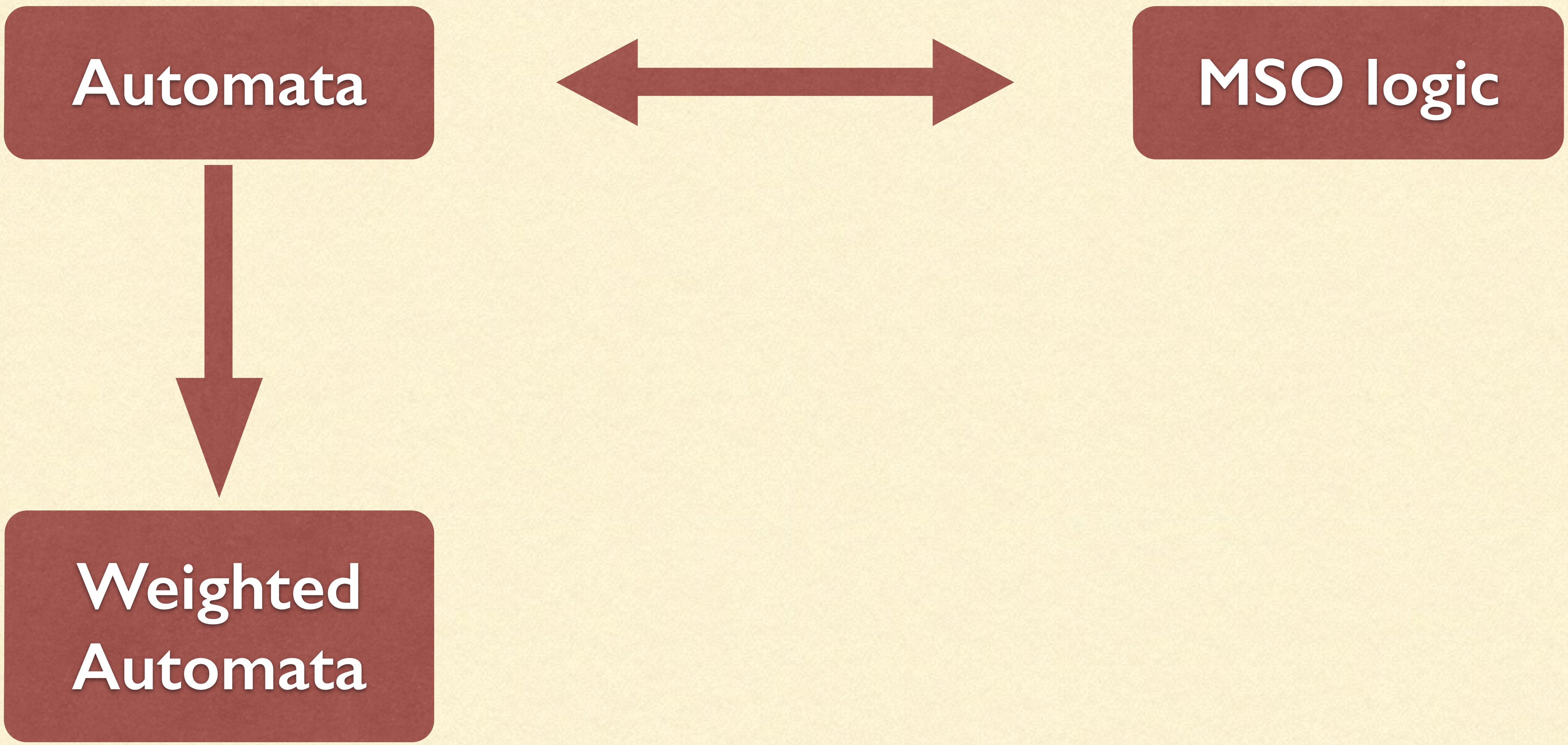
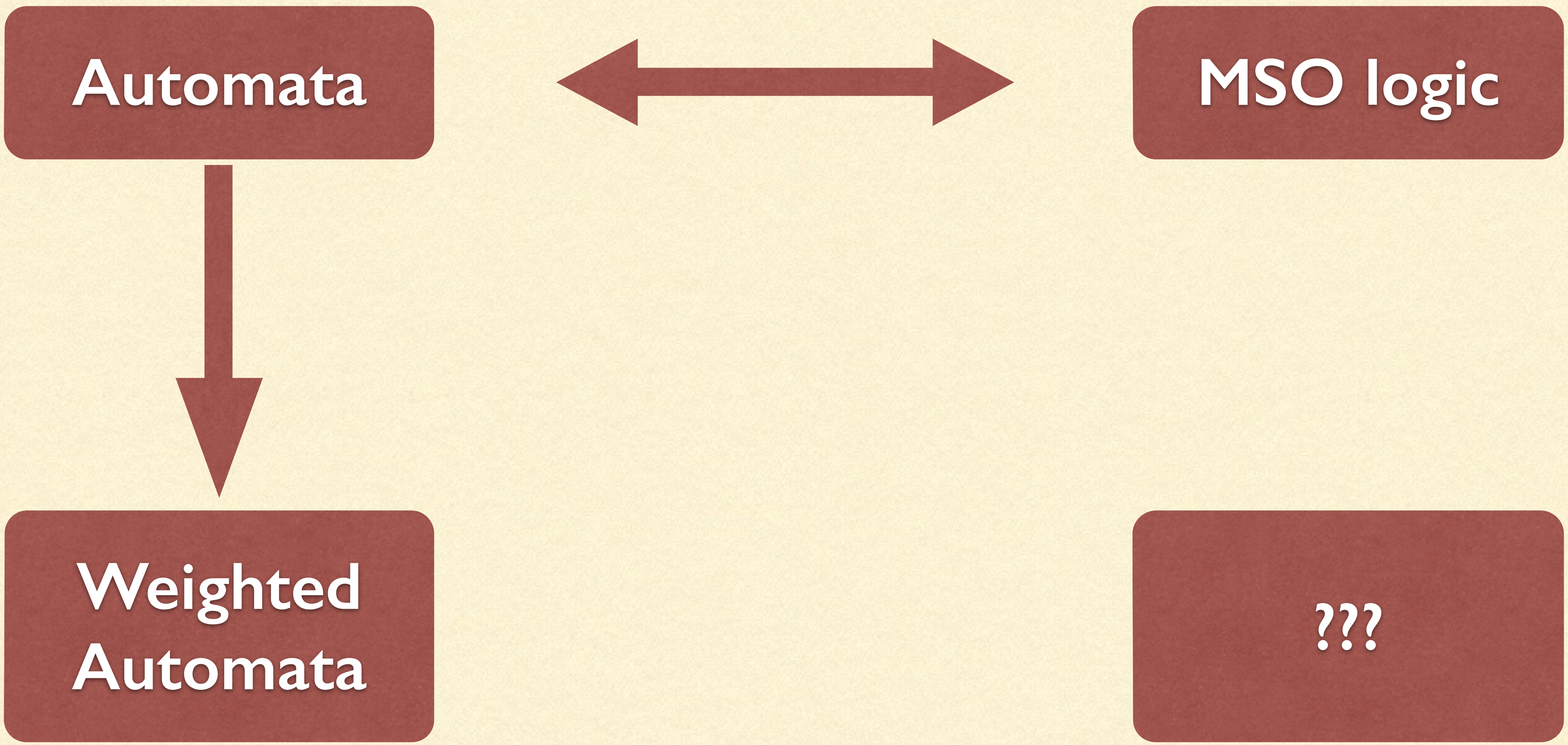


- Quantitative, weights

GOAL

- Qualitative, Boolean: [Büchi'60], [Elgot'61], [Trakhtenbrot'61]



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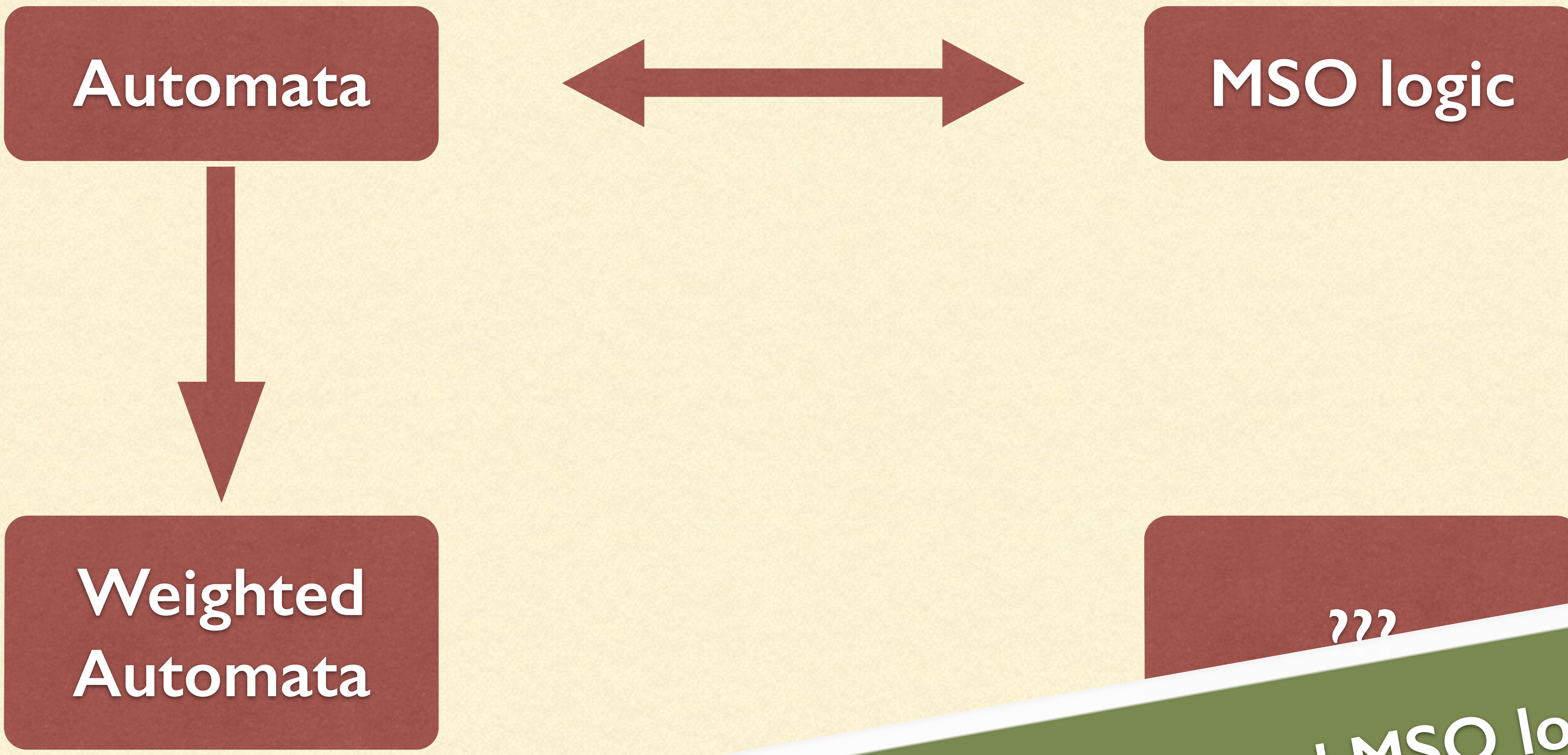

- Quantitative, weights

Find suitable weighted MSO logic

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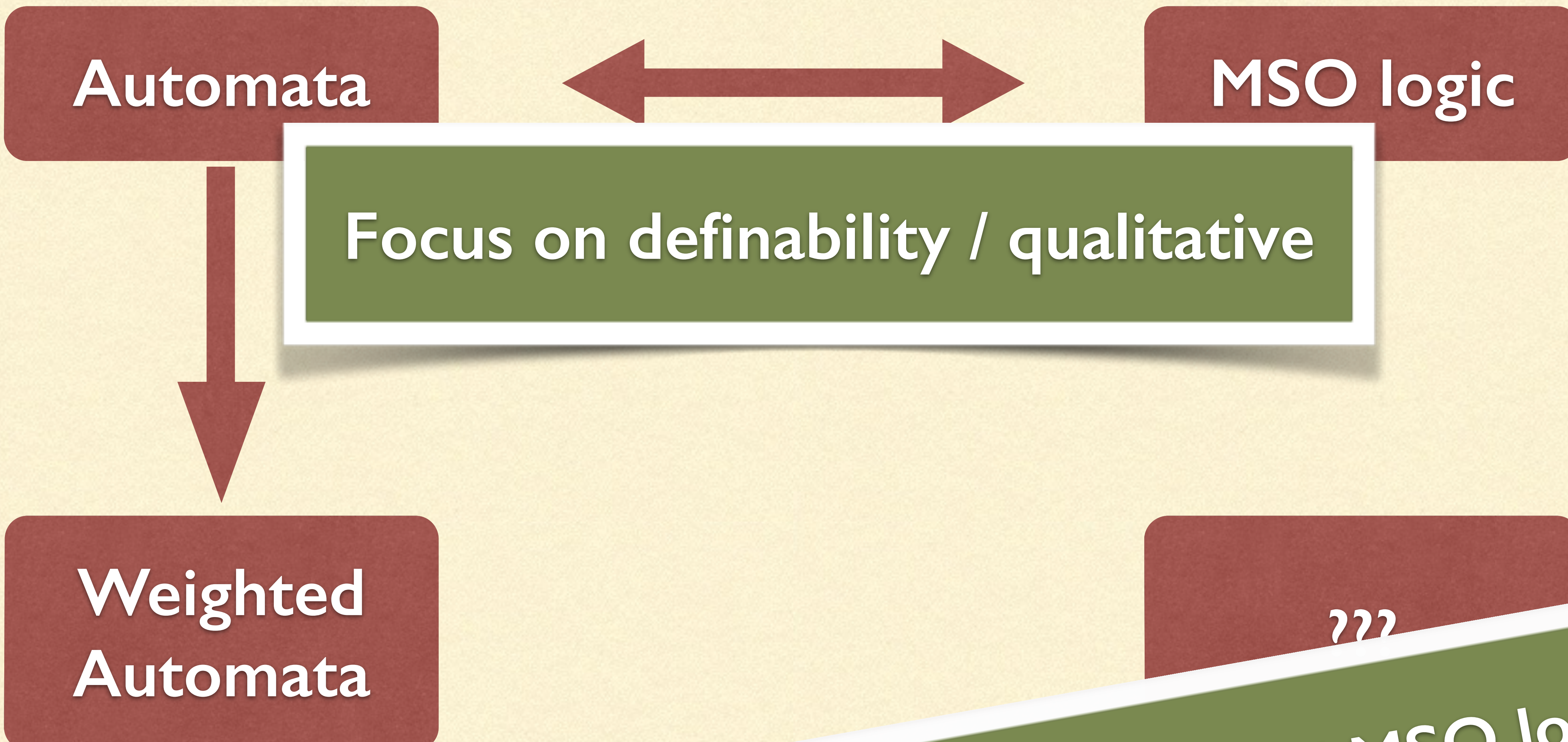


Focus on definability / qualitative

- Quantitative, weights

Find suitable weighted MSO logic

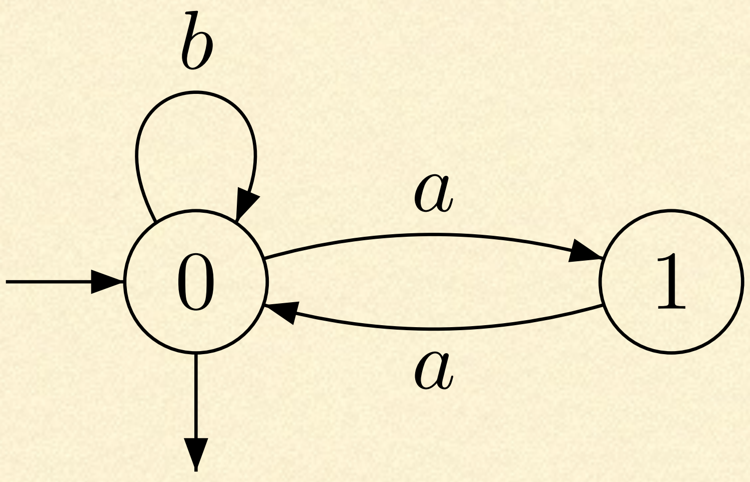
GOAL

- Qualitative, Boolean: [Büchi'60], [Elgot'61], [Trakhtenbrot'61]

- Quantitative, weights

AUTOMATA: FOCUS ON QUALITATIVE

- a's blocks are of even length



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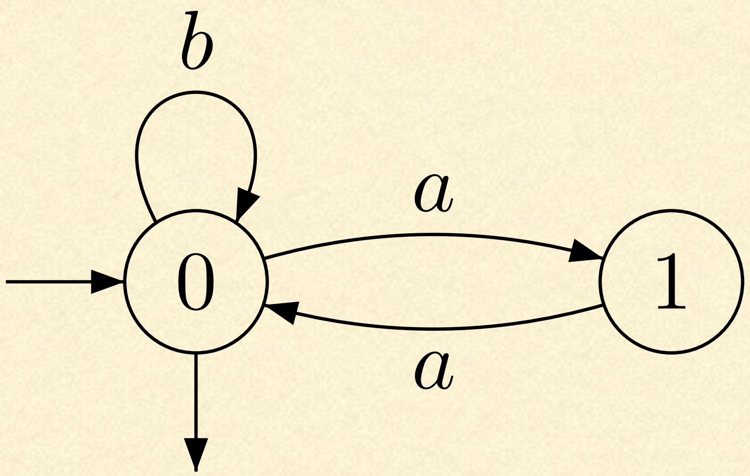

- # a's ≠ # b's

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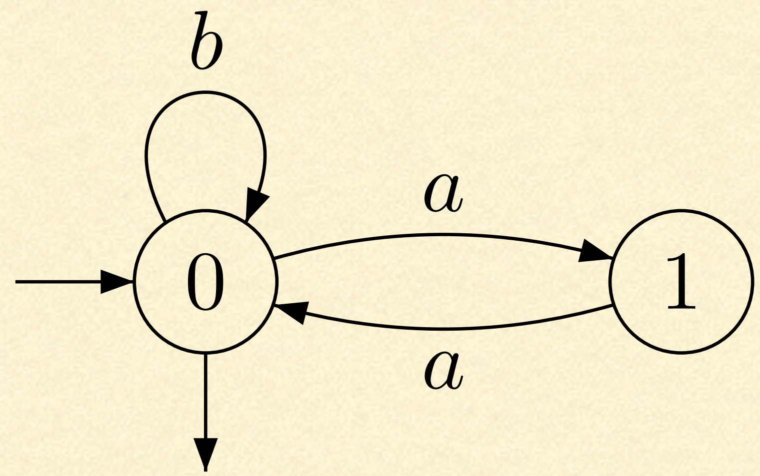
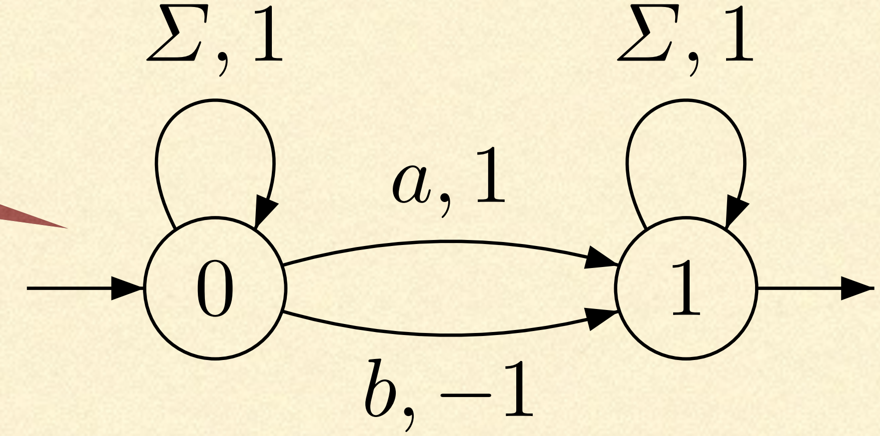


Support

- # a's ≠ # b's



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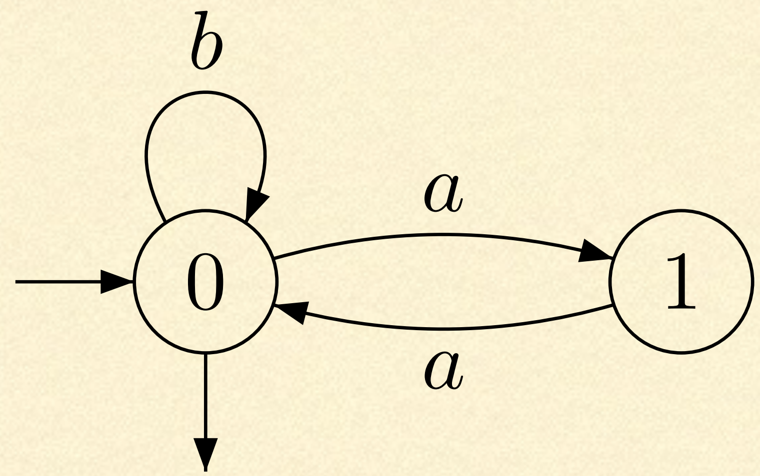

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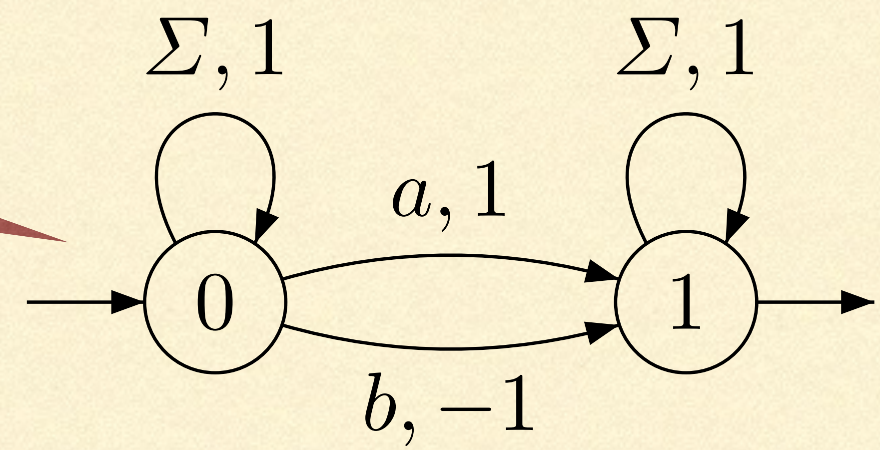
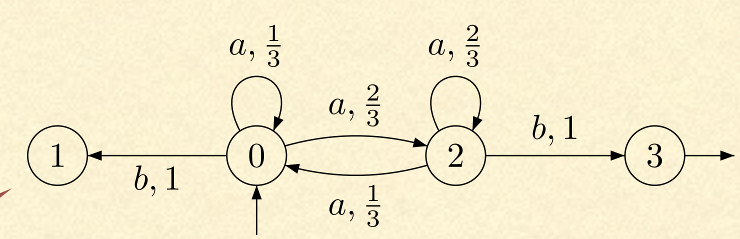


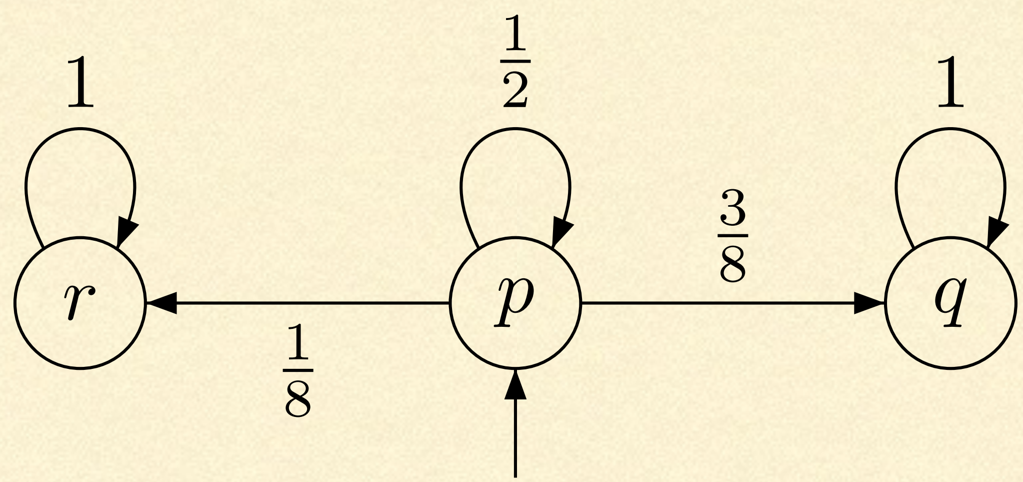
- probabilistic



Threshold

LOGIC: FOCUS ON QUALITATIVE

- Probabilistic CTL $\mathsf{Prob}(p \,\mathsf{U}\, q) \geqslant \frac{2}{3}$



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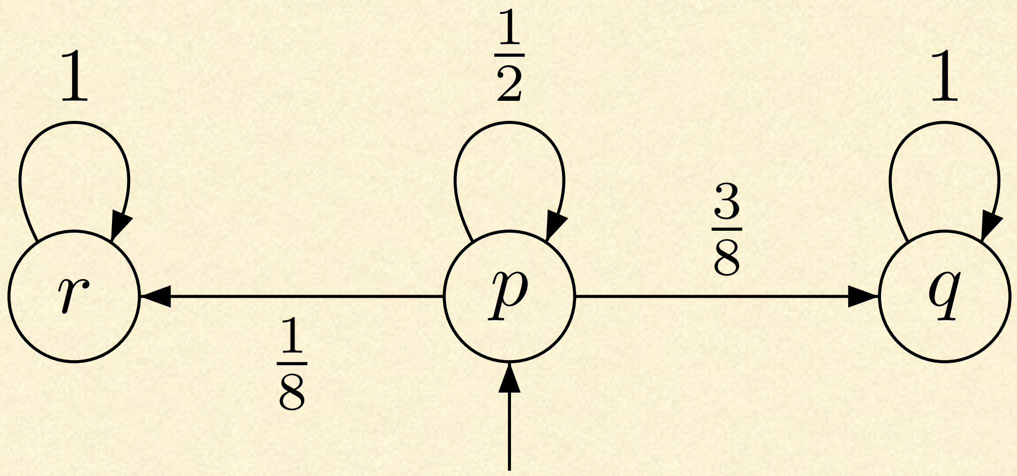
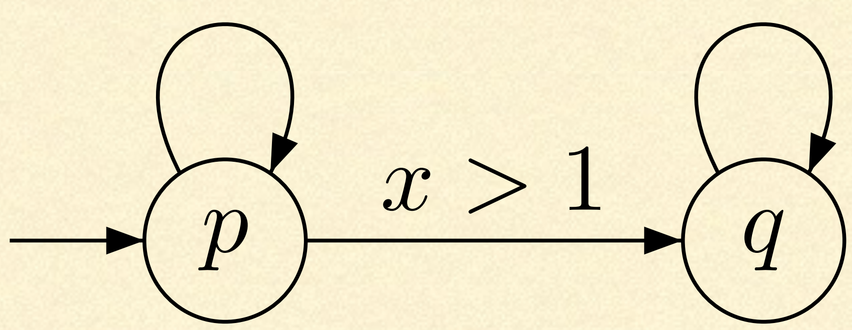


- Timed CTL

$\mathsf{E}\,p\,\mathsf{U}_{<3}\,q$

$\mathsf{A}\,p\,\mathsf{U}_{<3}\,q$



AUTOMATA: FOCUS ON QUANTITATIVE

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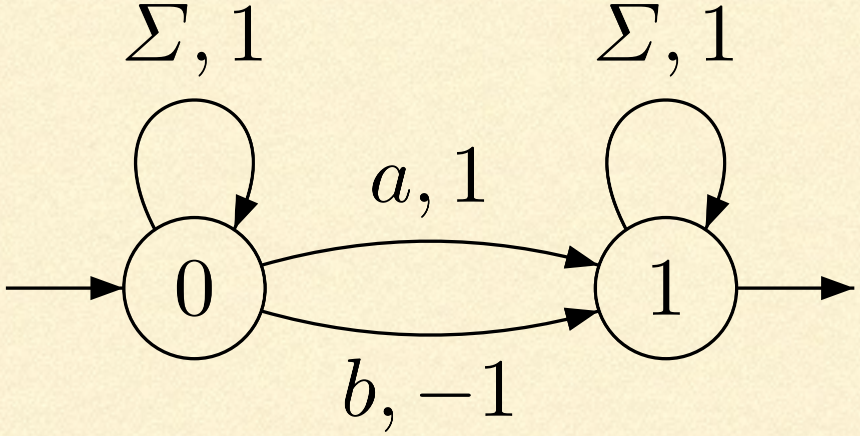

AUTOMATA: FOCUS ON QUANTITATIVE

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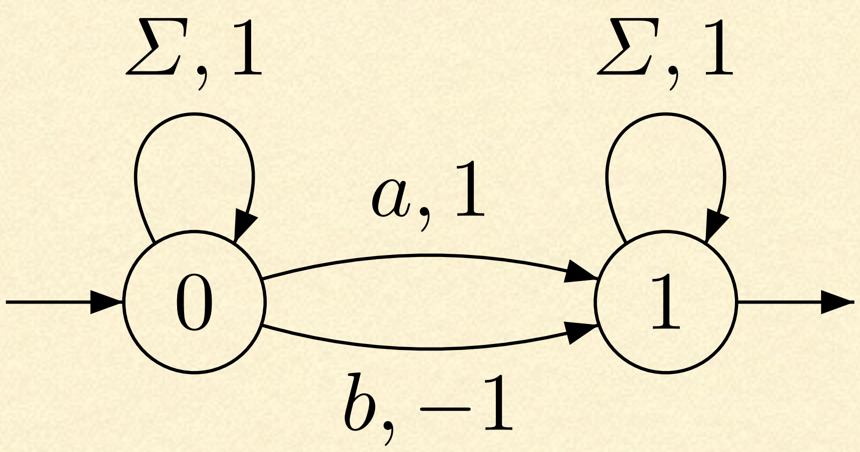
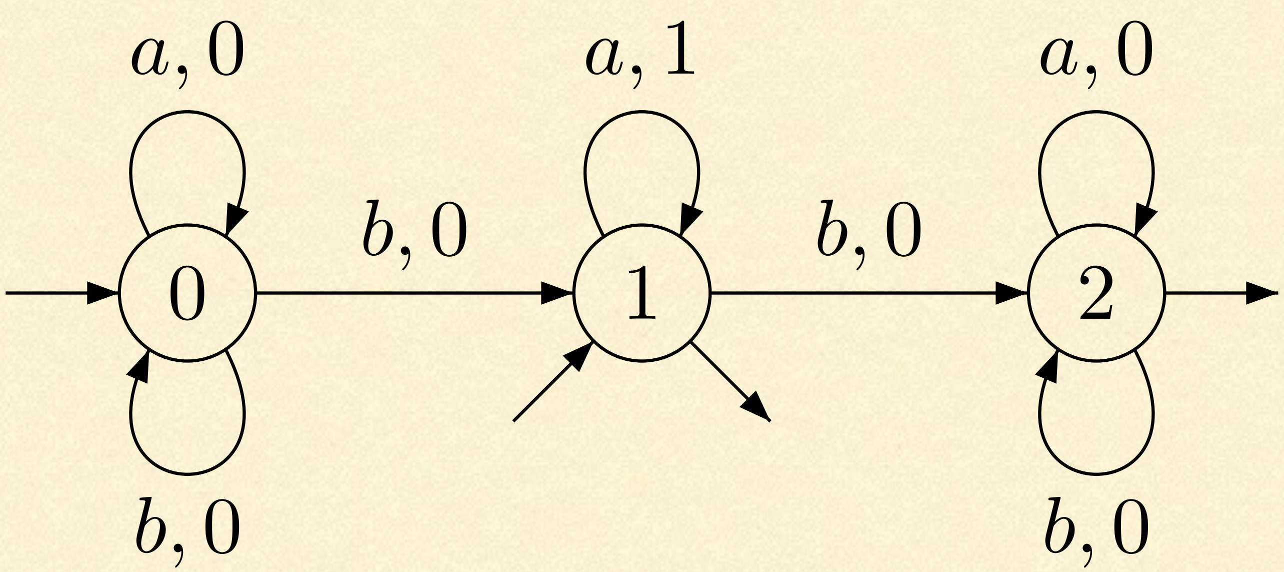
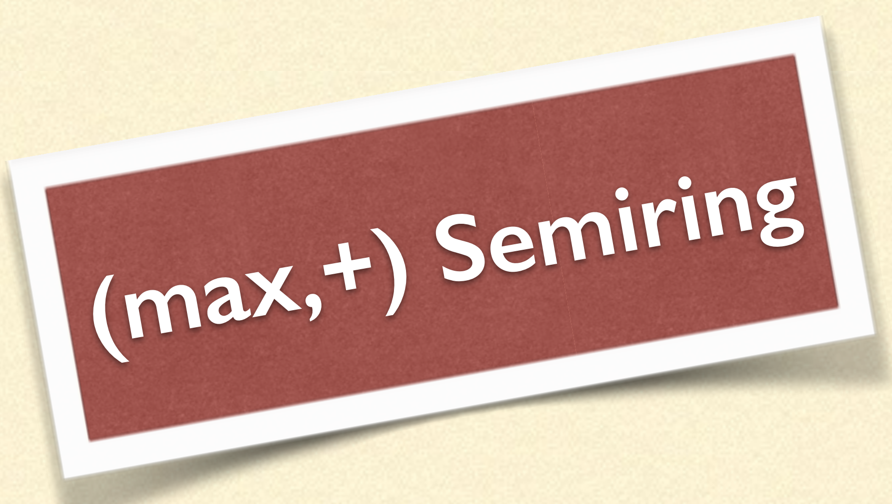


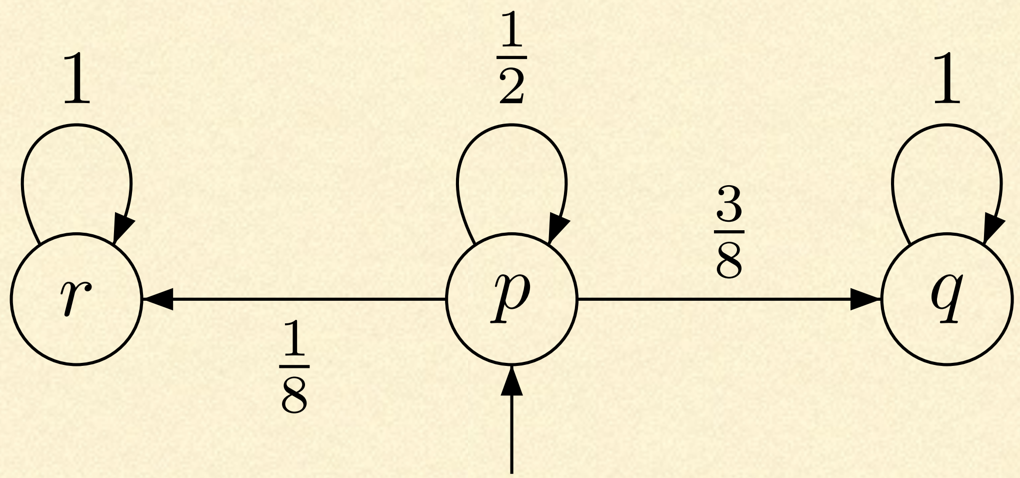
- Maximal length of a's blocks





LOGIC: FOCUS ON QUANTITATIVE

$\mathsf{Prob}(p \,\mathsf{U}\, q)$



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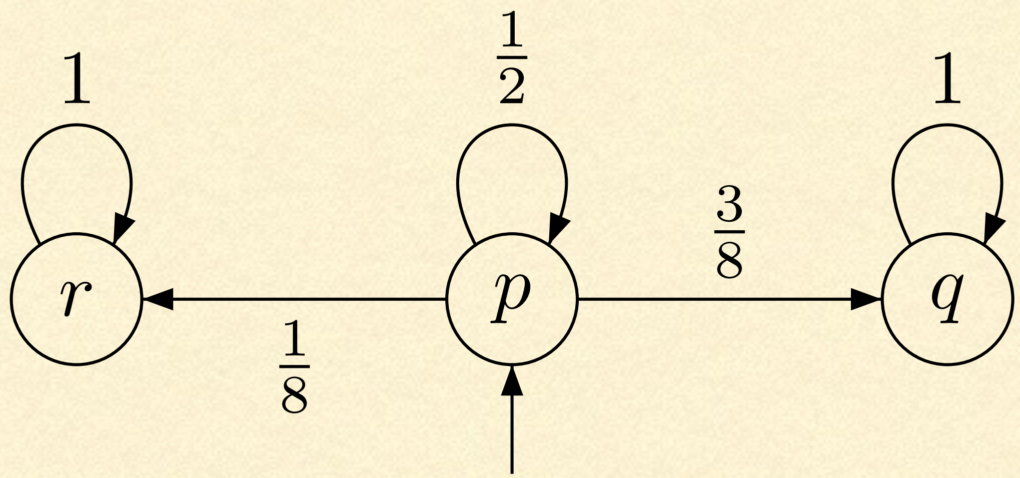
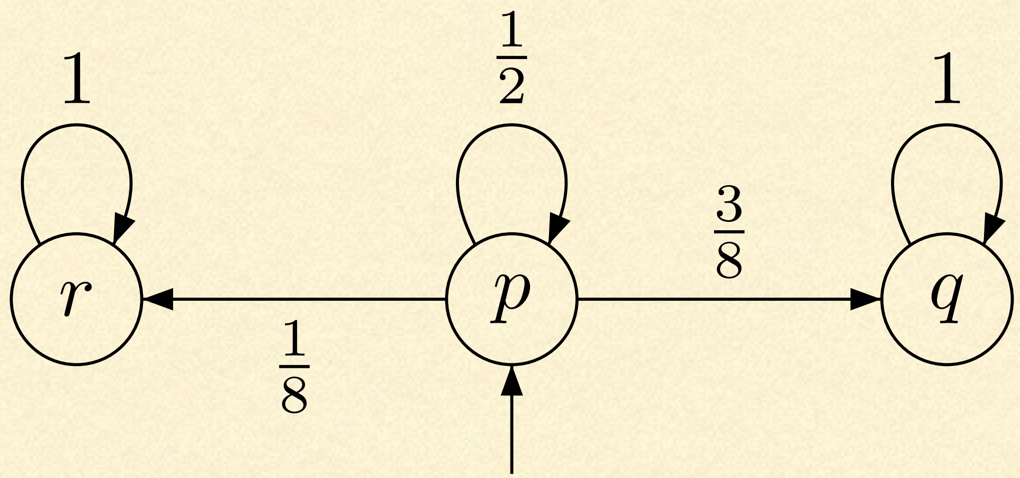


- aababbaaaababaaab

$$\sum_x P_a(x)\,?\,1 : 0$$

LOGIC: FOCUS ON QUANTITATIVE

Prob(p U q)



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$$\sum_x P_a(x)\,?\,1 : 0$$

$$\max_{x,y}\left(\forall z,\, x \leqslant z \leqslant y \to P_a(z)\right)\,?\,\left(\sum_z (x \leqslant z \leqslant y)\,?\,1 : 0\right) : -\infty$$

AUTOMATA vs LOGIC

- Qualitative: [Büchi'60], [Elgot'61], [Trakhtenbrot'61]

AUTOMATA = MSO

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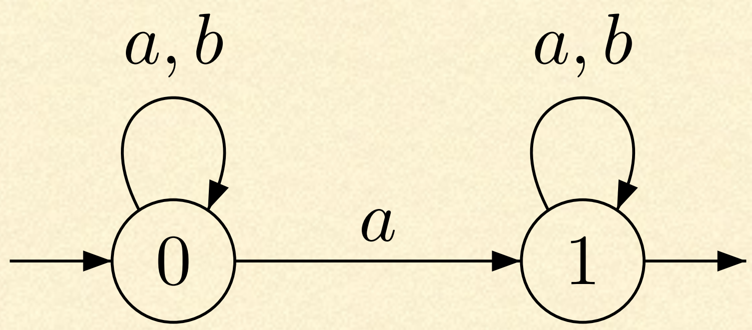
AUTOMATA = MSO

Goal: Find a weighted logic expressively equivalent to weighted automata

KEEP SYNTAX CHANGE SEMANTICS

- Inspired by weighted automata on semirings



  - Qualitative: existence of an accepting path
  - Quantitative: number of accepting paths

KEEP SYNTAX CHANGE SEMANTICS

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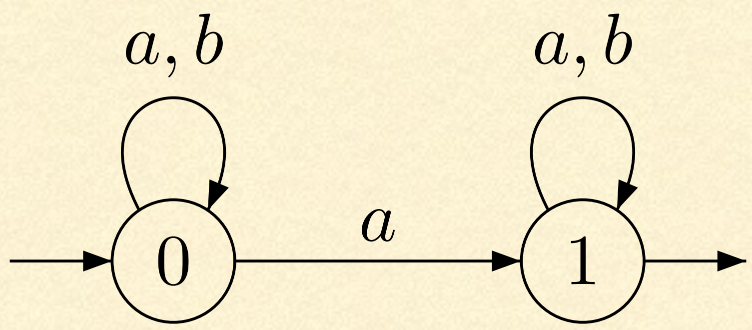

$$\mu(a) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \mu(b) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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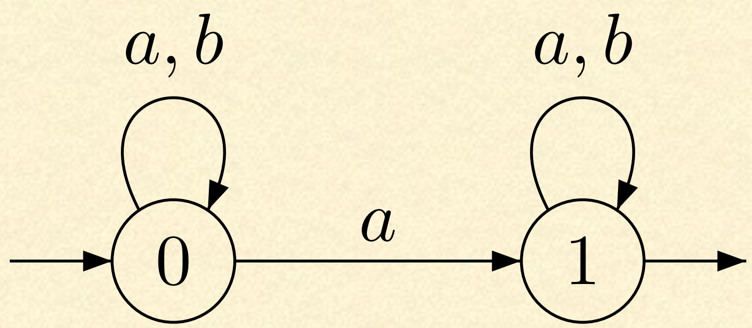

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- Qualitative: existence of an accepting path
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- Boolean semiring: $(\{0,1\}, \vee, \wedge, 0, 1)$ $\quad \mu(babaab) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

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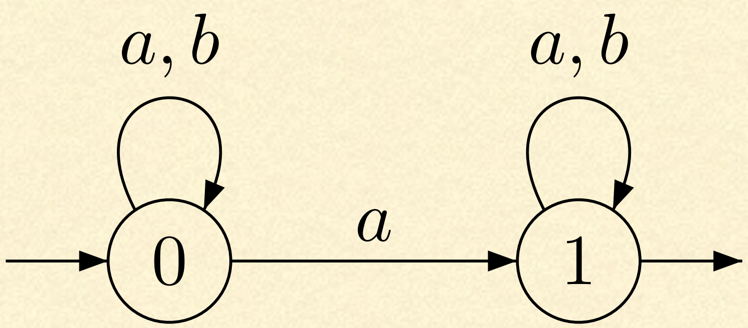

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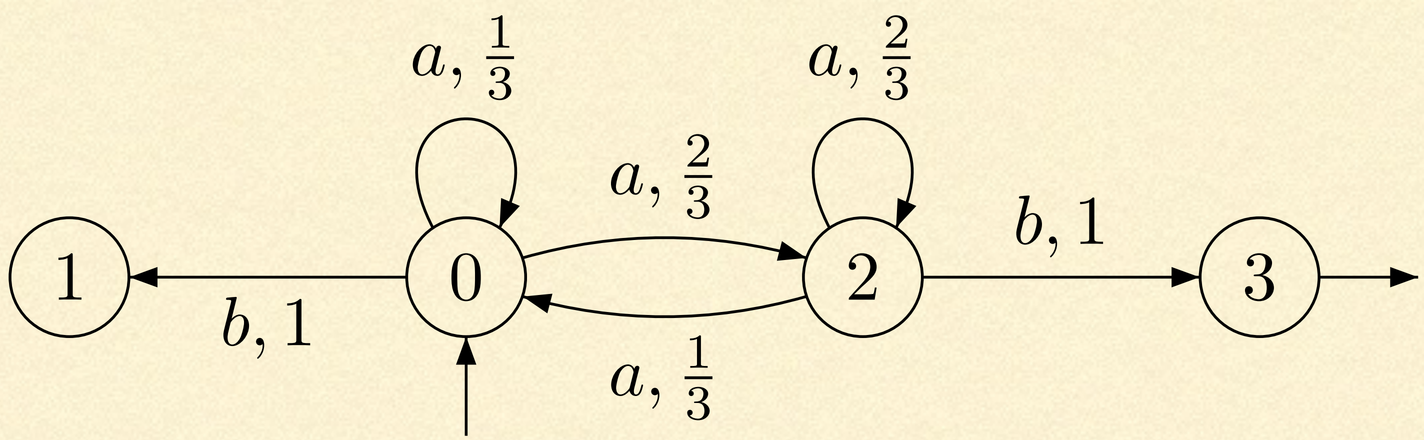
- Boolean semiring: $(\{0,1\}, \vee, \wedge, 0, 1)$ $\quad \mu(babaab) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
- Natural semiring: $(\mathbb{N}, +, \times, 0, 1)$ $\quad \mu(babaab) = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

WEIGHTED AUTOMATA ON SEMIRINGS

- Probabilistic semiring: $(\mathbb{R}_{\geqslant 0}, +, \times, 0, 1)$



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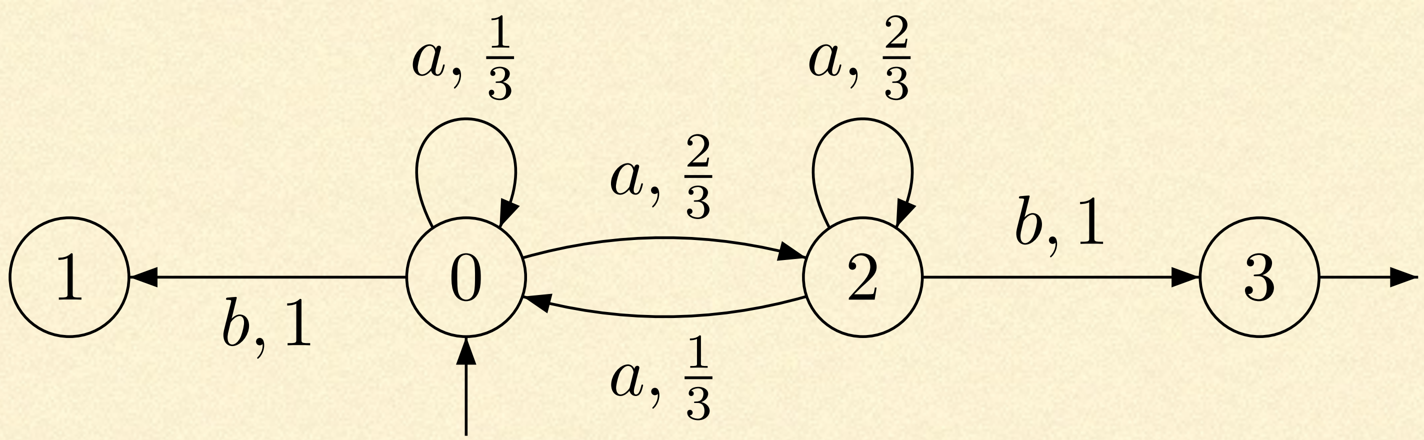
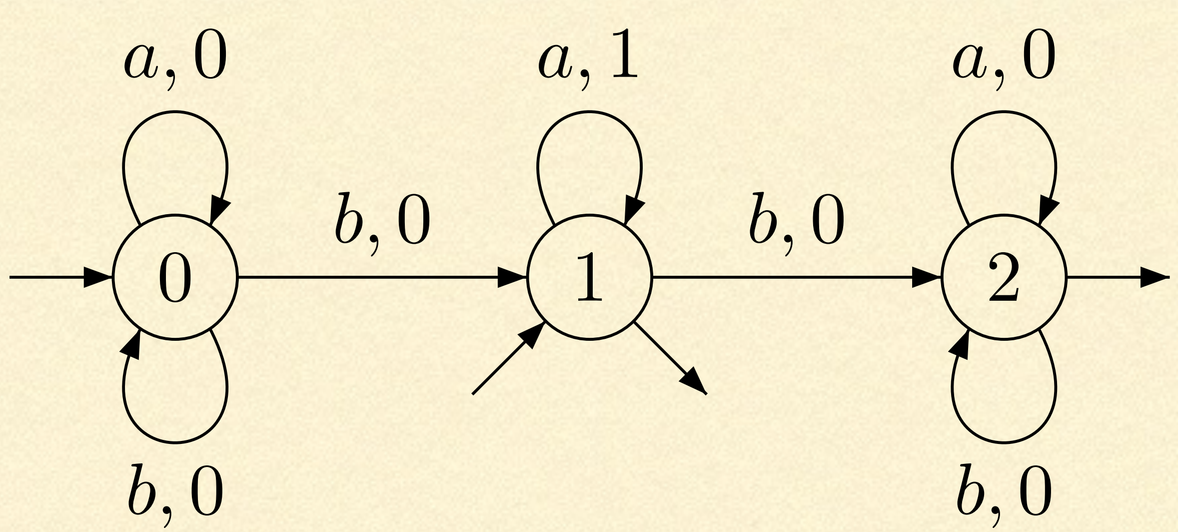


- (max,+) semiring: $(\mathbb{N} \cup \{-\infty\}, \max, +, -\infty, 0)$



THE FIRST SOLUTION

KEEP SYNTAX CHANGE SEMANTICS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

[Droste-Gastin, ICALP'05]

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Negation restricted to atomic formulae

[Droste-Gastin, ICALP'05]

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Arbitrary constants from a semiring

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- Semantics in a semiring $\mathbb{S} = (S, +, \times, 0, 1)$
  - Atomic formulae: 0,1
  - disjunction, existential quantifications: sum
  - conjunction, universal quantifications: product
- Inspired from the boolean semiring $\mathbb{B} = (\{0,1\}, \vee, \wedge, 0, 1)$

[Droste-Gastin, ICALP'05]

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- Examples

$$\varphi_1 = \exists x\, P_a(x)$$

$$[\![\varphi_1]\!](w) = |w|_a$$

[Droste-Gastin, ICALP'05]

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- Examples

$$\varphi_1 = \exists x\, P_a(x) \qquad \varphi_2 = \forall x\, \exists y\, (y \leqslant x \wedge P_a(y))$$

$$[\![\varphi_1]\!](w) = |w|_a \qquad [\![\varphi_2]\!](abaab) = 1 \times 1 \times 2 \times 3 \times 3$$

$$[\![\varphi_2]\!](a^n) = n!$$

[Droste-Gastin, ICALP'05]

KEEP SYNTAX CHANGE SEMANTICS

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$$[\![\varphi_1]\!](w) = |w|_a$$

$$\varphi_2 = \forall x\, \exists \ldots$$

$$\ldots 1 \times 2 \times 3 \times 3$$

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Too big to be computed by a weighted automaton

[Droste-Gastin, ICALP'05]

KEEP SYNTAX CHANGE SEMANTICS

$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P$ … $) \mid \neg(x \in X)$

$\mid \varphi \vee \varphi \mid$ …

We need to restrict weighted MSO

- Examples

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Too big to be computed by a weighted automaton

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Thm [Droste-Gastin] weighted automata = restricted wMSO

[Droste-Gastin, ICALP'05]

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φ almost boolean

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commutativity

φ almost boolean

Thm [Droste-Gastin] weighted automata = restricted wMSO

[Droste-Gastin, ICALP'05]

WEIGHTED AUTOMATA TO RESTRICTED WMSO

$$\exists \overline{X} = (X_1, \ldots, X_n) \ \mathsf{run}(\overline{X}) \land \forall x \, \mathsf{weight}(\overline{X}, x)$$

[Droste-Gastin, ICALP'05]

WEIGHTED AUTOMATA TO RESTRICTED WMSO

$$\exists \overline{X} = (X_1, \ldots, X_n)\ \mathsf{run}(\overline{X}) \wedge \forall x\, \mathsf{weight}(\overline{X}, x)$$

$$[\![\mathsf{run}(\overline{X})]\!](w, \sigma) = \begin{cases} 1 & \text{if } \overline{X} \text{ is a run} \\ 0 & \text{otherwise.} \end{cases}$$

[Droste-Gastin, ICALP'05]

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weight of the transition taken at x by run X

[Droste-Gastin, ICALP'05]

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sum over all runs

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weight of the transition taken at x by run X

[Droste-Gastin, ICALP'05]

WEIGHTED AUTOMATA TO RESTRICTED WMSO

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...nsition ...taken at x by run X

We need UNAMBIGUOUS formulas in the QUANTITATIVE semantics

[Droste-Gastin, ICALP'05]

UNAMBIGUOUS FORMULAS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

$$\exists x\, \exists y\ x < y \wedge P_a(x) \wedge P_b(y)$$

[Droste-Gastin, ICALP'05]

UNAMBIGUOUS FORMULAS

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$$\begin{aligned}\exists x\, \exists y\ x < y \wedge P_a(x) \wedge P_b(y) \\ \wedge \forall z\, (x \leqslant z \vee (z < x \wedge \neg P_a(z))) \\ \wedge \forall z\, (z \leqslant y \vee (y < z \wedge \neg P_b(z)))\end{aligned}$$

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$$\begin{aligned} &\exists x\, \exists y\;\, x < y \wedge P_a(x) \wedge P_b(y) \\ &\qquad \wedge \forall z\, (x \leqslant z \vee (z < x \wedge \neg P_a(z))) \\ &\qquad \wedge \forall z\, (z \leqslant y \vee (y < z \wedge \neg P_b(z))) \end{aligned}$$

$$\varphi \mapsto (\varphi^+, \varphi^-)$$

$$[\![\varphi^+]\!](w,\sigma) = \begin{cases} 1 & \text{if } w,\sigma \models \varphi \\ 0 & \text{otherwise.} \end{cases} \qquad [\![\varphi^-]\!](w,\sigma) = \begin{cases} 0 & \text{if } w,\sigma \models \varphi \\ 1 & \text{otherwise.} \end{cases}$$

[Droste-Gastin, ICALP'05]

RESTRICTED WMSO TO WEIGHTED AUTOMATA

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

φ almost boolean

- disjunction : closure under sum
- existential quantification : closure under projections/renamings
- universal quantification: main difficulty
    - φ almost boolean: finite image, pre-images are MSO-definable

MANY EXTENSIONS

$$\varphi ::= s \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg P_a(x) \mid \neg(x \leqslant y) \mid \neg(x \in X)$$
$$\mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x\, \varphi \mid \forall x\, \varphi \mid \exists X\, \varphi \mid \forall X\, \varphi$$

Thm: weighted automata = restricted wMSO

- Finite words: [Droste-Gastin, ICALP'05]
- Finite ranked trees: [Droste-Vogler, Theor. Comp. Sci.'06]
- Pictures: [Fichtner, STACS'06]
- Infinite words: [Droste-Rahonis, DLT'06]
- Finite unranked trees: [Droste-Vogler, Theor. of Computing Systems'09]
- Traces: [Fichtner-Kuske-Meinecke, Handbook of weighted automata '09]
- Nested words: [Mathissen, LMCS'10]
- …

CHANGING THE SYNTAX TOWARDS EASIER FORMULAE

CHANGING THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

[Bollig-Gastin, DLT'09] [Droste-Meinecke'12] [Fülöp-Stüber-Vogler'12]

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- No need to write unambiguous formulae

$$\exists x\, \exists y\ x < y \wedge P_a(x) \wedge P_b(y)$$

[Bollig-Gastin, DLT’09] [Droste-Meinecke’12] [Fülöp-Stüber-Vogler'12]

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$$\begin{aligned} \exists x\, \exists y\ x < y \wedge P_a(x) \wedge P_b(y) & \\ & \wedge \forall z\, (x \leqslant z \vee (z < x \wedge \neg P_a(z))) \\ & \wedge \forall z\, (z \leqslant y \vee (y < z \wedge \neg P_b(z))) \end{aligned}$$

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- No need to write unambiguous formulae

- Quantitative fragment

$$\Phi ::= s \mid \varphi \mid \Phi + \Phi \mid \Phi \times \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Phi$$

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- Formulae are more readable

$$\sum_x P_a(x)$$

$$\exists x\, P_a(x)$$

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$$\sum_x P_a(x) \qquad \prod_x \sum_y (y \leqslant x \wedge P_a(y))$$

$$\exists x\, P_a(x)$$

[Bollig-Gastin, DLT'09] [Droste-Meinecke'12] [Fülöp-Stüber-Vogler'12]

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$$\exists x\, P_a(x) \qquad \sum_x \forall y\, (y \leqslant x \to P_a(y))$$

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$\sum_x P_a(x)$ … $\leqslant x \wedge P_a(y))$

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We still need restrictions

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Φ has no Σ or Π

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- No nee[…] formulae

commutative

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Thm: weighted automata = restricted wMSO

BEYOND SEMIRINGS
AVERAGE, DISCOUNTED SUMS, …

FROM PRODUCTS TO VALUATIONS

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- We compute the weight of the run $\mathsf{Val}(s_1 s_2 \cdots s_n)$
- product $\mathsf{Val}(s_1 s_2 \cdots s_n) = s_1 \times s_2 \times \cdots \times s_n$
- average: $\mathsf{Val}(s_1 s_2 \cdots s_n) = \frac{s_1 + s_2 + \cdots + s_n}{n}$
- discounted: $\mathsf{Val}(s_1 s_2 \cdots s_n) = s_1 + \lambda s_2 + \cdots + \lambda^{n-1} s_n$

[Droste-Meinecke, CIAA'10, IC'12]

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Val: possibly non-associative, non distributive

[Droste-Meinecke, CIAA'10, IC'12]

FROM PRODUCTS TO VALUATIONS

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
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- Final semantics $$[\![\mathcal{A}]\!](w) = \sum_{\rho \text{ run on } w} \mathsf{Val}(\mathsf{wgt}(\rho))$$
- Valuation monoid $(S, +, 0, \mathsf{Val})$ $\quad \mathsf{Val}\colon S^+ \to S$

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- Semantics in a valuation monoid $(S, +, 0, \mathsf{Val})$

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$$[\![\prod_x \Phi]\!](w, \sigma) = \mathsf{Val}(([\![\Phi]\!](w, \sigma[x \mapsto i]))_i)$$

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Thm [Droste-Meinecke]
weighted automata = restricted wMSO

$$[\![\textstyle\prod_x \Phi]\!](w,\sigma) = \mathsf{Val}(([\![\Phi]\!](w,\sigma[x \mapsto i]))_i)$$

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EXTENDING THE SUM

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- We compute the weight of the run $\mathsf{Val}(s_1 s_2 \cdots s_n)$
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$$[\![\mathcal{A}]\!](w) = \mathsf{Average}\{\{\mathsf{Val}(\mathsf{wgt}(\rho)) \mid \rho \text{ run on } w\}\}$$

[Droste-Perevoshchikov, CSR'13]

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not associative

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- Valuation structure $(S, F, \mathsf{Val}) \quad F : \mathbb{N}\langle S \rangle \to S$

[Droste-Perevoshchikov, CSR'13]

ABSTRACT SEMANTICS
MULTISETS OF WEIGHT-STRUCTURES

MULTISETS OF WEIGHT STRUCTURES

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
- Abstract semantics $\{\!|\mathcal{A}|\!\}(w) = \{\!\{\mathsf{wgt}(\rho) \mid \rho \text{ run on } w\}\!\}$

multiset

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

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$$\{\!|\mathcal{A}|\!\} \colon \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$$

multiset

weights of A

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

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- Aggregation $\mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

MULTISETS OF WEIGHT STRUCTURES

Semiring: sum-product

$$\text{aggr}_{\texttt{sp}}(A) = \sum \prod A = \sum_{r_1 \cdots r_n \in A} r_1 \times \cdots \times r_n$$

$$\{\!|\mathcal{A}|\!\} : \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$$

multiset

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Valuation monoid: sum-valuation

$$\mathsf{aggr}_{\mathtt{sv}}(A) = \sum \mathsf{Val}(A) = \sum_{r_1 \cdots r_n \in A} \mathsf{Val}(r_1 \cdots r_n)$$

Valuation structure: F-valuation

$$\mathsf{aggr}_{\mathtt{fv}}(A) = F \circ \mathsf{Val}(A) = F(\{\{\mathsf{Val}(r_1 \cdots r_n) \mid r_1 \cdots r_n \in A\}\})$$

- Aggregation $\mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

MULTISETS OF WEIGHT STRUCTURES

- A run generates a sequence of weights $\mathsf{wgt}(\rho) = s_1 s_2 \cdots s_n$
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  multiset

  weights of A

- Aggregation $\mathsf{aggr} \colon \mathbb{N}\langle R^\star \rangle \to S$
- Concrete semantics $[\![\mathcal{A}]\!] = \mathsf{aggr} \circ \{\!|\mathcal{A}|\!\} \colon \Sigma^\star \to S$

[Gastin-Monmege '14][Droste-Perevoshchikov '14]

THE FINAL TOUCH
CHANGING AGAIN THE SYNTAX

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$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- [Gastin-Monmege, 14]

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- Step formulae $\Psi ::= s \mid \varphi \,?\, \Psi : \Psi$

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if … then … else …
no sum, no product

- [Gastin-Monmege, 14]

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$$P_a(x) \,?\, 1 : 0$$

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$$P_a(x)\,?\,1 : 0$$

$$P_a(x)\,?\,1 : (P_b(x)\,?\,-1 : 0)$$

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no sum, no product

$$P_a(x) \,?\, 1 : 0$$

$$P_a(x) \,?\, 1 : (P_b(x) \,?\, -1 : 0)$$

$$\varphi_1(x) \,?\, s_1 : (\varphi_2(x) \,?\, s_2 : \cdots (\varphi_{n-1}(x) \,?\, s_{n-1} : s_n) \cdots)$$

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$$P_a(x) \,?\, 1 : 0$$

$$P_a(x) \,?\, 1 : (P_b(x) \,?\, -1 : 0)$$

$$\varphi_1(x) \,?\, s_1 : (\varphi_2(x) \,?\, s_2 : \cdots (\varphi_{n-1}(x) \,?\, s_{n-1} : s_n) \cdots)$$

$$[\![\Psi]\!](w, \sigma) = s$$

some value occurring in Ψ

- [Gastin-Monmege, 14]

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$P_a(x) \,?\, 1 : 0$

$P_a($

$\varphi_1($ ... $(\varphi_2(x) \,?\, s_2 : \cdots (\varphi_{n-1}(x) \,?\, s_{n-1} : s_n) \cdots)$

se ...

duct

A step formula takes finitely many values

For each value, the pre-image is MSO-definable

$$[\![\Psi]\!](w, \sigma) = s$$

some value occurring in Ψ

- [Gastin-Monmege, 14]

CHANGING AGAIN THE SYNTAX

- Boolean fragment

$$\varphi ::= \top \mid P_a(x) \mid x \leqslant y \mid x \in X \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall x\, \varphi \mid \forall X\, \varphi$$

- Step formulae $\Psi ::= s \mid \varphi\, ?\, \Psi : \Psi$

- core wMSO $\Phi ::= \mathbf{0} \mid \varphi\, ?\, \Phi : \Phi \mid \Phi + \Phi \mid \sum_x \Phi \mid \sum_X \Phi \mid \prod_x \Psi$

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- Semantics

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sums over multiset

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Thm: weighted automata = core wMSO

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- Abstract semantics $\{\!| - |\!\} \colon \Sigma^\star \to \mathbb{N}\langle R^\star \rangle$
- Concrete semantics $[\![-]\!] = \mathsf{aggr} \circ \{\!| - |\!\} \colon \Sigma^\star \to S$

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Thm: weighted automata = core wMSO

- Abstract semantics $\{\!|\, - \,|\!\}: \Sigma^+ \ldots$
- Concrete $\ldots \to S$

Easy constructive proofs
preservation of the constants
no restriction on core wMSO
no hypotheses on weights

- [Gastin-Monmege, 14]

CONCLUDING REMARKS

- Similar approaches
  - M-expressions for multi-operator monoids [Fülöp-Stüber-Vogler'12]
  - Assignment logic [Perevoshchikov PhD'15]

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  - multisets of weight-structures
  - core-wMSO: only the boolean fragment changes

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  - Assignment logic [Perevoshchikov PhD'15]
- Other structures: ranked trees, unranked trees, …
  - multisets of weight-structures
  - core-wMSO: only the boolean fragment changes
- More operators in core-wMSO
  - any binary operation $\diamondsuit : S \times S \to S$ can be lifted to multisets of weight-structures and added to core-wMSO