TD 1: Models

Exercise 1 (Rendez-vous with Data)**.** Consider the synchronization of transition systems with variables through a rendez-vous mechanism. Such a system is of form $M = (S, \Sigma, \mathcal{V}, (D_v)_{v\in\mathcal{V}}, T, I, \mathrm{AP}, l)$ where $\mathcal{V}$ the set of (typed) variables v, each with domain D_v.

We want to extend the rendez-vous mechanism between systems with variables with the ability to exchange data values. For instance, a system M_i may transmit a value m by performing

$$s_i \xrightarrow{!m} s'_i \ ,$$

only if some system M_j is ready to receive the message, i.e. to perform

$$s_j \xrightarrow{?v} s'_j \ ,$$

where v is a variable of M_j and m is in D_v. Of course the synchronization is also possible if M_j performs instead

$$s_j \xrightarrow{?m} s'_j \ .$$

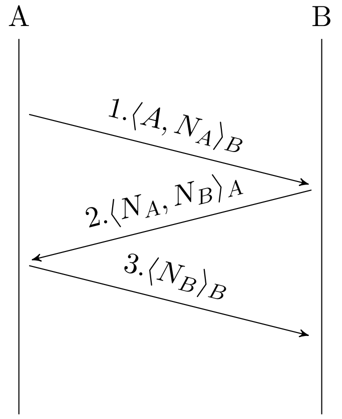
1. Propose Structural Operational Semantics for the rendez-vous with data synchronization.
2. Assume $D_v = D$ for all variables v in $\mathcal{V}$.

   Generalize these semantics to allow sending and receiving *terms* in $T(\Sigma, \mathcal{V})$ build from the variables and a finite set of symbols Σ that contains D.

Exercise 2 (Needham-Schroeder Protocol)**.** We consider the analysis of a public-key authentication protocol proposed by Needham and Schroeder in 1978. The protocol relies on

- the generation of *nounces* N_C: random numbers that should only be used in a single session, and
- on public key encryption: we denote the encryption of message M using C's public key by $\langle M \rangle_C$.

A(lice) and B(ob) try to make sure of each other's identity by the following (very simplified) exchange:



1. Alice first presents herself (the A part of the message) and challenges Bob with her nounce N_A. Assuming both cryptography and random number generation to be perfect, only Bob can decrypt $\langle A, N_A\rangle_B$ and find the correct number N_A.

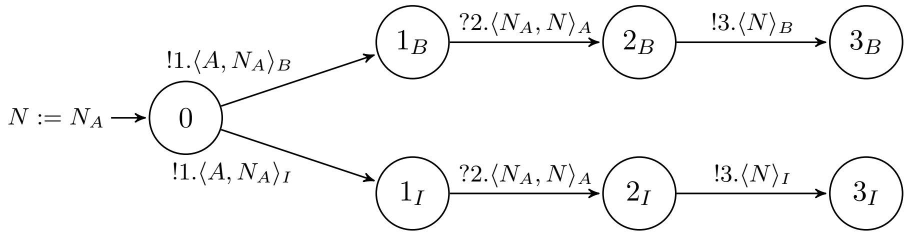
2. Bob responds by proving his identity (the N_A part) and challenges Alice with his own nounce N_B.

3. Finally, Alice proves her identity by sending N_B.

The nounces N_A and N_B are used by Alice and Bob as secret keys for their communications.

In order to account for the insecure channel, we have to add an intruder I to the model, who has his own nounce N_I, and can read and send any message it fancies, but can only decrypt $\langle M\rangle_I$ messages and cannot guess the nounces generated by Alice and Bob.

We can model the behaviour of Alice as a transition system M_A with variables and rendez-vous with data, using a single variable N ranging over $D_N = \{N_A, N_B, N_I\}$.



1. Provide a model M_B for Bob.

2. Provide a model M_I the intruder.

3. Unfold an execution path in the synchronized product of M_A, M_B, and M_I that unveils a flaw in the protocol.

Exercise 3 (Channel Systems)**.** The course notes present the semantics of FIFO channels. We consider here the case of a single finite system $M = \langle S, \Sigma, T, I, \mathrm{AP}, \ell\rangle$ along with n unbounded channels over a finite set Γ (i.e. each channel is declared as c_i: `channel[∞] of` Γ for each $1 \leq i \leq n$). Configurations of the full system $\hat{M}$ are thus in $S \times (\Gamma^*)^n$, i.e. of form $(s, \gamma_1, \dots, \gamma_n)$ where s is a state of S and channel i contains γ_i. Without loss of generality, we consider the channels to be empty in the initial configurations, i.e. $\hat{I} = \{(s_i, \varepsilon, \dots, \varepsilon) \mid s_i \in I\}$.

We are interested in the *control-state reachability problem*, i.e. given an n-channel system $\hat{M}$ and a state s, does there exist an initial state s_i in I and n strings $\gamma_1, \dots, \gamma_n$ in Γ^* s.t. $(s_i, \varepsilon, \dots, \varepsilon) \to^* (s, \gamma_1, \dots, \gamma_n)$?

1. Consider the case $\Gamma = \{a\}$ and $n = 1$. Show that the control-state reachability problem is decidable in PTime.

2. Show that it becomes undecidable for $n = 1$ and $|\Gamma| \geq 2$.

3. We allow the channel systems to test the contents of a channel for emptiness:

$$\frac{\nu(c_j) = \varepsilon \wedge s_i \xrightarrow{\text{empty}(c_j)} s'_i}{(\bar{s}, \nu) \xrightarrow{\text{empty}(c_j)} (\bar{s}', \nu)}$$

   Show that the control-state reachability problem is then undecidable for $n \geq 2$ even if $|\Gamma| = 1$. Hint: reduce from the control state reachability in 2-counters Minsky machines.