Homework 9

To hand in on December 6th at the beginning of the exercise session, or by mail (before 14:00) at marie.fortin@lsv.fr.

Answers can be written in french or in english.

Exercise 1. Draw the BDDs for the following functions, using the order of your choice on the variables $\{x_1, x_2, x_3\}$. You may omit the 0-node. No justification is necessary.

1. $(x_1 \Leftrightarrow x_2) \lor (x_1 \Leftrightarrow x_3),$ 2. $s(x_1, x_2, x_3) = \begin{cases} 1 & \text{if } x_1 + x_2 + x_3 = 1\\ 0 & \text{otherwise}. \end{cases}$

Exercise 2. Let x_1, \ldots, x_n , be Boolean variables, for some $n \ge 1$. We fix the ordering $x_1 < \cdots < x_n$. Given a function f, we let B(f) denote the number of nodes labelled with variables in the BDD for f. For instance, the figure below shows the BDD of $f := x_1 \Leftrightarrow x_3$, where we have B(f) = 3.



1. Let n = 2k for some $k \ge 1$, and

$$f(x_1, \dots, x_n) = \begin{cases} 1 & \text{if } \Sigma_{i=1}^k x_i = \Sigma_{i=k+1}^n x_i \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Draw the BDD for f for n = 4.
- (b) Let $1 \leq i \leq k$, and $\nu, \nu' : \{1, \ldots, i\} \to \{0, 1\}$ two possible valuations of variables x_1 to x_i . Show that ν and ν' lead to the same node in the BDD for f if and only if $\sum_{i=1}^{i} \nu(i) = \sum_{i=1}^{i} \nu'(i)$.
- (c) Give a similar condition for the case k < i < n.
- (d) Compute B(f), depending on k.

- 2. Depending on n, how many different functions f exist such that
 - (a) B(f) = 1?
 - (b) B(f) = 2?
 - (c) B(f) = 3?