Homework 8

To hand in on November 29th at the beginning of the exercise session, or by mail (before 14:00) at `marie.fortin@lsv.fr`.

Answers can be written in french or in english.

Reminder (conditions for ample sets):

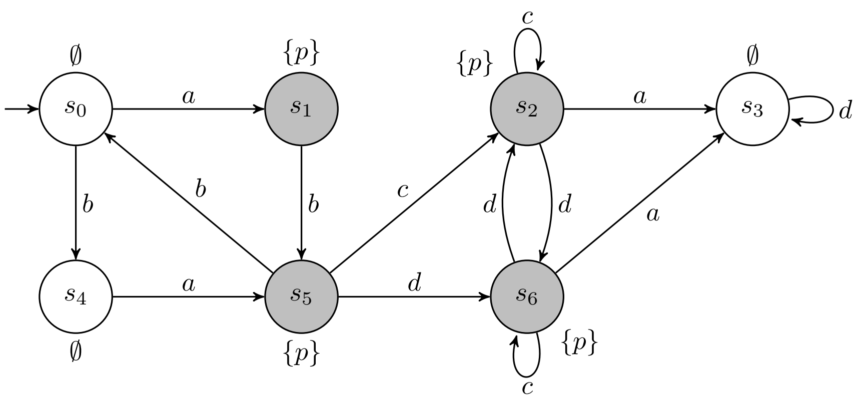
(C0) $red(s) = \emptyset$ iff $en(s) = \emptyset$.

(C1) For every path $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n \xrightarrow{a} t$ in $\mathcal{K}$ (for any $n \geq 0$), if $a \notin red(s)$ and a depends on some action in $red(s)$ (i.e. there exists $b \in red(s)$ such that $(a,b) \notin I$), then there exists $1 \leq i \leq n$ such that $a_i \in red(s)$.

(C2) If $red(s) \neq en(s)$, then all actions in $red(s)$ are invisible.

(C3) For all cycles in the reduced system $\mathcal{K}'$, the following holds: if $a \in en(s)$ for some state s in the cycle, then $a \in red(s')$ for some (possibly other) state s' in the cycle.

Exercise 1. Let $\mathcal{K} = (S, A, \rightarrow, s_0, \mathrm{AP}, \nu)$ be the Kripke structure below, with set of actions $A = \{a, b, c, d\}$ and atomic propositions $\mathrm{AP} = \{p\}$.



1. Compute the maximal independance relation I and the maximal set of invisible actions U. Justify your answers.

2. Give sets $red(s) \subseteq en(s)$ satisfying conditions C0 to C3, and such that for all state s, no action can be removed from $red(s)$ without breaking one of conditions C0 to C3. Justify your answer.

3. Draw the reduced system $\mathcal{K}'$ associated with your assignment red, after removing unreachable states. Is there a smaller system $\mathcal{K}''$, obtained by removing additional transitions, that is stuttering equivalent to $\mathcal{K}$?

Exercise 2. Consider the condition (C_1'): for any s with $red(s) \neq en(s)$, any a in $red(s)$ is independent from every b in $en(s) \backslash red(s)$.

1. Show that (C_1) implies (C_1').
2. Show that $(C_0), (C_1'), (C_2), (C_3)$ are not sufficient to ensure stuttering equivalence, i.e., that there exists a Kripke structure $\mathcal{K}$ and an assignment *red* satisfying conditions $(C_0), (C_1'), (C_2), (C_3)$ but such that the reduced system $\mathcal{K}'$ induced by *red* is not stuttering equivalent to $\mathcal{K}$.