Exercise 1 (Complexity of LTL($X$)). We want to show that LTL($X$) existential model checking is NP-complete (instead of PSPACE-complete for full LTL($SU$)).

1. Given $\varphi \in \text{LTL}(X)$, the *temporal depth* of $\varphi$ is defined as follows:

\[
\begin{align*}
    d(\top) = d(p) &= 0 & d(\neg \varphi) &= d(\varphi) \\
    d(\varphi \lor \varphi') &= \max\{d(\varphi), d(\varphi')\} & d(X \varphi) &= 1 + d(\varphi)
\end{align*}
\]

Show by induction on $\varphi$ that for all $\varphi \in \text{LTL}(X)$ and $w \in \Sigma^\omega$, if $u$ is the prefix of length $d(\varphi) + 1$ of $w$, we have $w, 0 \models \varphi$ iff $u\emptyset^\omega, 0 \models \varphi$.

2. Show that $\text{MC}_3(X)$ is in NP:

   Input: $\varphi \in \text{LTL}(X)$ and a finite Kripke structure $M$.

   Question: Does $M \models_3 \varphi$?

3. Reduce 3SAT to $\text{MC}_3(X)$ in order to prove NP-hardness.