## Homework 6

To hand in on November 8th during the partial exam, or by mail (before 14:00) at marie.fortin@lsv.fr.

Answers can be written in french or in english.

**Exercise 1** (Complexity of LTL(X)). We want to show that LTL(X) existential model checking is NP-complete (instead of PSPACE-complete for full LTL(SU)).

1. Given  $\varphi \in LTL(X)$ , the *temporal depth* of  $\varphi$  is defined as follows:

$d(\top) = d(p) = 0$	$d(\neg \varphi) = d(\varphi)$
$d(\varphi \lor \varphi') = \max\{d(\varphi), d(\varphi')\}$	$d(X\varphi) = 1 + d(\varphi)$

Show by induction on  $\varphi$  that for all  $\varphi \in LTL(X)$  and  $w \in \Sigma^{\omega}$ , if u is the prefix of length  $d(\varphi) + 1$  of w, we have  $w, 0 \models \varphi$  iff  $u \emptyset^{\omega}, 0 \models \varphi$ .

- 2. Show that  $MC^{\exists}(X)$  is in NP: Input:  $\varphi \in LTL(X)$  and a finite Kripke structure M. Question: Does  $M \models_{\exists} \varphi$ ?
- 3. Reduce 3SAT to  $MC^{\exists}(X)$  in order to prove NP-hardness.