Homework 4

To hand in on October 18th at the beginning of the exercise session, or by mail (before 14:00) at `marie.fortin@lsv.fr`.

Answers can be written in french or in english.

Exercise 1. Let $\mathrm{AP} = \{p, q\}$, and $\Sigma = 2^{\mathrm{AP}} = \{a, b, c, d\}$, where $a = \{p\}$, $b = \{q\}$, $c = \{p, q\}$, and $d = \emptyset$. We identify each letter in Σ with a boolean formula over AP, for instance, $a = p \wedge \neg q$.

For each LTL formula φ below, give a Büchi automaton acccepting the language $L(\varphi) = \{w \in \Sigma^\omega \mid w, 0 \models \varphi\}$.

1. $\mathsf{G}(a \rightarrow (\neg a \,\mathsf{SU}\, b))$
2. $(\mathsf{G}\,\mathsf{F}\, a) \wedge (\mathsf{F}\, b) \wedge \neg(\mathsf{G}\,\mathsf{F}\, c)$
3. $(\mathsf{F}\,\mathsf{G}\, a) \rightarrow (\mathsf{G}\,\mathsf{F}\, b)$
4. $G((a \vee b)\,\mathsf{U}\,(c \vee d))$

Exercise 2. A Büchi automaton $\mathcal{A} = (Q, \Sigma, I, T, F)$ is *deterministic* if $|I| \leq 1$, and for each state q in Q and symbol a in Σ, $|\{(q, a, q') \in T \mid q' \in Q\}| \leq 1$.

1. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under intersection.
2. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under union.