Homework 3

To hand in on October 11th at the beginning of the exercise session, or by mail (before 14:00) at marie.fortin@lsv.fr.

Answers can be written in french or in english.

Exercise 1. Let M be the Kripke structure below. For a state formula φ in CTL^{*}, we let $\llbracket \varphi \rrbracket = \llbracket \varphi \rrbracket^M = \{s \in \{1, \ldots, 8\} \mid M, s \models \varphi\}.$



Compute the following sets. Give a short explanation.

- 1. **[**EG *r*]
- 2. $[\![AX q]\!]$
- 3. $\llbracket \varphi_1 \rrbracket$ where $\varphi_1 = (\mathsf{EG} r) \lor (\neg q \land \mathsf{EX} q)$
- 4. $\llbracket \mathsf{E} \psi \rrbracket$ where $\psi = \mathsf{GF} \varphi_1 \to \mathsf{GF}(q \land \neg r)$.

Exercise 2. Recall that two state formulæ ψ and ψ' in CTL^{*} are equivalent if for all models M and all states s of M, we have $M, s \models \psi$ if and only if $M, s \models \psi'$.

1. Consider the CTL^{*} formulæ $\psi_1 = \mathsf{AF}(p \wedge \mathsf{G} q)$ and $\psi_2 = \mathsf{AF}(p \wedge \mathsf{AG} q)$. Show that ψ_2 implies ψ_1 , i.e., for all models M and all states s of M, we have

 $M, s \models \psi_2$ implies $M, s \models \psi_1$.

Show that the formulæ ψ_1 and ψ_2 are not equivalent by giving a model M with at most 4 states and a state s of M such that $M, s \models \psi_1$ but $M, s \not\models \psi_2$.

- 2. Prove that the CTL^{*} formula $\psi_3 = \mathsf{E}(p \mathsf{U}(q \mathsf{U} r))$ can be expressed in CTL, i.e., there exists a CTL formula ψ'_3 which is equivalent to ψ_3 .
- 3. Prove that the CTL^{*} formula $\psi_4 = \mathsf{E}((p \cup q) \land (r \cup s))$ can be expressed in CTL, i.e., there exists a CTL formula ψ'_4 which is equivalent to ψ_4 .