Homework 2

To hand in on October 4th at the beginning of the exercise session, or by mail (before 14:00) at marie.fortin@lsv.fr.

Exercise 1 (Equivalences). We fix a set of atomic propositions AP including $\{p, q\}$, and the time flow $(\mathbb{N}, <)$. With of the following equivalences are correct? Give a proof or a counter-example.

- 1. $(X p) \land (X q) \equiv X(p \land q)$
- 2. $(\mathsf{SF}\,p) \land (\mathsf{SF}\,q) \equiv \mathsf{SF}(p \land q)$
- 3. $(\mathbf{Y} p) \mathbf{S} (\mathbf{Y} q) \equiv \mathbf{Y}(p \mathbf{S} q)$
- 4. $(\operatorname{\mathsf{G}} p) \operatorname{\mathsf{U}} (\operatorname{\mathsf{G}} q) \equiv \operatorname{\mathsf{G}} (p \operatorname{\mathsf{U}} \operatorname{\mathsf{G}} q)$
- 5. $(X p) U q \equiv X(p U (p \land q))$
- 6. $(p \cup q) \cup q \equiv p \cup q$
- 7. $(\mathsf{GF} p) \to (\mathsf{GF} q) \equiv \mathsf{G}(p \to \mathsf{F} q)$
- 8. $\mathsf{G} p \to \mathsf{F} q \equiv p \mathsf{U} (q \lor \neg p)$

Exercise 2 (Specification). We fix a set of propositions $AP = \{ok, crash, alarm, reset\}$ and the time flow ($\mathbb{N}, <$). Provide formulæ for the following properties (a) in FO(AP, <), (b) in TL(AP, SU, SS) (your formula should use past modalities), and (c) in TL(AP, SU). For cases (b) and (c), your formula φ must be such that a temporal structure w satisfies the property described if and only if $w, 0 \models \varphi$.

- 1. "Whenever the alarm rings, there has been a crash immediately before."
- 2. "Whenever the alarm rings, there has been a crash some time before, and no reset in the meantime."