Homework 2

To hand in on October 4th at the beginning of the exercise session, or by mail (before 14:00) at marie.fortin@lsv.fr.

Exercise 1 (Equivalences). We fix a set of atomic propositions \( AP \) including \( \{p, q\} \), and the time flow \((\mathbb{N}, <)\). Which of the following equivalences are correct? Give a proof or a counter-example.

1. \( (\text{X} p) \land (\text{X} q) \equiv \text{X}(p \land q) \)
2. \( (\text{SF} p) \land (\text{SF} q) \equiv \text{SF}(p \land q) \)
3. \( (\text{Y} p) \text{S} (\text{Y} q) \equiv \text{Y}(p \text{S} q) \)
4. \( (\text{G} p) \text{U} (\text{G} q) \equiv \text{G}(p \text{U} q) \)
5. \( (\text{X} p) \text{U} q \equiv \text{X}(p \text{U} (p \land q)) \)
6. \( (p \text{U} q) \text{U} q \equiv p \text{U} q \)
7. \( (\text{G} \text{F} p) \rightarrow (\text{G} \text{F} q) \equiv \text{G}(p \rightarrow q) \)
8. \( \text{G} p \rightarrow \text{F} q \equiv p \text{U} (q \lor \neg p) \)

Exercise 2 (Specification). We fix a set of propositions \( AP = \{\text{ok, crash, alarm, reset}\} \) and the time flow \((\mathbb{N}, <)\). Provide formulæ for the following properties (a) in FO(\( AP, < \)), (b) in TL(\( AP, SU, SS \)) (your formula should use past modalities), and (c) in TL(\( AP, SU \)). For cases (b) and (c), your formula \( \varphi \) must be such that a temporal structure \( w \) satisfies the property described if and only if \( w, 0 \models \varphi \).

1. “Whenever the alarm rings, there has been a crash immediately before.”
2. “Whenever the alarm rings, there has been a crash some time before, and no reset in the meantime.”