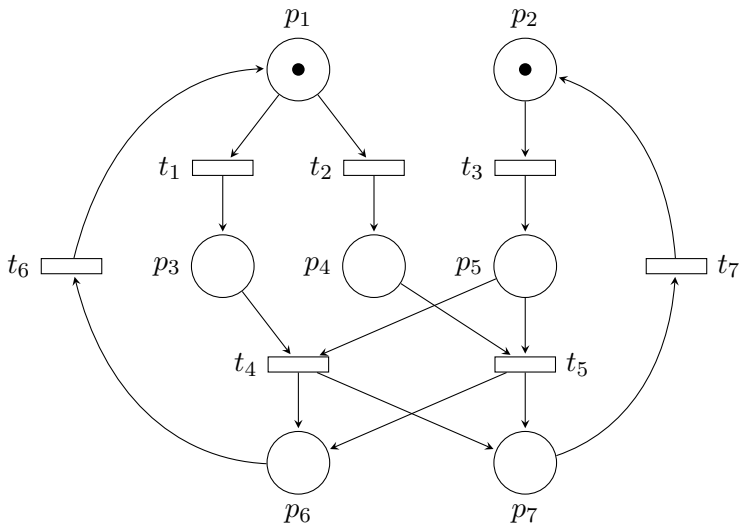
Homework 12

To hand in on January 10th at the beginning of the exercise session, or by mail (before 14:00) at `marie.fortin@lsv.fr`.

Answers can be written in french or in english.

Exercise 1. Give a finite complete prefix of the unfolding of the Petri net below. Indicate the order in which events are added, and the marking m_e associated with each event e.



Exercise 2.

1. Let $\mathcal{N} = \langle P, T, F, W, m_0 \rangle$ be Petri net, G its coverability graph, and m some marking in $\mathbb{N}^P$. The *effect* $\Delta(u)$ of a transition sequence u in T^* is defined by $\Delta(\varepsilon) = 0^P$ and $\Delta(ut) = \Delta(u) - W(P,t) + W(t,P)$.

   We want to show that the following are equivalent:

   (i) there exists an infinite execution $m_0 \xrightarrow{t_0}_{\mathcal{N}} m_1 \xrightarrow{t_1}_{\mathcal{N}} m_2 \cdots$ of $\mathcal{N}$ such that $m \leq m_i$ for infinitely many indices i;

   (ii) there exists an accessible loop $m' \xrightarrow{v}_G m'$ in G such that $v \in T^+$, $m \leq m'$ and $\Delta(v) \geq 0^P$.

   (a) Suppose that we have an execution as in (i). Show that there exists an infinite sequence $j_0 < j_1 < \cdots$ such that for all $k \in \mathbb{N}$ and $v_k = t_{j_k} t_{j_k+1} \dots t_{j_{k+1}-1}$, we have $\Delta(v_k) \geq 0^P$.
   Hint: use the fact that $(\mathbb{N}^P, \leq)$ is a wqo (cf. Exercise 1 of TD 11).

(b) Let $m_0 \xrightarrow{t_0}_G \hat{m}_1 \xrightarrow{t_1}_G \hat{m}_2 \cdots$ the path in G associated with the execution from (i): by construction, $m_i \leq \hat{m}_i$ for all i. Show that this path contains a loop satisfying the conditions of (ii).

(c) Prove that (ii) implies (i).

Hint: consider a marking $m_1 \in \mathbb{N}^P$ such that $m \leq m_1 \leq m'$ and v is enabled from m_1.

2. Let AP be a finite set of atomic propositions, and $\Sigma = 2^{\mathrm{AP}}$. We now consider labeled Petri nets $\mathcal{N} = \langle P, T, F, W, m_0, \lambda \rangle$, where the labeling function $\lambda : T \to \Sigma$ maps each transition to a set of atomic propositions. We define the language $L(\mathcal{N})$ of $\mathcal{N}$ as the set of infinite words $a_0 a_1 \cdots$ in Σ^ω such that there exists an execution $m_0 \xrightarrow{t_0}_{\mathcal{N}} m_1 \xrightarrow{t_1}_{\mathcal{N}} m_2 \cdots$ of $\mathcal{N}$ with $\lambda(t_i) = a_i$ for all i.

   Let $\mathcal{A} = \langle Q, \Sigma, \delta, q_0, Q_f \rangle$ be a Büchi automaton. Show that one can construct an (unlabeled) Petri net $\mathcal{N}'$ with places $P \uplus Q$ such that $L(\mathcal{A}) \cap L(\mathcal{N}) \neq \emptyset$ if and only if there exists an infinite computation of $\mathcal{N}'$ that covers some place in Q_f infinitely often.

3. Show that action-based LTL model-checking is decidable for labeled Petri nets. The problem asks, given a labeled Petri net $\mathcal{N}$ and an LTL formula φ, whether $\sigma \models \varphi$ for all $\sigma \in L(\mathcal{N})$.