Homework 10

To hand in on December 13th at the beginning of the exercise session, or by mail (before 14:00) at marie.fortin@lsv.fr.

Answers can be written in french or in english.

Exercise 1. Let $\mathcal{N}$ be the following Petri net:

1. Draw the reachability graph of $\mathcal{N}$. A marking $m$ will be denoted by the tuple $\langle m(p_1), m(p_2), m(p_3), m(p_4) \rangle$, for instance the initial marking is $\langle 1, 0, 0, 1 \rangle$.

2. Is $\mathcal{N}$ 1-safe ? 2-safe ? 3-safe ?

Exercise 2. We say that a Petri net $\mathcal{N} = \langle P, T, W, m_0 \rangle$ is acyclic if the directed graph $G_\mathcal{N} = (P \cup T, F)$ does not contain any cycle. Let $\mathcal{A}$ denote the class of Petri nets that are 1-safe and acyclic.

1. Let $\mathcal{N} = \langle P, T, F, m_0 \rangle$ be a Petri net.

   (a) Show that if $m \xrightarrow{t_1} m_1 \xrightarrow{t_2} m'$ in $\mathcal{N}$ and $t_1 \bullet \cap t_2 = \emptyset$, then there exists a marking $m_2$ such that $m \xrightarrow{t_2} m_2 \xrightarrow{t_1} m'$.

   (b) Let $m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \cdots \xrightarrow{t_k} m_{k+1}$ be an execution in $\mathcal{N}$ for some $k > 1$. Assume that for all $1 < i < k$, there exists a nonempty path from $t_1$ to $t_i$ in the graph $G_\mathcal{N}$, and that there is no nonempty path from $t_1$ to $t_k$ in $G_\mathcal{N}$. Show that there exists an execution $m_1 \xrightarrow{t_k} m'_1 \xrightarrow{t_1} m'_2 \xrightarrow{t_2} \cdots \xrightarrow{t_{k-1}} m'_k = m_{k+1}$.
2. Let \( \mathcal{N} = \langle P, T, F, W, m_0 \rangle \) be a 1-safe, acyclic Petri net. We assume that for all \( t \in T, t \neq \emptyset \) or \( t^* \neq \emptyset \).

(a) Show that there is no reachable marking \( m \) from which some transition \( t \) can fire twice, i.e. that there are no \( t, m, m', m'' \) such that \( m \) is reachable from \( m_0 \) and \( m \xrightarrow{t} m' \xrightarrow{t} m'' \).

(b) Show that for all executions \( m_0 \xrightarrow{t_1} m_1 \xrightarrow{t_2} \cdots \xrightarrow{t_n} m_n \) in \( \mathcal{N} \), we have \( t_i \neq t_j \) for all \( i \neq j \).

3. Show that the reachability problem for the class \( \mathcal{A} \) is in NP.