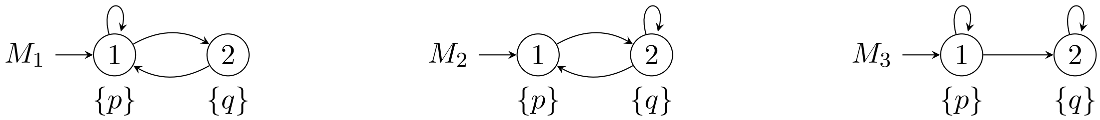
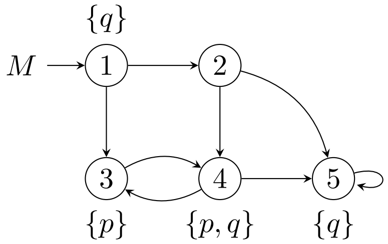
TD 3: CTL, CTL*

Exercise 1 (CTL*)**.** For each model and each formula, say if the model satisfies the formula.



1. $\phi_1 = \mathsf{AGF}q$
2. $\phi_2 = \mathsf{EGF}q$
3. $\phi_3 = \mathsf{AFEG}p$
4. $\phi_4 = \mathsf{AGEF}q$

Exercise 2 (Semantics of CTL*)**.** Compute $[\![\varphi]\!]$, where:



$\varphi = \mathsf{A}\big[(\mathsf{X}q) \lor \mathsf{FA}\big((\mathsf{EFG}p)\mathsf{U}(\mathsf{AG}q)\big)\big]$

Exercise 3 (CTL Model-Checking)**.** Let $M = (S, T, I, \mathrm{AP}, \ell)$ be a finite Kripke structure, and φ a CTL formula.

1. Let M_φ be the restriction of M to states satisfying φ: $M_\varphi = ([\![\varphi]\!], T \cap [\![\varphi]\!]^2, I \cap [\![\varphi]\!], \mathrm{AP}, \ell_{|[\![\varphi]\!]})$.

   Show that $s \in [\![\mathsf{EG}\,\varphi]\!]$ iff there exists a non-trival strongly connected component C of M_φ and $t \in C$ such that $s \to^* t$ in M_φ.

2. Deduce an algorithm to compute $[\![\mathsf{EG}\,\varphi]\!]$ from M and $[\![\varphi]\!]$. What is the complexity of your procedure?

Exercise 4 (CTL Equivalences)**.**

1. Are the two formulæ $\varphi = \mathsf{A}\,\mathsf{G}(\mathsf{E}\,\mathsf{F}\,p)$ and $\psi = \mathsf{E}\,\mathsf{F}\,p$ equivalent? Does one imply the other?

2. Same questions for $\varphi = \mathsf{E}\,\mathsf{G}\,q \vee (\mathsf{E}\,\mathsf{G}\,p \wedge \mathsf{E}\,\mathsf{F}\,q)$ and $\psi = \mathsf{E}(p \,\mathsf{U}\, q)$.

Exercise 5 (CTL$^+$)**.** CTL$^+$ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

$$f ::= \top \mid a \mid f \wedge g \mid \neg f \mid \mathsf{E}\,\varphi \mid \mathsf{A}\,\varphi \qquad \text{(state formulæ } f, g)$$
$$\varphi ::= \varphi \wedge \psi \mid \neg\varphi \mid \mathsf{X}\,f \mid f \,\mathsf{U}\, g \qquad \text{(path formulæ } \varphi, \psi)$$

where a is an atomic proposition. The associated semantics is that of CTL*.

We want to prove that, for any CTL$^+$ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for

$$\mathsf{E}((a_1 \,\mathsf{U}\, b_1) \wedge (a_2 \,\mathsf{U}\, b_2)) \; .$$

2. Generalize your translation for any formula of form

$$\mathsf{E}\left(\bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\,\varphi\right) \; . \tag{1}$$

What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL$^+$ formula:

$$\mathsf{E}(\mathsf{X}\,a \wedge (b \,\mathsf{U}\, c)) \; .$$

4. Using subformulæ of form (1) and E modalities, give an equivalent CTL formula to

$$\mathsf{E}(\mathsf{X}\,\varphi \wedge \bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\,\varphi') \; . \tag{2}$$

What is the complexity of your translation?

5. We only have to transform any CTL$^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:

$$\mathsf{A}((\mathsf{F}\,a \vee \mathsf{X}\,a \vee \mathsf{X}\,\neg b \vee \mathsf{F}\,\neg d) \wedge (d \,\mathsf{U}\, \neg c)) \; .$$