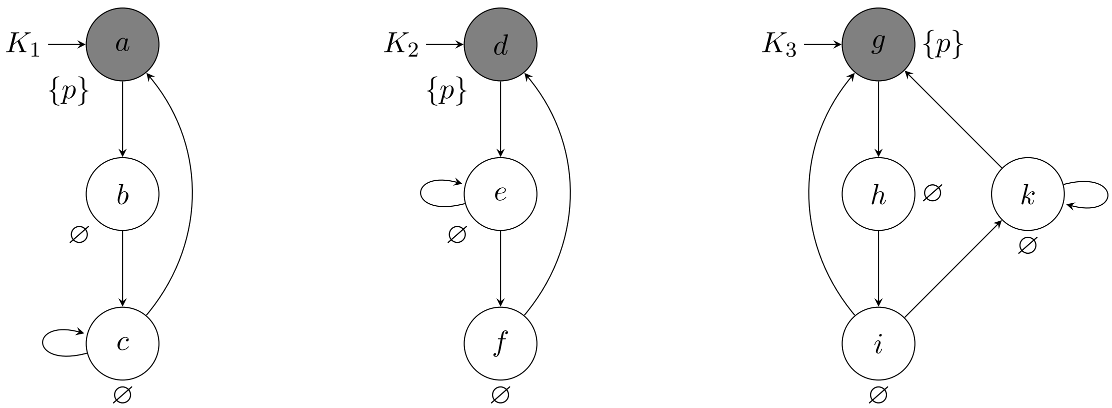
Homework 9

To hand in on December 14th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

Exercise 1. For each pair $(\mathcal{K}_i, \mathcal{K}_j)$ of Kripke structures below, determine whether $K_i \preccurlyeq K_j$, $\mathcal{K}_j \preccurlyeq K_i$, $K_i \equiv K_j$. In each case, give a (bi)simulation or a proof that none exists.



Exercise 2. Let AP be a finite set of atomic propositions, and $\mathcal{K}_1 = (S, \rightarrow_1, s_0, \mathrm{AP}, \nu)$, $\mathcal{K}_2 = (T, \rightarrow_2, t_0, \mathrm{AP}, \mu)$ two finite Kripke structures without deadlocks.

1. Given $s \in S$ and $t \in T$, we write $s \equiv_{\text{CTL}} t$ when for all CTL formulas φ, we have $\mathcal{K}_1, s \models \varphi$ iff $\mathcal{K}_2, t \models \varphi$.

   (a) Show that if $s \equiv_{\text{CTL}} t$, then $\nu(s) = \mu(t)$.

   (b) Let $s, s' \in S$ such that $s \rightarrow_1 s'$ and $t \in T$. Let $t'_1, \ldots, t'_n$ be the successors of t, i.e. $\{t'_1, \ldots, t'_n\} = \{t' \in T \mid t \rightarrow t'\}$. Assume that for all $1 \leq i \leq n$, $s' \not\equiv_{\text{CTL}} t'_i$. Show that there exists a CTL formula φ such that $\mathcal{K}_1, s \models \varphi$ and $\mathcal{K}_2, t \not\models \varphi$.

   (c) Assume that $s_0 \equiv_{\text{CTL}} t_0$. Show that $\{(s,t) \in S \times T \mid s \equiv_{\text{CTL}} t\}$ is a bisimulation between $\mathcal{K}_1$ and $\mathcal{K}_2$.

2. Assume that there exists a bisimulation $\mathcal{R}$ between $\mathcal{K}_1$ and $\mathcal{K}_2$. We write $s \mathrel{\mathcal{R}} t$ when $(s, r) \in \mathcal{R}$. Given two infinite paths $\sigma = s_1, s_2, \ldots$ and $\tau = t_1, t_2, \ldots$ in $\mathcal{K}_1$ and $\mathcal{K}_2$, we write $\sigma \mathrel{\mathcal{R}} \tau$ when for all $i \geq 1$, $s_i \mathrel{\mathcal{R}} t_i$.

   (a) Let $s \in S$ and $t \in T$ such that $s \mathrel{\mathcal{R}} t$, and $\sigma = s, s_2, \ldots$ a path starting at s in $\mathcal{K}_1$. Show that there exists an infinite path τ in $\mathcal{K}_2$ starting at t such that $\sigma \mathrel{\mathcal{R}} \tau$.

(b) Recall the syntax of CTL*:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \,\mathsf{SU}\, \varphi \mid \mathsf{E}\,\varphi$$

Show by induction on φ that for all CTL* formulas φ, the following holds:

$$\forall \sigma, \tau \text{ such that } \sigma \mathrel{\mathcal{R}} \tau,\ \forall i \geq 0, \quad \mathcal{K}_1, \sigma, i \models \varphi \text{ iff } \mathcal{K}_2, \tau, i \models \varphi\,.$$

3. Show that the following are equivalent:

   (a) $\mathcal{K}_1$ and $\mathcal{K}_2$ are bisimilar.

   (b) For all CTL formulas φ, $\mathcal{K}_1 \models \varphi$ iff $\mathcal{K}_2 \models \varphi$.

   (c) For all CTL* formulas φ, $\mathcal{K}_1 \models_\forall \varphi$ iff $\mathcal{K}_2 \models_\forall \varphi$.