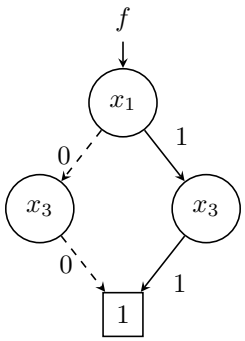
Homework 8

To hand in on December 7th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

Exercise 1. Draw the BDDs for the following functions, using the order of your choice on the variables $\{x_1, \ldots, x_5\}$. You may omit the 0-node. No justification is necessary.

1. $x_2 \Rightarrow (x_1 \wedge x_3)$
2. $\neg x_1 \wedge (x_2 \Leftrightarrow \neg x_4) \wedge (x_3 \Leftrightarrow x_5)$
3. $f(x_1, \ldots, x_5) = \begin{cases} 1 & \text{if } x_1 + \cdots + x_5 > 3 \\ 0 & \text{otherwise} \end{cases}$

Exercise 2. Let $x_1, \ldots, x_n$, be Boolean variables, for some $n \geq 1$. We fix the ordering $x_1 < \cdots < x_n$. Given a function f, we let $B(f)$ denote the number of nodes labelled with variables in the BDD for f. For instance, the figure below shows the BDD of $f := x_1 \Leftrightarrow x_3$, where we have $B(f) = 3$.



1. Let f_p be the parity function, i.e. f_p is 1 iff there is an even number of variables with value 1 among $x_1, \ldots, x_n$.
   (a) Draw the BDD for f_p for $n = 3$.
   (b) Compute $B(f_p)$, depending on n.
2. Depending on n, how many different functions f exist such that
   (a) $B(f) = 1$?
   (b) $B(f) = 2$?