# Homework 7

To hand in on November 23th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

**Reminder (conditions for ample sets):**

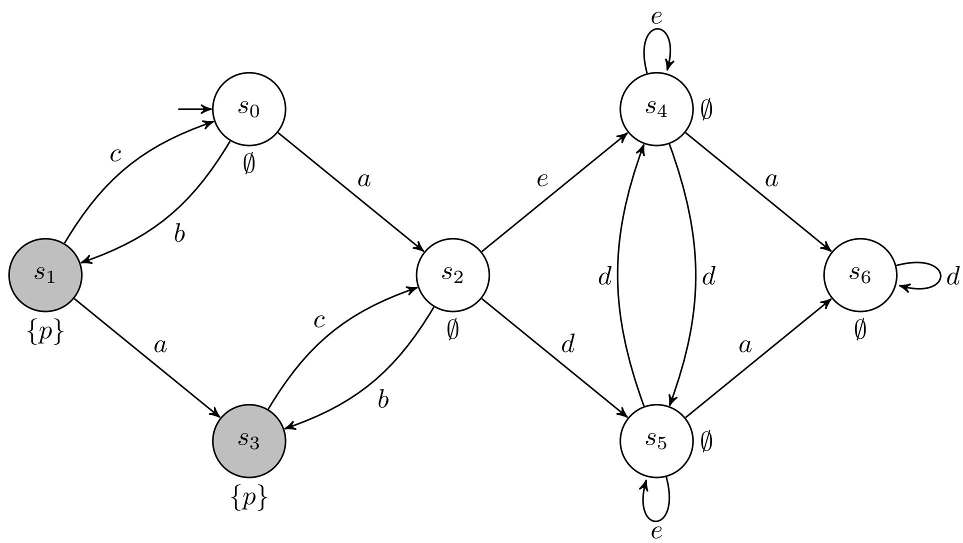
(C0) $red(s) = \emptyset$ iff $en(s) = \emptyset$.

(C1) For every path $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n \xrightarrow{a} t$ in $\mathcal{K}$ (for any $n \geq 0$), if $a$ depends on some action in $red(s)$ (i.e. there exists $b \in red(s)$ such that $(a, b) \notin I$), then $a \in red(s)$ or there exists $1 \leq i \leq n$ such that $a_i \in red(s)$.

(C2) If $red(s) \neq en(s)$, then all actions in $red(s)$ are invisible.

(C3) For all cycles in the reduced system $\mathcal{K}'$, the following holds: if $a \in en(s)$ for some state $s$ in the cycle, then $a \in red(s')$ for some (possibly other) state $s'$ in the cycle.

**Exercise 1.** Let $\mathcal{K} = (S, A, \rightarrow, r, \mathrm{AP}, \nu)$ be the Kripke structure below, with set of actions $A = \{a, b, c, d, e\}$ and atomic propositions $\mathrm{AP} = \{p\}$.

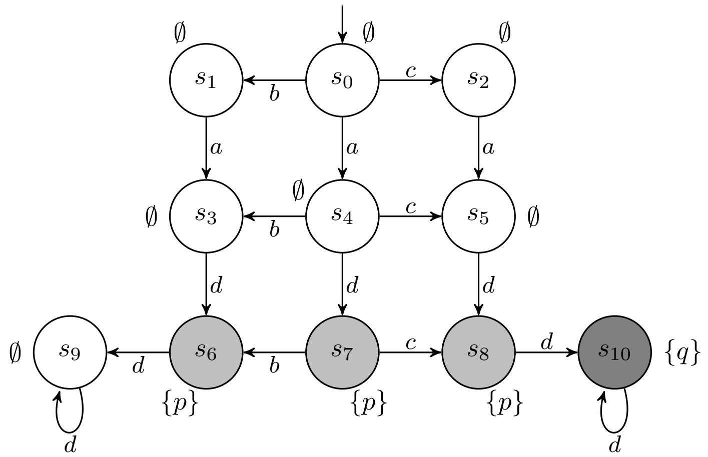



1. Compute the maximal independance set $I$ and the maximal set of invisible actions $U$. Justify your answers.

2. Give sets $red(s) \subseteq en(s)$ satisfying conditions C0 to C3, and such that for all state $s$, no action can be removed from $red(s)$ without breaking one of conditions C0 to C3. Justify your answer.

3. Draw the reduced system $\mathcal{K}'$ associated with your assignment *red*, after removing unreachable states. Is there a smaller system $\mathcal{K}''$, obtained by removing additional transitions, that is stuttering equivalent to $\mathcal{K}$?

**Exercise 2.** Consider the system below, with $A = \{a, b, c, d\}$ and $\text{AP} = \{p, q\}$:




1. Let $red(s_0) = \{b, c\}$ and $red(s) = en(s)$ for $s \neq s_0$. Show that this ample set assignment is compatible with C0–C3.

2. Exhibit a CTL(U) formula that distinguishes between the original system and its reduction.

3. CTL(U)-equivalence between the original system and the reduced one can be ensured by adding a condition C4: if $red(s) \neq en(s)$, then $|red(s)| = 1$. Give an assignment $red \neq en$ compatible with C0–C4.