

Homework 6

To hand in on October 26th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

Exercise 1 (Complexity of $LTL(X)$). We want to show that $LTL(X)$ existential model checking is NP-complete (instead of PSPACE-complete for full $LTL(SU)$).

1. Given $\varphi \in LTL(X)$, the *temporal depth* of φ is defined as follows:

$$\begin{aligned}d(\top) = d(p) &= 0 & d(\neg\varphi) &= d(\varphi) \\d(\varphi \vee \varphi') &= \max\{d(\varphi), d(\varphi')\} & d(X\varphi) &= 1 + d(\varphi)\end{aligned}$$

Show by induction on φ that for all $\varphi \in LTL(X)$ and $w \in \Sigma^\omega$, if u is the prefix of length $d(\varphi) + 1$ of w , we have $w, 0 \models \varphi$ iff $u0^\omega, 0 \models \varphi$.

2. Show that $MC^\exists(X)$ is in NP:
Input: $\varphi \in LTL(X)$ and a finite Kripke structure M .
Question: Does $M \models \exists \varphi$?
3. Reduce 3SAT to $MC^\exists(X)$ in order to prove NP-hardness.