Homework 5

To hand in on October 17th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

Exercise 1. Fix $\mathrm{AP} = \{p, q\}$.

1. Give a deterministic synchronous Büchi transducer $\mathcal{A}_1$ with 2 states for the formula $\varphi_1 = \mathsf{Y}\, p$. Prove that the transducer is correct, i.e. that $[\![\mathcal{A}_1]\!] = [\![\mathsf{Y}\, p]\!]$.
2. Give a deterministic synchronous Büchi transducer $\mathcal{A}_2$ with 2 states for the formula $\varphi_2 = p\, \mathsf{SS}\, q$. Prove that the transducer is correct, i.e. that $[\![\mathcal{A}_2]\!] = [\![p\, \mathsf{SS}\, q]\!]$.
3. Give a synchronous Büchi transducer $\mathcal{A}_3$ with 3 states for the formula $\varphi_3 = \mathsf{G}\, p$. Prove that the transducer is correct, i.e. that $[\![\mathcal{A}_3]\!] = [\![\mathsf{G}\, p]\!]$.

Exercise 2. Fix $\mathrm{AP} = \{p, q\}$ and $\Sigma = 2^{\mathrm{AP}}$. Let $L \subseteq \Sigma^\omega$ be the set of words satisfying the following $\mathrm{MSO}(\mathrm{AP}, <)$ sentence:

$$\begin{aligned}\varphi = \forall x.\ \forall y.\ x < y \wedge p(x) \wedge p(y) \to \exists X_0.\ \exists X_1.\ (\forall z. \forall z'.\ (z \in X_0 \wedge z' \in X_1 \to z \neq z')) \\ \wedge\ (\forall z.\ ((z \in X_0 \vee z \in X_1) \leftrightarrow (x < z < y \wedge \neg p(z) \wedge q(z)))) \\ \wedge\ (\forall z.\ z \in X_1 \to \exists z'.\ z' < z \wedge z' \in X_0) \\ \wedge\ (\forall z.\ z \in X_0 \to \exists z'.\ z < z' \wedge z' \in X_1) \\ \wedge \bigwedge_{i \in \{0,1\}} (\forall z.\ \forall z'.\ (z < z' \wedge z \in X_i \wedge z' \in X_i) \to \exists z''.\ z < z'' < z' \wedge z'' \in X_{1-i})\end{aligned}$$

1. Give a simple informal description of L.
2. Give a Büchi automaton $\mathcal{A}$ such that $L(\mathcal{A}) = L$.