Homework 5

To hand in on October 17th at 14:00, during the exercise session or by mail at marie.fortin@lsv.fr.

Exercise 1. Fix \( AP = \{ p, q \} \).

1. Give a deterministic synchronous Büchi transducer \( A_1 \) with 2 states for the formula \( \varphi_1 = Y p \). Prove that the transducer is correct, i.e. that \( J_{A_1} = J_{Y p} \).

2. Give a deterministic synchronous Büchi transducer \( A_2 \) with 2 states for the formula \( \varphi_2 = p \text{SS} q \). Prove that the transducer is correct, i.e. that \( J_{A_2} = J_{p \text{SS} q} \).

3. Give a synchronous Büchi transducer \( A_3 \) with 3 states for the formula \( \varphi_3 = G p \). Prove that the transducer is correct, i.e. that \( J_{A_3} = J_{G p} \).

Exercise 2. Fix \( AP = \{ p, q \} \) and \( \Sigma = 2^{AP} \). Let \( L \subseteq \Sigma^\omega \) be the set of words satisfying the following MSO\((AP, <)\) sentence:

\[
\varphi = \forall x. \forall y. x < y \land p(x) \land p(y) \rightarrow \exists X_0. \exists X_1. (\forall z. (z \in X_0 \lor z \in X_1) \leftrightarrow (x < z < y \land \neg p(z) \land q(z)))
\]

\[
\land (\forall z. (z \in X_0 \lor z \in X_1) \leftrightarrow (x < z < y \land \neg p(z) \land q(z)))
\]

\[
\land (\forall z. z \in X_1 \rightarrow \exists z'. z' < z \land z' \in X_0)
\]

\[
\land (\forall z. z \in X_0 \rightarrow \exists z'. z < z' \land z' \in X_1)
\]

\[
\land \bigwedge_{i \in \{0, 1\}} (\forall z. (z < z' \land z \in X_i \land z' \in X_i) \rightarrow \exists z''. z < z'' < z' \land z'' \in X_{1-i})
\]

1. Give a simple informal description of \( L \).

2. Give a Büchi automaton \( A \) such that \( L(A) = L \).