

## Homework 4

To hand in on October 12th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

**Exercise 1.** Let  $\Sigma = \{a, b, c\}$ . For each of the following LTL formulæ  $\varphi$ , give a Büchi automaton accepting the language  $\{w \in \Sigma^\omega \mid w, 0 \models \varphi\}$  (where a word  $w \in \Sigma^\omega$  can be seen as a temporal structure  $(\mathbb{N}, <, \lambda)$  in which each letter in  $\Sigma$  stands for a set of atomic propositions).

1.  $(GFa) \rightarrow (GFb)$
2.  $G(a \rightarrow (\neg a \text{ SU } b))$
3.  $G(Xb \rightarrow a)$
4.  $(GFa) \wedge (Fb) \wedge (Fc)$

**Exercise 2.** A Büchi automaton  $\mathcal{A} = (Q, \Sigma, I, T, F)$  is *deterministic* if  $|I| \leq 1$ , and for each state  $q$  in  $Q$  and symbol  $a$  in  $\Sigma$ ,  $|\{(q, a, q') \in T \mid q' \in Q\}| \leq 1$ .

1. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under intersection.
2. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under union.