Homework 10

To hand in on December 21th at 14:00, during the exercise session or by mail at marie.fortin@lsv.fr.

Exercise 1. Let $\mathcal{N}$ be the following Petri net:

$$
\begin{array}{c}
p_1 \bullet \\
\downarrow t_1 \\
p_3 \circ \\
\downarrow t_3 \\
p_2 \circ \\
\downarrow t_2 \\
p_4 \bullet \\
\end{array}
$$

1. Draw the reachability graph of $\mathcal{N}$. A marking $m$ will be denoted by the tuple $\langle m(p_1), m(p_2), m(p_3), m(p_4) \rangle$, for instance the initial marking is $\langle 1, 0, 0, 1 \rangle$.

2. Is $\mathcal{N}$ 1-safe ? 2-safe ? 3-safe ?

Exercise 2. We say that a Petri net $\mathcal{N} = \langle P, T, F, W, m_0 \rangle$ is acyclic if the directed graph $G_{\mathcal{N}} = (P \cup T, F)$ does not contain any cycle. Let $\mathcal{A}$ denote the class of Petri nets that are 1-safe and acyclic.

1. Let $\mathcal{N} = \langle P, T, F, W, m_0 \rangle$ be a Petri net.

   (a) Show that if $m \xrightarrow{t_1} m_1 \xrightarrow{t_2} m'$ in $\mathcal{N}$ and $t_1 \bullet \cap t_2 = \emptyset$, then there exists a marking $m_2$ such that $m \xrightarrow{t_2} m_2 \xrightarrow{t_1} m'$.

   (b) Let $m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \ldots \xrightarrow{t_k} m_{k+1}$ be an execution in $\mathcal{N}$ for some $k > 1$. Assume that for all $1 < i < k$, there exists a nonempty path from $t_1$ to $t_i$ in the graph $G_{\mathcal{N}}$, and that there is no nonempty path from $t_1$ to $t_k$ in $G_{\mathcal{N}}$. Show that there exists an execution $m_1 \xrightarrow{t_k} m'_1 \xrightarrow{t_1} m'_2 \xrightarrow{t_2} \ldots \xrightarrow{t_{k-1}} m'_k = m_{k+1}$. 

2. Let $\mathcal{N} = (P, T, F, W, m_0)$ be a 1-safe, acyclic Petri net. We assume that for all $t \in T$, $t^* \neq \emptyset$ or $t^* \neq \emptyset$.

(a) Show that there is no reachable marking $m$ from which some transition $t$ can fire twice, i.e. that there are no $t, m, m', m''$ such that $m$ is reachable from $m_0$ and $m \xrightarrow{t} m' \xrightarrow{t} m''$.

(b) Show that for all executions $m_0 \xrightarrow{t_1} m_1 \xrightarrow{t_2} \cdots \xrightarrow{t_n} m_n$ in $\mathcal{N}$, we have $t_i \neq t_j$ for all $i \neq j$.

3. Show that the reachability problem for the class $\mathcal{A}$ is in NP.