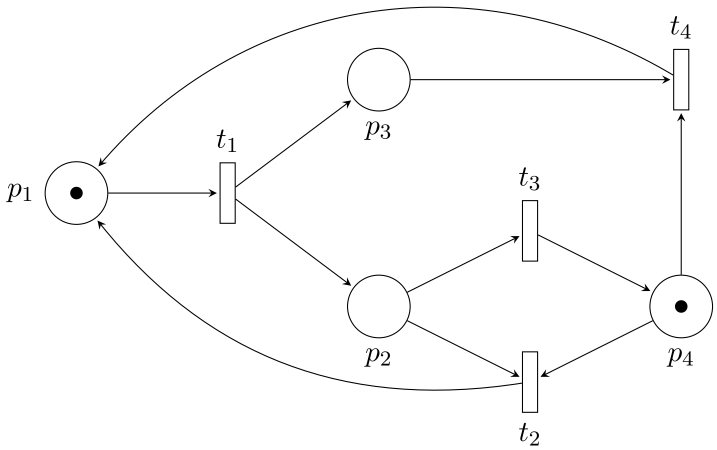
Homework 10

To hand in on December 21th at 14:00, during the exercise session or by mail at `marie.fortin@lsv.fr`.

Exercise 1. Let $\mathcal{N}$ be the following Petri net:



1. Draw the reachability graph of $\mathcal{N}$. A marking m will be denoted by the tuple $\langle m(p_1), m(p_2), m(p_3), m(p_4)\rangle$, for instance the initial marking is $\langle 1, 0, 0, 1\rangle$.
2. Is $\mathcal{N}$ 1-safe ? 2-safe ? 3-safe ?

Exercise 2. We say that a Petri net $\mathcal{N} = \langle P, T, F, W, m_0\rangle$ is *acyclic* if the directed graph $G_\mathcal{N} = (P \cup T, F)$ does not contain any cycle. Let $\mathcal{A}$ denote the class of Petri nets that are 1-safe and acyclic.

1. Let $\mathcal{N} = \langle P, T, F, W, m_0\rangle$ be a Petri net.
   (a) Show that if $m \xrightarrow{t_1} m_1 \xrightarrow{t_2} m'$ in $\mathcal{N}$ and $t_1{}^\bullet \cap {}^\bullet t_2 = \varnothing$, then there exists a marking m_2 such that $m \xrightarrow{t_2} m_2 \xrightarrow{t_1} m'$.
   (b) Let $m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \cdots \xrightarrow{t_k} m_{k+1}$ be an execution in $\mathcal{N}$ for some $k > 1$. Assume that for all $1 < i < k$, there exists a nonempty path from t_1 to t_i in the graph $G_\mathcal{N}$, and that there is no nonempty path from t_1 to t_k in $G_\mathcal{N}$. Show that there exists an execution $m_1 \xrightarrow{t_k} m'_1 \xrightarrow{t_1} m'_2 \xrightarrow{t_2} \cdots \xrightarrow{t_{k-1}} m'_k = m_{k+1}$.

2. Let $\mathcal{N} = \langle P, T, F, W, m_0 \rangle$ be a 1-safe, acyclic Petri net. We assume that for all $t \in T$, ${}^\bullet t \neq \varnothing$ or $t^\bullet \neq \varnothing$.

   (a) Show that there is no reachable marking m from which some transition t can fire twice, i.e. that there are no t, m, m', m'' such that m is reachable from m_0 and $m \xrightarrow{t} m' \xrightarrow{t} m''$.

   (b) Show that for all executions $m_0 \xrightarrow{t_1} m_1 \xrightarrow{t_2} \cdots \xrightarrow{t_n} m_n$ in $\mathcal{N}$, we have $t_i \neq t_j$ for all $i \neq j$.

3. Show that the reachability problem for the class $\mathcal{A}$ is in NP.