

Fully Homomorphic Encryption

Implementation Progresses and Challenges

Chiffrement homomorphe : une révolution en marche

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Outline

- 1 Context and Introduction
- 2 Applications and Practical Issues
 - Security
 - How to express high-level algorithms?
 - Huge expansion of ciphertexts
 - Complexity
- 3 Conclusion

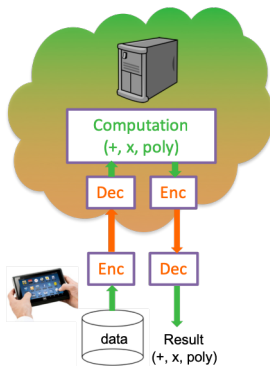
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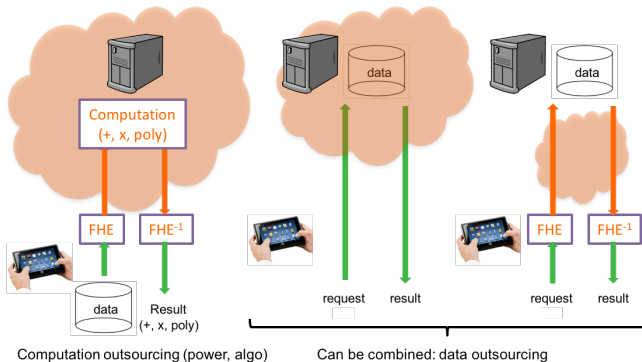
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Data is encrypted for transmission and storage but processed in clear :-)

Homomorphic Encryption : we are dreaming of ...



A revolution : data and/or services outsourcing without losing confidentiality!

Impact : citizens, administrations, companies, military, ...

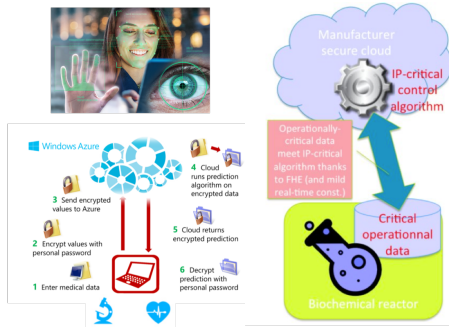
Domains : health care, power plants, multimedia content delivery, ...

Computations : comparing, sorting/filtering, clustering, compressing, ...

Also “Intelligent” and “Evolving” algorithms :-)

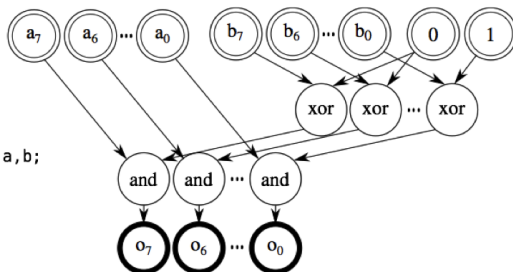
- + **privacy** concerns for the **end-user**
- + **IP** concerns and **software update** for the **service provider**

- Targeted **advertising**
- **Access Control** with respect to user profile
- **Biometric** authentication
- **Medical Diagnosis**
- **Critical engine** (reactor) **control**
- **Machine Learning** (deep learning)



Program's output = Circuit Eval = Polynomial Eval

```
#include <iostream>
#include <stdint.h>
#include "integer.h"
void f
    (std::istream &i,
     std::ostream &o)
{
    SlicedInteger<int8_t> a,b;
    i >> a >> b;
    b = b ^ 0x01;
    a &= b;
    o << a;
}
```



$$\begin{aligned} F_i(x) &= x_i x_{i+8} \quad i = 1, \dots, 7 \\ F_0(x) &= x_8 (x_{16} + 1) \end{aligned}$$

Program's output = Circuit Eval = Polynomial Eval

$$F_0(x) = x_8(x_{16} + 1)$$

Then needing $Enc()$ and $Dec()$ satisfying

$$Dec(F(Enc(x_1), \dots, Enc(x_n))) = F(x_1, \dots, x_n)$$

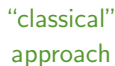
F computed on plaintexts

```
graph TD; a[a] --- XOR1((xor)); b[b] --- XOR1; XOR1 --- NOT1(not); XOR1 --- AND1((and)); NOT1 --- AND1; AND1 --- XOR2((xor)); c[c] --- AND2((and)); d[d] --- AND2; AND2 --- XOR2; XOR2 --- output[output]
```

F computed on ciphertexts

Diagram illustrating a neural network architecture for message processing. The input consists of two stacked blocks (orange, blue, green) representing a 'Message' and a key P_{key} plus noise. These are processed by a function f_+ . The output of f_+ is then processed by two parallel functions f_x , each preceded by a '+1' operation. The outputs of f_x are then combined by a final f_+ function to produce the final output block.

(post-quantum)



Lattice based S/FHE in a nutshell ...

Ex : FHE over the integers [vDGHV 10]

- Secret key (symmetric version here) : s
- Encryption of $m \in \{0, 1\}$: α, β random
- Decryption : $c \bmod s = m + 2\alpha$

$$c = m + 2\alpha + \beta s$$

$$m = (c \bmod s) \bmod 2$$

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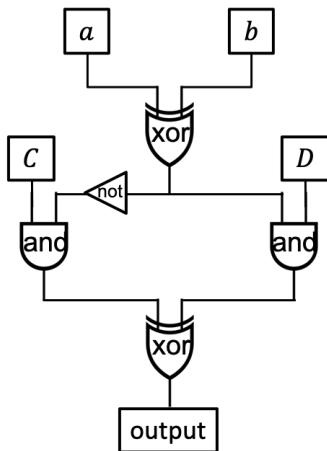
And it is even worse in the case of homomorphic multiplication!

The challenge is to keep control of this noise during computation.

Noise grows gate after gate...

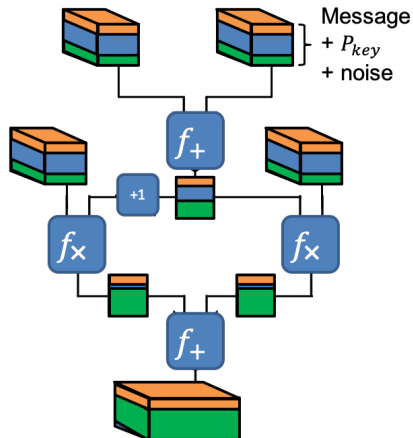
F computed on **plaintexts**

$$F(x_1, \dots, x_n)$$



F computed on **ciphertexts**

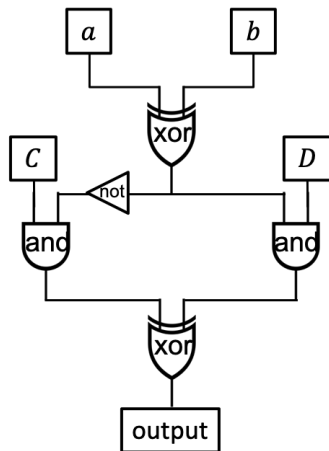
$$F(Enc(x_1), \dots, Enc(x_n))$$



If noise grows too much...

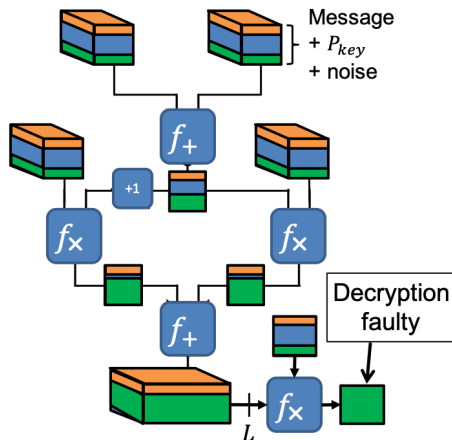
F computed on **plaintexts**

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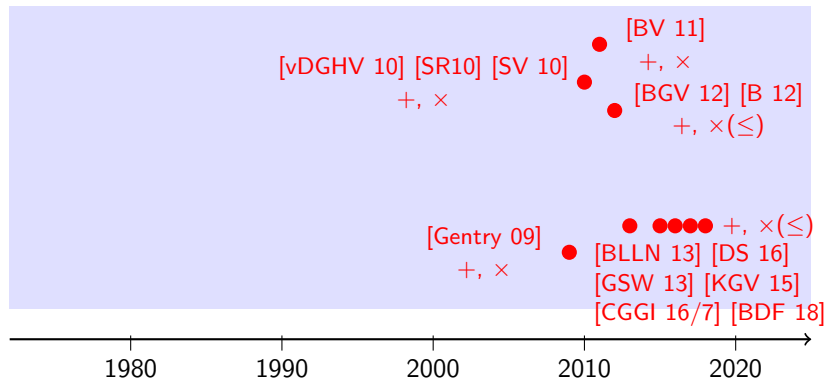


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$$F(Enc(x_1), \dots, Enc(x_n))$$

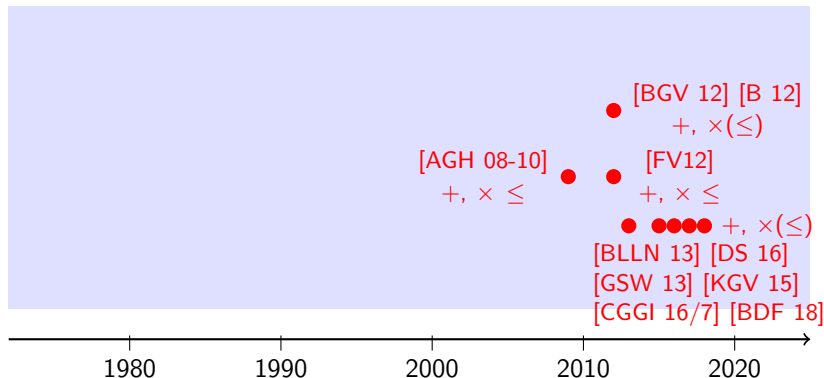


How to handle this noise ? (1/2)



- FHE : \times unbounded \rightarrow using bootstrapping
 - once the setting is fixed, "any" circuit can be evaluated
 - 2009-2014 : too complex to be used in practice
 - BUT recent improvements, e.g. [PV15] to opt. bootstrapping use, [CGGI16/7] to accelerate it...

How to handle this noise ? (2/2)



- **Leveled FHE schemes** : \times bounded \rightarrow without bootstrapping
 - a limited (but often sufficient) number of multiplications
 - maximum mult. depth is related to the setting
(cannot be modified afterwards)
 - a lower complexity

How to handle this noise ? (genealogy)

Central problem : Noisy encryption \longrightarrow noise management

Several noise management strategies \Rightarrow 4 generations of ciphers :

- ① 2009 [Gen 09] exponential in the circuit size
- ② 2010-2012 (DGHV, BGV, FV, YASHE [BLLN 13]), polynomial
- ③ 2013-2016 (GSW, SHIELD [KGV 15] , F-NTRU [DS 16]), linear
- ④ since 2014 (FHEW, TFHE [CGGI 16] , HE6 [BDF 18]), constant

Remark : settings depend on target security level and circuit size

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Want to play? (1/2)

- 2011 : open-source implementation of [SV10] by [PBS11]
<http://www.hcrypt.com>
- 2012 : private implem. of [BGV12] dedicated to AES homo. eval. [GHS12]
- 2013-* : platform at CEA [AFFGS13,FSFAG13], home-made implem. of [BGV12] (vect and poly) and [FV12] + HElib and more recent open-source libraries + compilation chain \Rightarrow private platform at beginning, open-source since Jan 2018!
<https://github.com/CEA-LIST/Cingulata>
- 2013 : open-source implementation of [vDGHV10] with the improvements from [CNT12] : <https://github.com/coron/fhe>
- 2013 : private implementation in [CLT 13] dedicated to AES homomorphic evaluation using an improved version of [vDGHV10]
- 2013 : private implementation of [BLLN 13], with good performances with 2 or 3 multiplicative depth
- 2013-* : open-source implem. of [SV10] and [BGV12] called HElib by Halevi et al. <http://shaih.github.io/HElib/>

Want to play? (2/2)

- 2014 : [open-source](#) implem. of [FV12] and [BLLN13] YASHE, compared in [LN14] <https://github.com/tlepoint/homomorphic-simon>
- 2015 : open-source library called SEAL1.0, based on YASHE' <http://sealcrypto.codeplex.com/>
- 2016-* : SEAL1.0 is replaced by SEAL2.1, and now SEAL3.1.0 based on another implementation of [FV12] <http://sealcrypto.org/>
- 2016 : [open-source](#) library to efficiently handle polynomials, called NFLlib <https://github.com/quarkslab/NFLlib>
- 2016 : [open-source](#) implementation of [FV12] based on NFLlib <https://github.com/CryptoExperts/FV-NFLlib>
- 2016 : [open-source](#) multi-precision moduli library, called HELib-MP <https://github.com/tricosset/HElib-MP>, based on HELib
- 2016 : private implem. of FV with RNS [BEHZ16]
- 2017-* : [open-source](#) implem. of TFHE [CGG16] <https://github.com/tfhe/tfhe>
- 2018 : [open-source](#) implem. of HE6 [BDF18] <https://github.com/gbonnoron/Borogrove>

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Which kind of security ?

Semantic Security

Semantic security is necessary !

(and as S/FHE schemes are malleable, IND-CCA2 can never be achievable).

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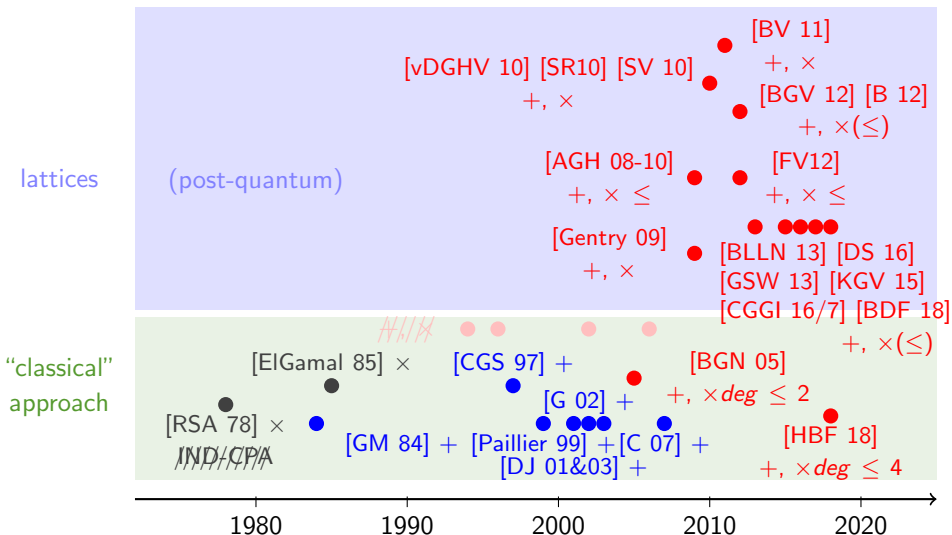
⇒ **expansion** (ciphertexts are longer than plaintexts)

and parameters setting has a huge impact on expansion !

e.g. for 128-bits security level, expansion is (without batching) :

- equal to 2 with Paillier cryptosystem (only +)
- around 5,000 with elliptic curve based solution BGN-F-CF [HBF18]
(+, $\times_{deg} \leq 4$)
- between 50,000 and 1,000,000 for lattice-based S/FHE ! (+, $\times(\leq)$)

It has been a long quest to handle polynomials



Which security level ?

Security Analysis of elliptic curve based schemes

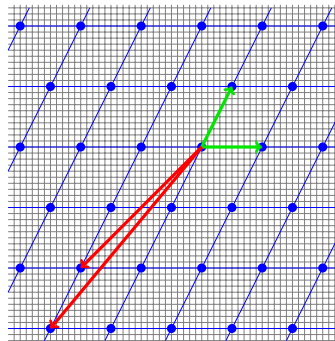
Computational Security (w.r.t. DLP). Well understood and studied.

Security Analysis of lattice based schemes

Computational Security (w.r.t. hard problems as LWE, R-LWE, ...)

Theoretical studies essentially focus on asymptotic and generic estimations (may be not so close to real S/FHE situations).

Some experiments (based on LLL, BKZ, ...) provide estimations (but may remain too optimistic today).



See e.g. [Alb15,ABD16][Peik16][KF17][BF17][Alb17][AN17] .

⇒ Due to some of them, YASHE AND F-NTRU are down !

Which security level for lattice based S/FHE?

See the (online) estimator provided by Martin Albrecht (always evolving) :
<https://bitbucket.org/malb/lwe-estimator>



Type some Sage code below and press Evaluate.

```
1 load("https://bitbucket.org/malb/lwe-estimator/raw/HEAD/estimator.py")
2 n, alpha, q = 256, 0.0009765625000000000, 65537
3 set_verbose(1)
4 _ = estimate_lwe(n, alpha, q)
```



Evaluate

Language: Sage

Share

```
mitn bop: ~2^917.3, oracle: 370, mem: ~2^904.3, rop: ~2^913.3
bkw bop: ~2^148.7, oracle: ~2^131.5, m: ~2^124.7, mem: ~2^132.5, rop: ~2^144.7, b: 8, t1: 8, t2: 15, l: 1
sis sieve: ~2^142.9, oracle: ~2^28.6, delta_0: 1.0045951, bks2: ~2^224.3, betas: 325, lp: ~2^212.2, quantum_sieve: ~2^134.1, |v|: 1360.4505
dec rop: ~2^122.8, oracle: ~2^18.8, sieve: ~2^121.7, delta_0: 1.0050017, bks2: ~2^183.4, beta: 286, lp: ~2^180.4, quantum_sieve: ~2^113.9
kannan sieve: ~2^114.4, oracle: ~2^15.0, delta_0: 1.0051468, bks2: ~2^170.4, beta: 274, lp: ~2^169.6, quantum_sieve: ~2^107.0, m: 744
```

Help | Powered by SageMath

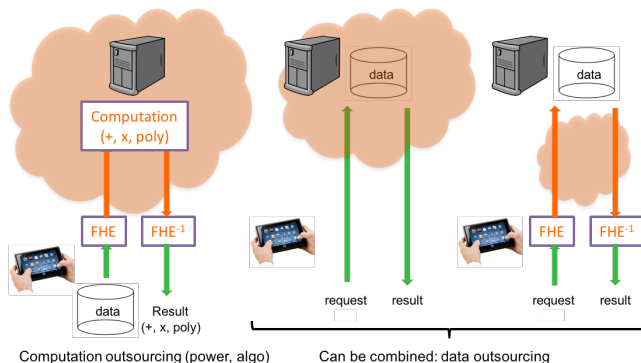
⇒ it is really hard today to know how to choose the right parameters to ensure a given security level (e.g. 128) and we really need more targeted attacks and studies to derive precise guidelines for the choice of parameters (see [MBF18] for an attempt, based on the current state-of-the-art).

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How to express high-level algorithms ?

Applications : we are dreaming of ...



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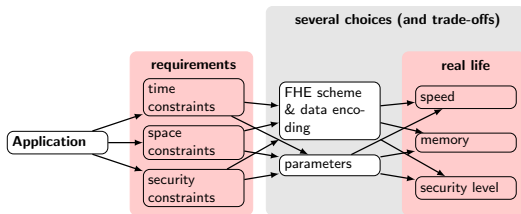
How to express high-level algorithms ?

How to help programmers ?

Our goal

To help programmers (not crypto specialists !) to use S/FHE in the development of their software/hardware stuff [AFF+13][FAR+13][CS14]...

- 1 Cryptographers are necessary to help choosing the most appropriate S/FHE scheme & data encoding & parameters :

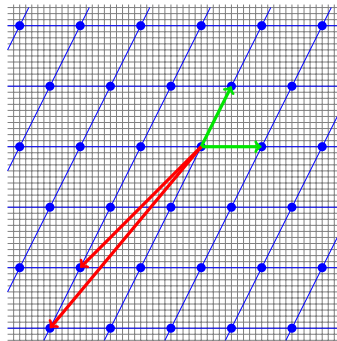


- 2 This being done, programmers must be able to go further alone, without interacting with cryptographers !

How to express high-level algorithms ?

With cryptographers : choosing data encoding (1/3)

Your (sliced) data (bits, integers/floats) \longleftrightarrow



Each piece of (sliced) data has to be related with one plaintext (a point of the lattice, *i.e.* integers or polynomials)

How to express high-level algorithms ?

With cryptographers : choosing data encoding (2/3)

Your data : managing bits or integers/floats ? (slicing)

Processing integers/floats may seem more interesting at a first glance, BUT in some cases using integers/floats will reduce the set of algorithms one can execute in the encrypted domain, e.g. if-then-else implies a management at the bit-level for Generations 1-2-3.

| Operations | bit-level | | integer/float-level | |
|-----------------------|-----------|----------------|---------------------|----------------|
| | ? | \times depth | ? | \times depth |
| addition | yes | $n - 1$ | yes | 0 |
| multiplication | yes | $n - 1$ | yes | 1 |
| scalar division | yes | dep. on scalar | yes | 1 |
| scalar multiplication | yes | dep. on scalar | yes | 0 |
| shift | yes | 0 | yes | 1 |
| comparison | yes | $\log_2 n$ | no | - |
| cond. assignment | yes | $\log_2 n$ | no | - |

With cryptographers : choosing data encoding (3/3)

In case we choose an encoding at the bit-level, we need to redefine integers/floats encoding to get operators on integers/floats (based on those on bits, with 2's complement, sign bit, ...), for :

addition multiplication subtraction << >>

Batching (packing several plaintexts into one)

To process several bits (resp. integers/floats) at the same time, e.g. using Chinese Remaining Theorem.

Programers are not obliged to implem. S/FHE

From Armadillo platform [AFF+13][FAR+13][CS14] , now Cingulata :

Definition of C++ classes `ClearBit` and `CryptoBit` written with the help of cryptographers (link with data encoding and S/FHE scheme) :

```
class C++ template<typename bit, int size>
```

Any programmer can then use them :

Example

Applying a bubble sort on data in clear :

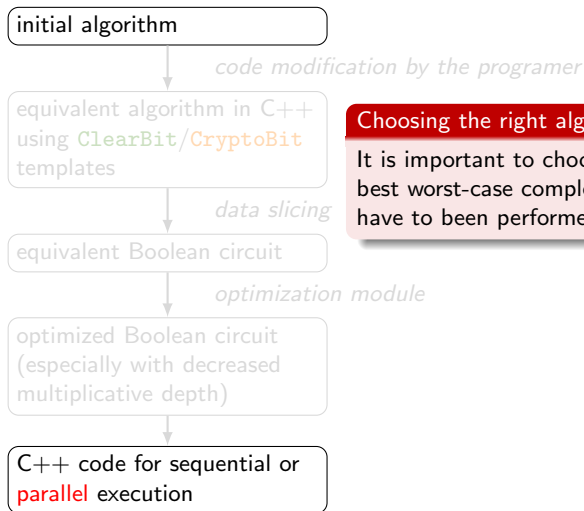
```
bssort<Integer<ClearBit,8> >(arr,n);
```

Applying the **same** bubble sort on encrypted data :

```
bssort<Integer<CryptoBit,8> >(arr,n);
```

How to express high-level algorithms ?

Software Compilation Process and Optimization

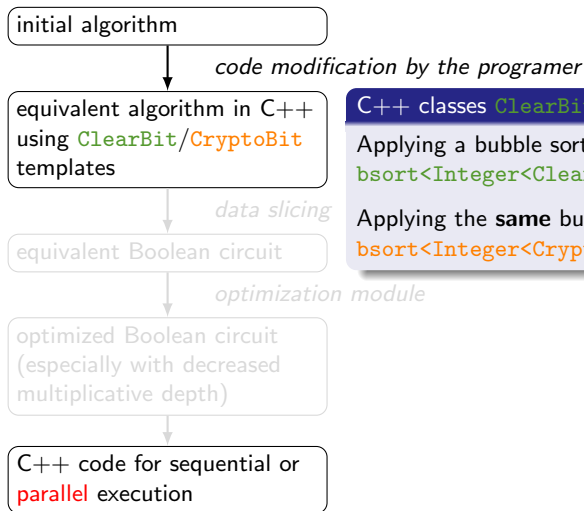


Choosing the right algorithm

It is important to choose the algorithm with the best worst-case complexity (not usual!) if tests have to be performed over the encrypted data.

How to express high-level algorithms ?

Software Compilation Process and Optimization



C++ classes **ClearBit** and **CryptoBit**

Applying a bubble sort on data in clear :

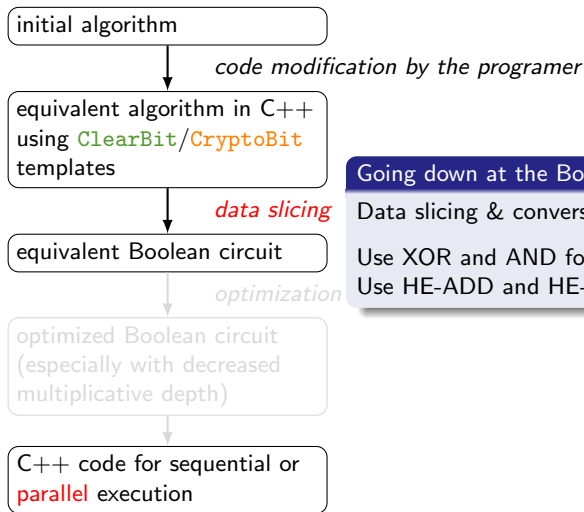
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How to express high-level algorithms ?

Software Compilation Process and Optimization



Going down at the Boolean level

Data slicing & conversion Pgm → Boolean circuit.

Use XOR and AND for ClearBit

Use HE-ADD and HE-MULT for CryptoBit

How to express high-level algorithms ?

Program \rightarrow Boolean circuit

Comparisons of Encrypted Data

How to perform tests and express if-then-else ?

Boolean bitwise operators :
$$\begin{cases} a < b : \text{MSB of } a+(-b) \\ a > b : \text{MSB of } b+(-a) \\ a = b : (a < b) \text{ NOR } (a > b) \end{cases}$$

“if c then x = a else x = b” can be achieved through the following operator :
$$x = \text{select}(c,a,b) = \begin{cases} a & \text{if } c=1 \\ b & \text{otherwise} \end{cases}$$

$$x = \text{select}(c,a,b) = (c \text{ AND } a) \text{ XOR } ((\text{NOT } c) \text{ AND } b)$$

- no data leakage;-)
- BUT bit-level encoding + worst-case complexity as we have to evaluate the whole circuit (all the branches of the circuit)

How to express high-level algorithms ?

Bubble sort : a meaningful example

Classical bubble sort :

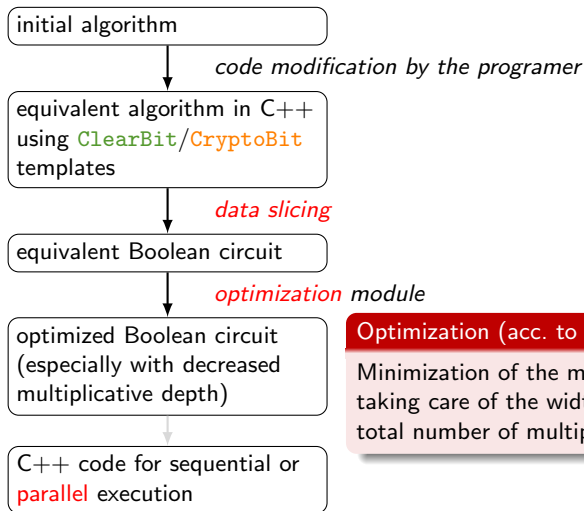
```
void bsort(int *arr,int n)
{
    for(int i=0;i<n-1;i++)
    {
        for(int j=1;j<n-i;j++)
            if(arr[j-1]>arr[j])
            {
                int t=arr[j-1];
                arr[j-1]=arr[j];
                arr[j]=t;
            }
    }
}
```

Rewritten bubble sort :

```
void bsort(int *arr,int n)
{
    for(int i=0;i<n-1;i++)
    {
        for(int j=1;j<n-i;j++)
        {
            int gt=arr[j-1]>arr[j];
            int t=gt*arr[j-1]^(!gt*arr[j]);
            arr[j-1]=gt*arr[j]^(!gt*arr[j-1]);
            arr[j]=t;
        }
    }
}
```

How to express high-level algorithms ?

Software Compilation Process and Optimization



Optimization (acc. to Generations 1-2-3-4)

Minimization of the multiplicative length (also taking care of the width of the circuit and the total number of multiplications and additions).

How to express high-level algorithms ?

Optimizing the Boolean circuit

Characterization of # add, # mul, \times depth

Estimation and optimization possible with the help of **ClearBit**.

Some values for classical algorithms (before optimization) :

| | $\sum_{i=1}^{10} t[i]$ (4 bits) | threshold (4 bits) | $b^2 - 4ac$ (4 bits) | bubble sort (10x4 bits) | FFT (256x32 bits) |
|----------------|------------------------------------|-----------------------|-------------------------|----------------------------|----------------------|
| # add | 99 | 390 | 126 | 2372 | 7291592 |
| # mul | 27 | 60 | 32 | 238 | 5296128 |
| \times depth | 4 | 5 | 7 | 69 | 166 |
| | (16 bits) | | (16 bits) | (10x8 bits) | |
| # add | 423 | | 1188 | 3240 | |
| # mul | 279 | | 1126 | 2790 | |
| \times depth | 16 | | 32 | 136 | |

\Rightarrow **ClearBit** class helps to debug the implementation and to optimize it !

How to express high-level algorithms ?

Circuit optimization (included in Cingulata)

Multiplicative depth should be kept < 30

- Main goal : to reduce multiplicative depth (the most critical)
- Secondary goal : to reduce the number of multiplicative gates

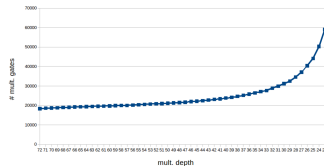
By iteratively applying local circuit rewriting operators.

E.g. Medical diagnosis :

- reducing multiplicative depth from around 20 to 8 !
- then we can add transcription (Kreyvium adds \times depth of 12

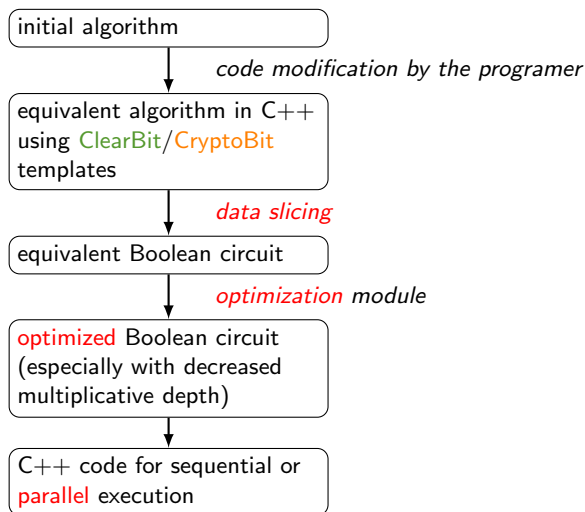
E.g. Running Length Encoding :

- reducing multiplicative depth from 70 to 20 ! Here at a cost in terms of total multiplicative gates.



How to express high-level algorithms ?

Software Compilation Process and Optimization



Outline

- 1 Context and Introduction
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 - Complexity
- 3 Conclusion

An awful expansion factor !

Expansion (without batching)

Current estimations of security parameters lead to an expansion factor

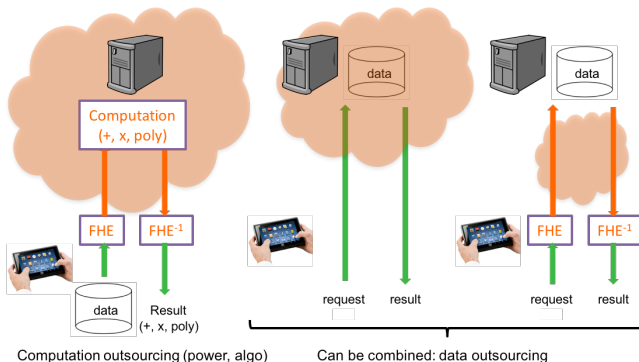
- equal to 2 with Paillier cryptosystem (only +)
- around 5,000 with elliptic curve based solution BGN-F-CF [HBF18]
(+, $\times_{deg} \leq 4$)
- between 50,000 and 1,000,000 for lattice-based S/FHE! (+, $\times(\leq)$)

⇒ pb to store and process, and to transmit data encrypted with S/FHE !

- ① it would be very nice to design new schemes with a lower expansion,
- ② we can help by choosing a good data representation and pack several plaintexts together (batching : CRT, SIMD, RNS),
- ③ we also have to do our best to manage huge ciphertexts, e.g. properly combining classical symmetric encryption with S/FHE.

Huge expansion of ciphertexts

Applications : we are dreaming of ...



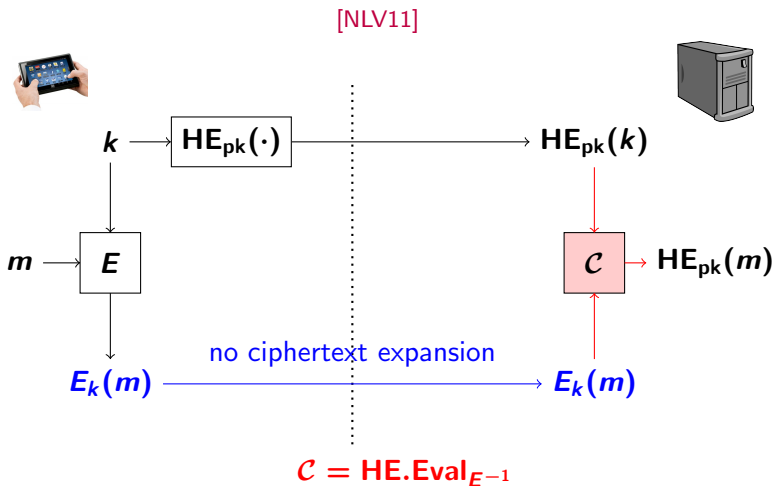
A revolution : data and/or services outsourcing without losing confidentiality !

Impact : citizens, administrations, companies, military, ...

Domains : health care, power plants, multimedia content delivery, ...

Computations : comparing, sorting/filtering, clustering, compressing, ...

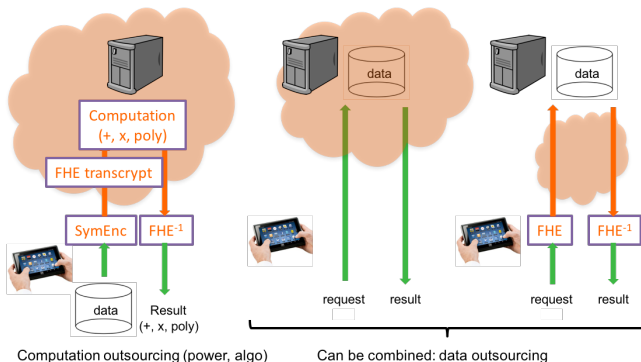
How to efficiently upload S/FHE ciphertext?



What kind of symmetric encryption is the most appropriate?

Huge expansion of ciphertexts

This leads to...



A revolution : data and/or services outsourcing without losing confidentiality !

Impact : citizens, administrations, companies, military, ...

Domains : health care, power plants, multimedia content delivery, ...

Computations : comparing, sorting/filtering, clustering, compressing, ...

HE-friendly ciphers ? (1/2)

Main goal

To **minimize** the **multiplicative depth** of the decryption function.

First concrete proposals have been block ciphers

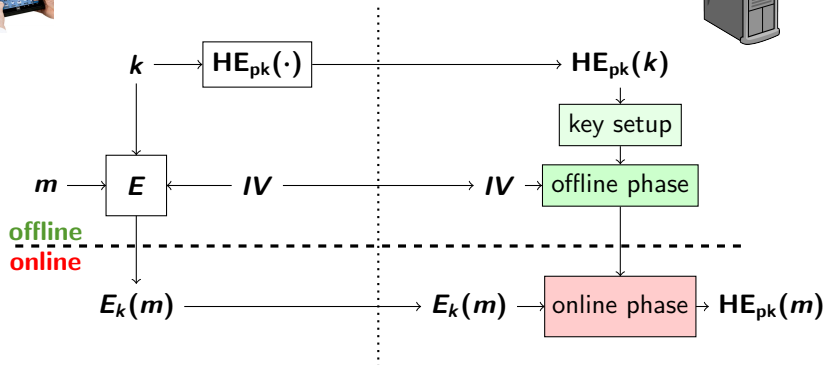
- Already existing block ciphers :
 - Optimized implementations of AES [GHS12][CCKL+13][DHS14]
 - but AES's \times depth remains too large (→ too slow)
 - Lightweight block ciphers : SIMON [LN14] , PRINCE [DSES14]
 - SIMON behaves better than AES
 - PRINCE behaves better than SIMON, but remains too slow
- Dedicated block cipher : Low-MC-80 and Low-MC-128 [ARSTZ15]
 - but subject to some interpolation attacks (sparse ANF)
 - ⇒ a tweaked version has been presented at FSE 2016's rump session (more rounds), but security remains not clear (≤ 118)

Huge expansion of ciphertexts

Ciphertext decompression with IV-based encryption

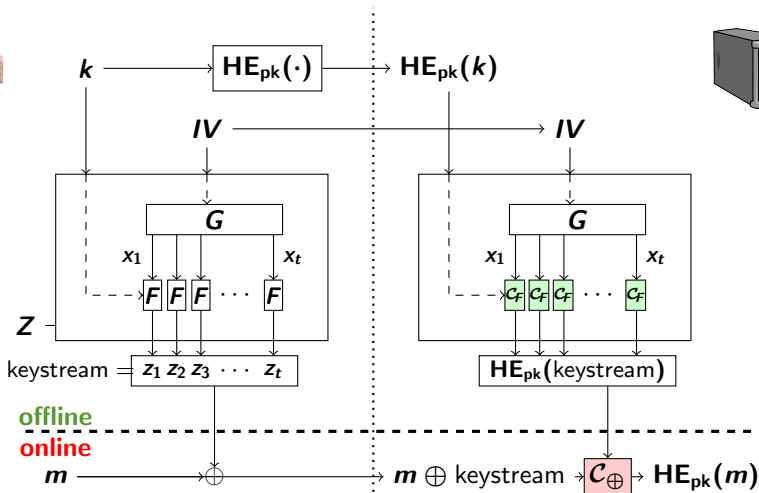
A new approach [CCCF+16,CCCF+18]

to reduce the online phase to a minimum ...



Ciphertext decompression with IV-based encryption

... with an additive stream cipher;-)



HE-friendly ciphers ? (2/2)

**Using a stream cipher reduces on-line phase to the minimum.
Current candidates for function F are :**

[CCCF+16,CCCF+18] :

- Trivium : coming from eSTREAM (2008), firmly established security, 80 bits security
- Kreyvium : based on Trivium, same security confidence, 128 bits security

[MJSC 16] :

- Flip : lower complexity, but security should be more deeply analyzed [DLR 16]

According to today's state-of-the-art, Kreyvium seems to be the best available solution (but may be replaced by Flip if new security analysis is good).

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Complexity issues

Complexity

High computation complexity related to the noise management.

Cryptographic issues :

⇒ it should be nice to have **less complex S/FHE schemes**, even if a huge effort has still been done and complexity already decreased a lot, and to **optimize the use of bootstrapping, modulus switching, re-linearization, etc** (e.g. see [PV15] for bootstrapping opt. and the hope arising from [CGGI16/7][BDF18]).

Application related issues :

⇒ for a given target, we need to carefully choose the right algorithm (with the **best worst-case complexity !**)

⇒ we need to **optimize the implementation** (circuit optimization, bits/integers & batching, software/hardware implementation).

Examples of FHE Practical Achievements

... order of magnitude (dep. on security level, batching, optimization) ...


- Energy-consumption profile classification : < 1 second
- Various medical diagnosis : 3 seconds < 2 minutes
- Genome-based diagnosis : < 10 minutes
- Running Length Encoding (step for image/video compression) : $\simeq 30$ minutes with 48 cores
- ...

o2a tech
Example of demonstration

- A dummy-yet-realistic « Wikipedia-inspired » medical diagnosis.
 - Setup:
 - Algorithm implementation, compilation and deployment on a server.
 - Homomorphic precalculation of Frequent-keystream on the server.
 - The Android tablet sends the Frequent-keystream-encrypted private user health data.
 - The server receives and homomorphically « homomorphically » to FHE.
 - The server homomorphically executes the diagnostic algorithm and sends back the encrypted answer to the tablet.
 - As the FHE secret key remains, the tablet is the only party able to decrypt and thus interpret the server reply.
 - Characteristics:
 - Fully-secure FHE.
 - Full-blown end-to-end 128 bits security.
 - 3-3 secs for program execution on the server.
 - < 8 secs RTD towards servers.
 - Claim: practicality achieved for not-too-big-data realistic algorithms!

• Facteur de risque cardiovasculaire :

- si homme d'âge > 50 ans.
- si femme d'âge > 60 ans.
- si antécédents familiaux.
- si fumeur.
- si diabète.
- si hypertension.
- si taux HDL < 40 .
- si poids > 90 kg.
- si activité physique/jour < 30 .
- si homme consommant plus de 3 verres/jour.
- si femme consommant plus de 2 verres/jour.



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Conclusion 1/2

Nice

Very nice applications + post-quantum encryption :-)
A lot of efforts and progresses (everything is moving really fast).
Quite a lot of implementations available now.

Making small applications affordable ! We are on the right way :-)



Conclusion 2[/2

BUT still a lot of (theoretical and practical) work to be done :

- **security** (to be better understood)
- **expansion** (to be better decreased and managed)
- **complexity** (new schemes, worst-case complexity, bootstrapping optimization, etc)
- implementation **optimization** (Boolean circuit, software & hardware)
- help programers to **choose** the right **scheme** with an adapted **setting**
/* comparison is not easy at all ! */

And beyond...

Functional Encryption

Similar but different paradigm to compute over encrypted data while giving access in clear to some computation results, according to different public keys.

Obfuscation

Strong links between FHE and indistinguishable Obfuscation.

Software certification in FHE context

Not yet efficiently addressed (only starting), but important in real applications if we want to trust computation !

Questions ?

Thanks to all co-authors and collaborators (academic & industry)



French activities :

- design (S/FHE + friendly symmetric)
- security analysis
- batching
- compilation : software, hardware
- benchmarking and parameters setting

Selected personal contributions

[CCCF+18] Stream Ciphers : A Practical Solution for Efficient Homomorphic-Ciphertext Compression, A. Canteaut, S. Carpov, C. Fontaine, T. Lepoint, M. Naya-Plasencia, P. Paillier, R. Sirdey. Journal of Cryptology, Volume 31, Issue 3, pp 885-916, July 2018.

[MBF18] Practical Parameters for Somewhat Homomorphic Encryption Schemes on Binary Circuits, V. Migliore, G. Bonnoron, C. Fontaine. IEEE Transactions on Computers, Special Section on Cryptographic Engineering in a Post-Quantum World, 2018.

[HBF18] Design and Implementation of Low Depth Pairing-based Homomorphic Encryption Scheme, V. Herbert, B. Biswas, C. Fontaine. Journal of Cryptographic Engineering, 2018.

[MMRL+18] Hardware/Software co-Design of an Accelerator for FV Homomorphic Encryption Scheme using Karatsuba Algorithm, V. Migliore, M. Méndez Real, V. Lapôtre, A. Tisserand, C. Fontaine, G. Gogniat. IEEE Transactions on Computers 67(3) :335-347, IEEE (2018).

[AFF+13] Recent advances in homomorphic encryption : a possible future for signal processing in the encrypted domain, C. Aguilar-Melchor, S. Fau, C. Fontaine, G. Gogniat, R. Sirdey. IEEE Signal Processing Magazine, Number 2, Volume 30, pp. 108-117 (2013), special issue "Signal Processing in the Encrypted Domain : When Cryptography Meets Signal Processing".