Automates d'arbre

TD n°3 : WSkS

Exercise 1 :

Produce formulae of WSkS for the following predicates :

- the set X has exactly two elements.
- the set X contains at least one string beginning with a 1.
- $x \leq_{lex} y$ where $\leq_{lex}$ is the lexicographic order on $\{1, ...k\}^*$.
- given a formula of WSkS ϕ with one free first-order variable, produce a formula of WSkS expressing that there is an infinity of words on $\{1, ..., k\}^*$ satisfying ϕ.
- the set X has an even number of elements.

Exercise 2 :

Prove that the predicate $x = 1y$ is not definable in WSkS.

Exercise 3 :

Let $n \in \mathbb{N}^*$, $n = \prod_{i=0}^{\infty} p_i^{e_i}$ its prime factorization and $e_i = \sum_{j=0}^{\infty} b_{i,j} 2^j$ the binary representation of e_i. We code n in WS2S by the set $S_n = \{1^i 2^j \mid b_{i,j} = 1\}$. Produce formulae of WS2S for the predicates $X = S_n$, $\exists n. X = S_n$ and $\exists n, m. X = S_n \wedge Y = S_m \wedge Z = S_{nm}$. Deduce that the first order theory of integers with multiplication and equality is decidable.

Exercise 4 :

Let L be a recognizable language on $\mathcal{F} = \{f_1, \ldots, f_n\}$. Let ϕ be a WSkS formula defining L, i.e., a formula with $n+1$ second order variables X, X_1, ..., X_n such that for every $t \in T(\mathcal{F})$,

$$Pos(t), Pos_{f_1}(t), \ldots, Pos_{f_n}(t) \models \phi \text{ iff } t \in L$$

where $Pos_{f_i}(t)$ is the set of positions of t labelled by f_i.
We want to prove that the language $\bar{L}$ of trees t such that for every decomposition $t = C[t']$ there is a decomposition $t' = C'[t'']$ with $C'[C[t'']] \in L$ is also recognizable. From ϕ, construct a WSkS formula $\bar{\phi}$ defining $\bar{L}$.