Automates d’arbre

TD n°2 : Decision problems & tree homomorphisms

Exercise 1 :
We consider the (GII) problem (ground instance intersection) :

Instance : t a term in \(T(F,X)\) and \(A\) a NFTA

Question : Is there at least one ground instance of \(t\) accepted by \(A\) ?

1) Suppose that \(t\) is linear. Prove that (GII) is P-complete.
   hint : you may use ideas from exercise 3 of TD1. For the hardness, reduce the emptiness.

2) Suppose that \(A\) is deterministic. Prove that (GII) is NP-complete.
   hint : for the hardness, reduce \((SAT)\).

3) Prove that \((GII)\) is EXPTIME-complete. 
   hint : for the hardness, reduce the intersection non-emptiness problem.

4) Deduce that the complement problem :
   Instance : \(t\) a term in \(T(F,X)\) and linear terms \(t_1, ..., t_n\)
   Question : Is there a ground instance of \(t\) which is not an instance of any \(t_i\) ?
   is decidable.

Exercise 2 :
A bottom-up tree transducer (NUTT) is a tuple \(U = (Q,F,F',Q_f,\Delta)\) where \(Q\) is a finite set (of states), \(F\) and \(F'\) are finite ranked sets (of input and output), \(Q_f \subseteq Q\) (final states) and \(\Delta\) is a finite set of rules of the form :

- \(f(q_1(x_1), ..., q_n(x_n)) \to q(u)\) where \(f \in F\) and \(u \in T(F', \{x_1, ..., x_n\})\)
- \(q(x_1) \to q'(u)\) where \(u \in T(F', \{x_1\})\).

We say that \(U\) is linear when the right side of the rules of \(\Delta\) are. This defines a rewrite system \(\to_U\) on \(T(F \cup F' \cup \{\}\)\). The relation induced by \(U\) is then \(R(U) = \{ (t, t') \mid t \in T(F), t' \in T(F'), t \to_U^* q(t'), q \in Q_f \}\).

1) Prove that tree morphisms are a special case of NUTT that is if \(\mu : T(F) \to T(F')\) is a morphism, then there exists a NUTT \(U_\mu\) such that \(R(U_\mu) = \{ (t, \mu(t)) \mid t \in T(F) \}\). Be sure that if \(\mu\) is linear then \(U_\mu\) is too.

2) Prove that the domain of a NUTT \(U\), that is \(\{ t \in T(F) \mid \exists t' \in T(F'), (t, t') \in U\}\), is recognizable.

3) Prove that the image of a recognizable tree language \(L\) by a linear NUTT \(U\), that is \(\{ t' \in T(F') \mid \exists t \in L, (t, t') \in U\}\), is recognizable.

Exercise 3 :
1) We can see the set of runs of an NFTA \(A = (Q,F,Q_f,\Delta)\) as a tree language on \(F \times Q = \{(f,q)(n) \mid f(n) \in F, q \in Q\}\) as the smallest set \(Run(A)\) included in \(T(F \times Q)\) such that :
   - if \(a \to q \in \Delta\), then \((a,q) \in Run(A)\)
   - if \(f(q_1, ..., q_n) \to q \in \Delta\) and \(t_1, ..., t_n \in Run(A)\) with \(t_i(\varepsilon) = (_, q_i)\) then \((f, q)(t_1, ..., t_n) \in Run(A)\).

Then the set of accepting runs can be seen as \(Acc(A) = \{ t \in Run(A) \mid t(\varepsilon) = (_, q), q \in Q_f \}\).

Prove that \(Acc(A)\) is in the smallest class \(Stab\) of sets which contains all the \(T(F)\) for any finite ranked set \(F\) and which is stable by image of linear morphisms and inverse image of morphisms. For example, you should be able to prove that \(Acc(A) = \beta^{-1}(\gamma(\delta^{-1}(T(F'))))\) where \(\gamma\) is linear.

2) Deduce that \(Stab = Rec\).