Automates d'arbre

TD n°2 : Decision problems & tree homomorphisms

Exercise 1 :

We consider the **(GII)** problem (ground instance intersection) :
Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and $\mathcal{A}$ a NFTA
Question : Is there at least one ground instance of t accepted by $\mathcal{A}$?

1) Suppose that t is linear. Prove that **(GII)** is P-complete.
hint : you may use ideas from exercise 3 of TD1. For the hardness, reduce the emptiness.
2) Suppose that $\mathcal{A}$ is deterministic. Prove that **(GII)** is NP-complete.
*hint : for the hardness, reduce **(SAT)**.*
3) Prove that **(GII)** is EXPTIME-complete.
hint : for the hardness, reduce the intersection non-emptiness problem.
4) Deduce that the complement problem :
Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and linear terms t_1, ..., t_n
Question : Is there a ground instance of t which is not an instance of any t_i ?
is decidable.

Exercise 2 :

A bottom-up tree transducer (NUTT) is a tuple $U = (Q, \mathcal{F}, \mathcal{F}', Q_f, \Delta)$ where Q is a finite set (of states), $\mathcal{F}$ and $\mathcal{F}'$ are finite ranked sets (of input and output), $Q_f \subseteq Q$ (final states) and Δ is a finite set of rules of the form :

- $f(q_1(x_1), ..., q_n(x_n)) \to q(u)$ where $f \in \mathcal{F}$ and $u \in T(\mathcal{F}', \{x_1, ..., x_n\})$
- $q(x_1) \to q'(u)$ where $u \in T(\mathcal{F}', \{x_1\})$.

We say that U is linear when the right side of the rules of Δ are. This defines a rewrite system $\to_U$ on $T(\mathcal{F} \cup \mathcal{F}' \cup Q)$. The relation induced by U is then $\mathcal{R}(U) = \big\{(t, t') \mid t \in T(\mathcal{F}),\, t' \in T(\mathcal{F}'),\, t \to_U^* q(t'),\, q \in Q_f\big\}$.

1) Prove that tree morphisms are a special case of NUTT that is if $\mu : T(\mathcal{F}) \longrightarrow T(\mathcal{F}')$ is a morphism, then there exists a NUTT U_μ such that $\mathcal{R}(U_\mu) = \big\{(t, \mu(t)) \mid t \in T(\mathcal{F})\big\}$. Be sure that if μ is linear then U_μ is too.
2) Prove that the domain of a NUTT U, that is $\big\{t \in T(\mathcal{F}) \mid \exists t' \in T(\mathcal{F}'),\, (t, t') \in U\big\}$, is recognizable.
3) Prove that the image of a recognizable tree language L by a linear NUTT U, that is $\big\{t' \in T(\mathcal{F}') \mid \exists t \in L,\, (t, t') \in U\big\}$, is recognizable.

Exercise 3 :

1) We can see the set of runs of an NFTA $\mathcal{A} = (Q, \mathcal{F}, Q_f, \Delta)$ as a tree language on $\mathcal{F} \times Q = \big\{(f, q)(n) \mid f(n) \in \mathcal{F},\, q \in Q\big\}$ as the smallest set $Run(\mathcal{A})$ included in $T(\mathcal{F} \times Q)$ such that :
   - if $a \to q \in \Delta$, then $(a, q) \in Run(\mathcal{A})$
   - if $f(q_1, ..., q_n) \to q \in \Delta$ and $t_1, ..., t_n \in Run(\mathcal{A})$ with $t_i(\epsilon) = (\_, q_i)$ then $(f, q)(t_1, ..., t_n) \in Run(\mathcal{A})$.

   Then the set of accepting runs can be seen as $Acc(\mathcal{A}) = \big\{t \in Run(\mathcal{A}) \mid t(\epsilon) = (\_, q),\, q \in Q_f\big\}$.
   Prove that $Acc(\mathcal{A})$ is in the smallest class **Stab** of sets which contains all the $T(\mathcal{F})$ for any finite ranked set $\mathcal{F}$ and which is stable by image of linear morphisms and inverse image of morphisms. For example, you should be able to prove that $Acc(\mathcal{A}) = \beta^{-1}(\gamma(\delta^{-1}(T(\mathcal{F}'))))$ where γ is linear.
2) Deduce that **Stab** = **Rec**.