Automates d'arbre

TD n°1 : Recognizable Tree Languages and Finite Tree Automata*

Exercise 1 :

Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down DFTA for the set $G(t)$ of ground instances of the term $t = f\big(f(a,x), g(y)\big)$ which is defined by :

$$G(t) = \Big\{ f\big(f(a,u), g(v)\big) \mid u, v \in T(\mathcal{F}) \Big\}$$

Exercise 2 :

Are the following tree languages recognizable (by a bottom-up FTA) ?

- $\mathcal{F} = \{f(2), a(0)\}$ and $L = \{f(t,t) \mid t \in T(\mathcal{F})\}$.
- $\mathcal{F} = \{g(1), a(0)\}$ and L the set of ground terms of even height.
- $\mathcal{F} = \{f(2), g(1), a(0)\}$ and L the set of ground terms of even height.

Exercise 3 :

1) Let $\mathcal{E}$ be a finite set of linear terms on $T(\mathcal{F}, \mathcal{X})$. Prove that $Red(\mathcal{E}) = \{C[t\sigma] \mid C \in \mathcal{C}(\mathcal{F}), t \in \mathcal{E}, \sigma \text{ ground substitution}\}$ is recognizable.
2) Prove that if $\mathcal{E}$ contains only ground terms, then one can construct a DFTA recognizing $Red(\mathcal{E})$ whose number of states is at most $n+2$, where n is the number of nodes of $\mathcal{E}$.

Exercise 4 :

Let $\mathcal{F} = \{f(2), a(0), b(0)\}$.

1) Let L_1 be the smallest set such that :
   - $f(a,b) \in L_1$
   - $t \in L_1 \Rightarrow f(a, f(t,b)) \in L_1$

   Prove that L_1 is recognizable.
2) Prove that $L_2 = \{t \in T(\mathcal{F}) \mid |t|_a = |t|_b\}$ is not recognizable.
3) Let L be recognizable on $\mathcal{F}$ and $C(L)$ be the closure of L by the congruence generated by the equation $f(x,y) = f(y,x)$. Prove that $C(L)$ is recognizable.
4) Let L be recognizable on $\mathcal{F}$ and $AC(L)$ be the closure of L by the congruence generated by the equations $f(x,y) = f(y,x)$ and $f(x, f(y,z)) = f(f(x,y),z)$. Prove that $AC(L)$ is not recognizable in general.
5) Let L be recognizable on $\mathcal{F}$ and $A(L)$ be the closure of L by the congruence generated by the equation $f(x, f(y,z)) = f(f(x,y),z)$. Prove that $A(L)$ is not recognizable in general.

Exercise 5 :

Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down NFTA for the set $M(t)$ of terms which have a ground instance of the term $t = f\big(a, g(x)\big)$ as a subterm, ie. $M(t) = \Big\{C\big[f\big(a, g(u)\big)\big] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F})\Big\}$.

Prove that $M(t)$ is not recognizable by a top-down DFTA or even a finite union of languages recognizable by a top-down DFTA.

hint : you can use without proof the following fact (prove it if you have time) : let t be a tree. The path language $\pi(t)$ is defined by :

- *if t is a constant, $\pi(t) = \{t\}$*
- *if $t = f(t_1, ..., t_n)$, $\pi(t) = \cup_{i=1}^{n} \{fiw \mid w \in \pi(t_i)\}$*

Let L be a tree language. The path language of L is $\pi(L) = \cup_{t \in L} \pi(t)$. The path closure of L is defined by $pathclosure(L) = \{t \mid \pi(t) \subseteq \pi(L)\}$. L is recognizable by a top-down DFTA iff L is recognizable and path closed, ie. $L = pathclosure(L)$.

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*taken from *Tree Automata Techniques and Applications*