
Robot Games

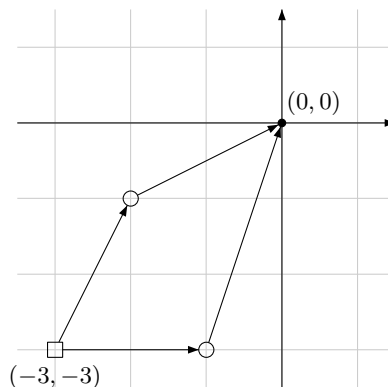
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We present robot games, and we give the simplest definitions for which decidability is not known.

1. Definition. Let $U, V \subseteq \mathbb{Z}^2$ be two finite sets of two-dimensional integer vectors. A *robot game* is played in rounds from an initial configuration $x_0 \in \mathbb{Z}^2$ as follows. In each round, player 2 chooses a vector $v \in V$, then player 1 chooses a vector $u \in U$, and the configuration in the next round is $x + v + u$ where x is the configuration in the current round. The objective of player 1 is to reach the configuration $(0, 0)$. A *strategy* for player 1 is a function $\sigma : \mathbb{Z}^2 \rightarrow U$ and a *strategy* for player 2 is a function $\pi : \mathbb{Z}^2 \rightarrow V$. The *play* according to σ and π from initial configuration x_0 is the infinite sequence $x_0 x_1 \dots$ such that for all $i \geq 0$, we have $x_{i+1} = x_i + v + u$ where $v = \pi(x_i)$ and $u = \sigma(x_i + v)$.

A configuration x_0 is *winning* for player 1 if there exists a strategy σ such that for all strategies π , in the resulting play from x_0 there exists $i \geq 0$ such that $x_i = (0, 0)$.

2. Example. Let $U = \{(1, 3), (2, 1)\}$ and $V = \{(2, 0), (1, 2)\}$. The initial configuration $(-3, -3)$ is winning for player 1. The set of winning configurations for player 1 is $\{(-3k, -3k) \mid k \geq 0\}$. Note that the set of winning configurations is closed under sum (i.e., $x + y$ is a winning configuration if x and y are winning configurations).



3. Decision problem. Given an initial configuration $x_0 \in \mathbb{Z}^2$ and two finite sets $U, V \subseteq \mathbb{Z}^2$, the problem is to decide whether x_0 is a winning configuration in the robot game defined by U, V . Whether this problem is decidable and what is its complexity are open questions.

The problem is undecidable if the game is played on a graph *with states* of player 1 and states of player 2, with \mathbb{Z}^2 or \mathbb{N}^2 as the vector space (as in games on VASS, vector-addition systems with states) [1, 2].

The one-player version of robot games (i.e., where $V = \{(0, 0)\}$) is decidable in polynomial time by a reduction to linear programming. The robot games defined in one dimension (with $U, V \subseteq \mathbb{Z}$ and $x_0 \in \mathbb{Z}$) are also decidable.

4. Extension. Extensions can be considered in several directions:

- Robot games in dimension $d \geq 3$.
- Reachability objectives can be defined by a (possibly upward-closed) set of target configurations.
- Players have internal states (e.g., for player 1 the set U of available moves may change as the game is played, according to some finite-state machine).
- And many others.

5. Partial results.

- The problem is undecidable in dimension $d \geq 9$, and in dimension $d \geq 3$ if player 1 has internal states [3]. In general, robot games in dimension d and internal states for player 1 can be reduced to games in dimension $d + 6$ and no states [3].

References

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- [2] T. Brázdil, P. Jancar, and A. Kucera. Reachability games on extended vector addition systems with states. In *Proc. of ICALP*, LNCS 6199, pages 478–489. Springer, 2010.
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