Thursday

Exercise 1 *Eliminate the cuts in the proof*

$$\dfrac{\dfrac{\dfrac{}{\exists x\ (P(x) \Rightarrow P(x)) \vdash \exists x\ (P(x) \Rightarrow P(x))}\ \text{axiom}}{\vdash \exists x\ (P(x) \Rightarrow P(x)) \Rightarrow \exists x\ (P(x) \Rightarrow P(x))}\ \Rightarrow\text{-intro} \qquad \dfrac{\dfrac{\dfrac{}{P(c) \vdash P(c)}\ \text{axiom}}{\vdash P(c) \Rightarrow P(c)}\ \Rightarrow\text{-intro}}{\vdash \exists x\ (P(x) \Rightarrow P(x))}\ \exists\text{-intro}}{\vdash \exists x\ (P(x) \Rightarrow P(x))}\ \Rightarrow\text{-elim}$$

What is the proof-term associated to this proof?
Reduce it to its irreducible form.

Exercise 2 *Find a proof π that contains a single cut but such that eliminating this cut in π creates other cuts.*

Exercise 3

1. *Prove that a proof that is constructive, cut-free and without any axioms, ends with an introduction rule.*
2. *Show that each hypothesis is necessary.*

Exercise 4 *Consider a set E and the set $R = \{x \in E \mid \neg x \in x\}$. Consider the rule*

$$x \in R \longrightarrow x \in E \wedge \neg x \in x$$

1. *Prove the sequent $R \in R \vdash \bot$.*
2. *Prove the sequent $\vdash \neg R \in R$.*
3. *Prove the sequent $R \in E \vdash R \in R$.*
4. *Prove the sequent $R \in E \vdash \bot$.*
5. *Prove the sequent $\vdash \neg R \in E$.*
6. *Eliminate the cuts in this proof.*