Wednesday

Exercise 1

1. *Prove that if the sequent* $\Gamma \vdash \neg\neg A$ *has a proof then so does the sequent* $\Gamma \vdash A$*.*
2. *Give a constructive proof of the proposition*

$$\neg\neg(A \vee \neg A)$$

3. *Let* $\mathcal{ND}$ *be the set of rules of Natural deduction and* $\mathcal{ND}'$ *be the set of rules obtained by replacing the excluded middle with the rule*

$$\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}$$

Show that a sequent $\Gamma \vdash B$ *is provable in one system if and only if it is provable in the other.*

Exercise 2

1. *In arithmetic, give a proof of the proposition* $N(100)$*.*
2. *In arithmetic, give a proof of the sequent* $P(0), \forall x\ (P(x) \Rightarrow P(S(x))) \vdash P(100)$*.*
3. *Give a proof of this sequent in the empty theory.*