Tuesday

Exercise 1 *Consider a language with three sorts of terms:* point*,* line *and* scalar*, two predicate symbols* $=$ *with arity* $\langle \text{scalar}, \text{scalar} \rangle$ *and* $\in$ *with arity* $\langle \text{point}, \text{line} \rangle$ *and two function symbols* d*,* distance*, with arity* $\langle \text{point}, \text{point}, \text{scalar} \rangle$ *and* b*,* bisector*, with arity* $\langle \text{point}, \text{point}, \text{line} \rangle$*. Let* Γ *be the set containing the propositions*

$$\forall x \forall y \forall z \; (x \in b(y,z) \Leftrightarrow d(x,y) = d(x,z))$$

$$\forall x \forall y \forall z \; ((x = y \wedge y = z) \Rightarrow x = z)$$

and A *a proposition stating that if two bisectors of the triangle* xyz *intersect at a point* w*, then the three bisectors intersect at this point:*

$$\forall w \forall x \forall y \forall z \; ((w \in b(x,y) \wedge w \in b(y,z)) \Rightarrow w \in b(x,z))$$

Write a proof of the sequent $\Gamma \vdash A$*.*

Exercise 2 *Prove that if the sequents* $\Gamma \vdash B$ *is provable in Natural deduction, then so is the sequent* $\Gamma, A \vdash B$*.*

Exercise 3

1. *Prove that if the sequents* $\Gamma, A \vdash B$ *and* $\Gamma \vdash A$ *are provable in Natural deduction, then so is the sequent* $\Gamma \vdash B$*.*
2. *Prove that if the sequent* $\Gamma, A, B \vdash C$ *is provable, then the sequent* $\Gamma, A \wedge B \vdash C$ *is provable.*

Exercise 4 *Prove that if the sequents* $\Gamma, \neg A \vdash \bot$ *is provable in Natural deduction, then so is the sequent* $\Gamma \vdash A$*.*