

$$\overline{\Gamma \vdash A} \text{ axiom } A \in \Gamma$$

$$\overline{\Gamma \vdash \top} \top\text{-intro}$$

$$\frac{\Gamma \vdash \perp}{\overline{\Gamma \vdash A}} \perp\text{-elim}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\overline{\Gamma \vdash A \wedge B}} \wedge\text{-intro}$$

$$\frac{\Gamma \vdash A \wedge B}{\overline{\Gamma \vdash A}} \wedge\text{-elim}$$

$$\frac{\Gamma \vdash A \wedge B}{\overline{\Gamma \vdash B}} \wedge\text{-elim}$$

$$\frac{\Gamma \vdash A}{\overline{\Gamma \vdash A \vee B}} \vee\text{-intro}$$

$$\frac{\Gamma \vdash B}{\overline{\Gamma \vdash A \vee B}} \vee\text{-intro}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\overline{\Gamma \vdash C}} \vee\text{-elim}$$

$$\frac{\Gamma, A \vdash B}{\overline{\Gamma \vdash A \Rightarrow B}} \Rightarrow\text{-intro}$$

$$\frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\overline{\Gamma \vdash B}} \Rightarrow\text{-elim}$$

$$\frac{\Gamma, A \vdash \perp}{\overline{\Gamma \vdash \neg A}} \neg\text{-intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\overline{\Gamma \vdash \perp}} \neg\text{-elim}$$

$$\frac{\Gamma \vdash A}{\overline{\Gamma \vdash \forall x A}} \forall\text{-intro } x \text{ not free in } \Gamma$$

$$\frac{\Gamma \vdash \forall x A}{\overline{\Gamma \vdash (t/x)A}} \forall\text{-elim}$$

$$\frac{\Gamma \vdash (t/x)A}{\overline{\Gamma \vdash \exists x A}} \exists\text{-intro}$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\overline{\Gamma \vdash B}} \exists\text{-elim } x \text{ not free in } \Gamma, B$$

$$\overline{\overline{\Gamma \vdash A \vee \neg A}} \text{ excluded middle}$$