What means "being a Turing machine"?

Gilles Dowek

A cell, a brain, a ribosome, the universe, a computer... is a Turing machine

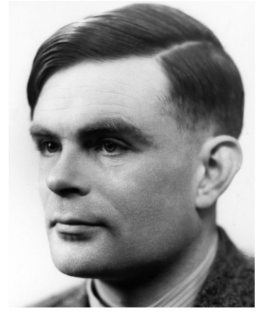
What does this mean?

A single or a plurality of meanings?

I. What is a Turing machine?

To which question does the Turing machine answer?

Does there exist algorithm to solve the problem X?



No

When you answer "yes": just provide an algorithm
E.g. Is there an algorithm to find a word in the dictionary?
Yes: compare this word to every word in the dictionary

But proving that there is no algorithm to solve a problem requires to characterise all functions that can be computed by an algorithm

Turing's characterisation

An abstract machine mimicking the computing human

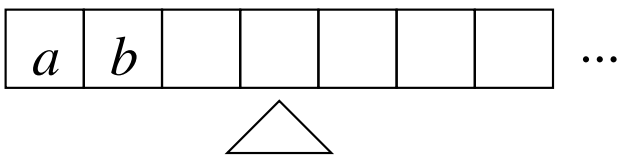
Squared sheet of a notebook

The tape: 1D, 2D...: immaterial

One tape, two tapes...: immaterial

A symbol in each square

A writing and reading head / hand / eye



Step by step

The head reads the symbol on the tape, and in function of this symbol and of its internal state

- writes a symbol
- moves the head left, right, or not
- evolves to another internal state

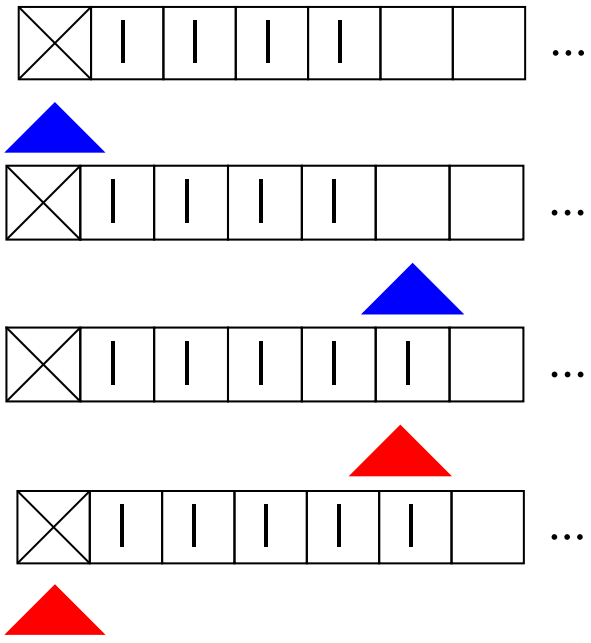
Each Turing machine defined by a transition function that tells

- what to write, how to move, in which state to evolve

in function of

- the symbol read, the current state

An example



$\times, \bullet \longrightarrow \times, \text{right}, \bullet$

$|, \bullet \longrightarrow |, \text{right}, \bullet$

$\text{blank}, \bullet \longrightarrow |, \text{same}, \bullet$

$|, \bullet \longrightarrow |, \text{left}, \bullet$

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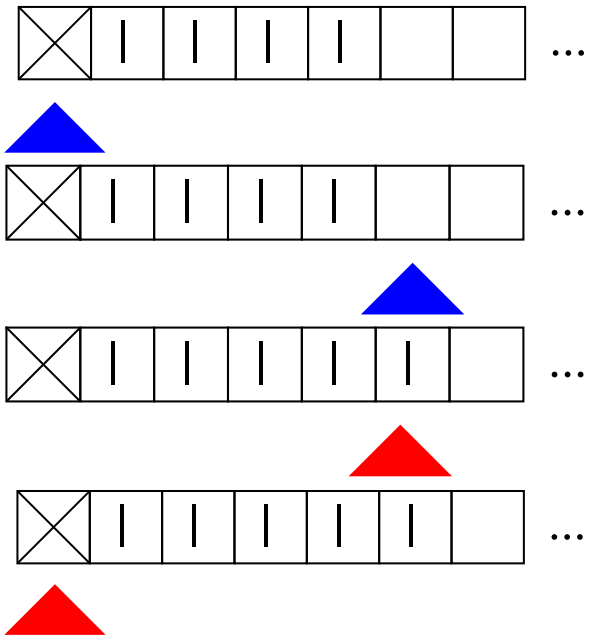

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Adding one to a number represented by stokes

(non) Determinism

$$
\begin{array}{rcl}
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\end{array}
$$

In this example: only one rule for each situation (e.g. $|, \bullet$)
Deterministic Turing machine

In others: several
Non deterministic Turing machine
A set of possible executions and results

II. The diversity and equivalence of computation models

Turing's characterisation: Mimic the computing human

Gandy's characterisation: Mimic nature (physical system, machine)

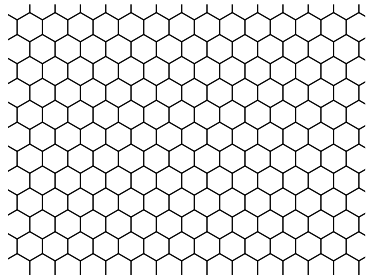
Gandy's hypotheses (about nature)

(1.) Space and time are homogeneous

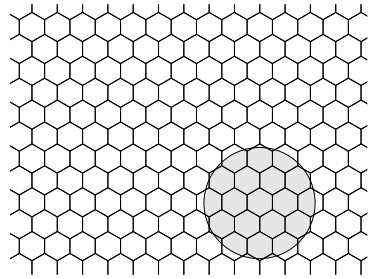
(2.) Information has a bounded velocity

(3.) Information has a bounded density

Discretise space and time in an arbitrary way



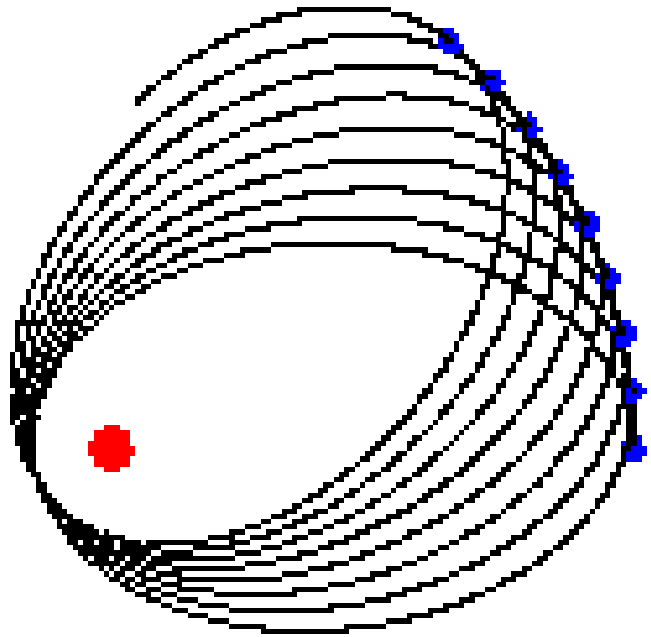
- Each cell has a finite state space (3.)
- The state of a cell depends of the state of a finite number of cells the previous time step (2.)



- Local evolution function finite and homogeneous (1.)

Cellular automata

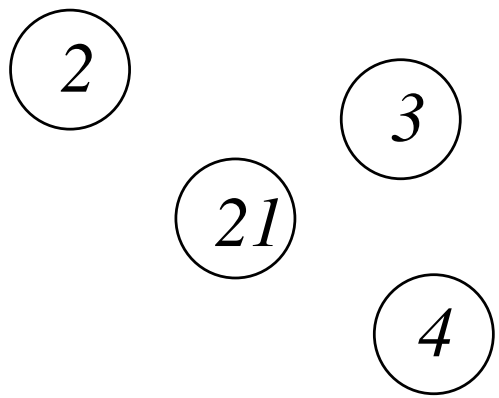
A cellular automaton to compute the geodesics of a gravitational field



With Pablo Arrighi

Many other computational models

Moving, interacting / communicating objects
Chemistry, swarm computing...



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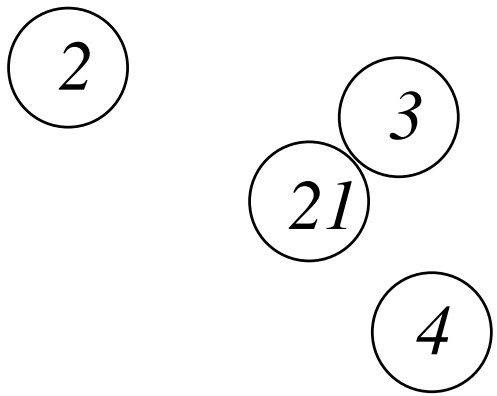

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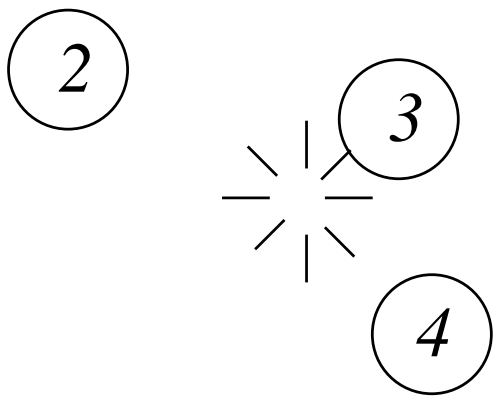

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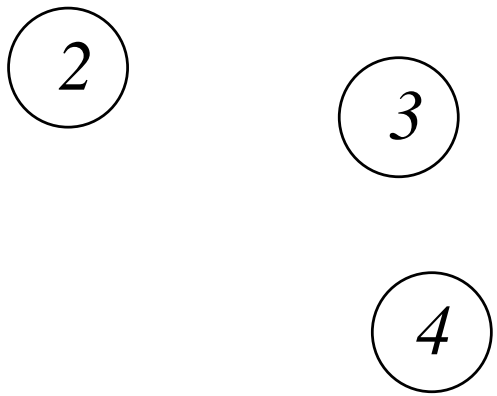

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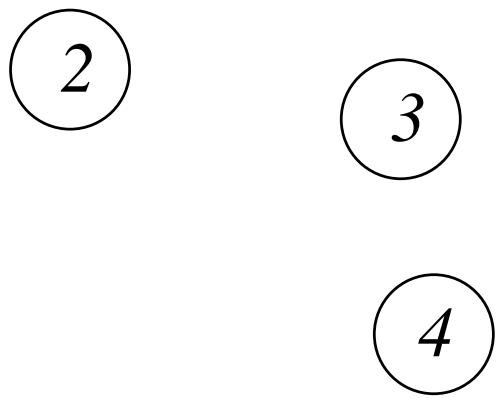


$(n)(n \times p) \longrightarrow n$ survival to the fittest

The main idea of the talk

All these computational models are equivalent

All these computational models are equivalent

They compute the same functions

Each can simulate each other "step by step"

(The Church-Turing thesis)

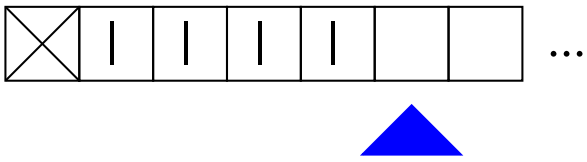
III. Programs, data, programs as data

Data and algorithm

In most of computational models: a distinction between

- the data
- the algorithm

An example: Turing machines: Data



Data stored in two types of memory: tape and internal state

Metaphor of the notebook: premonition of the theme of externalization of memory (clay tablets, notebooks, libraries, disc storage...)

An example: Turing machines: Algorithm

In the "hardware" of the abstract machine

$$
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$$

Itself a bunch of symbols

Algorithms as data

Can be expressed in the tape of (another) Turing machine

Turing machines interpreting, analysing, building, transforming Turing machines

Like commercial computers (RAM machines) where data and programs are stored in the same place
(Unlike Harvard machines, more resistant to computer viruses)

Porosity between data and algorithms: a common point between complex machines and life?

IV. What means "being a Turing machine"?

Literal meaning

A cell, a brain, a ribosome, the universe, a computer...

has a tape, made of squares, on which symbols are written, a head...

Always false
Most of the time a stupid question

Turing machine as a metonymy for "Computational model"

All computational models equivalent, name all by the name of one

A cell, a brain, a ribosome, the universe, a computer...

- cannot compute more than a Turing machine
- could be simulated "step by step" by a Turing machine

True if the Church-Turing thesis holds
Always an interesting question (How?)

Intermediate meanings (1)

A Turing machine is sequential...
A cellular automaton is parallel...

The universe is more a cellular automaton than a Turing machine

Intermediate meanings (2)

A Turing machine

- uses symbols
- structured in words
- with a moving focus on one symbol

Like others (weaker) computational models: finite state automata, finite state transducers...

Like a ribosome?

Intermediate meanings (3)

A Turing machine: algorithms, data and a porosity between them

Allowing machines interpreting, analysing, building, transforming... machines

Say what you mean

By default: the weaker meaning: does not compute more than a Turing machine

But many interesting more specific meanings, this meeting may explore