Induction and computation

The language of arithmetic contains two sorts of terms: ι for the natural numbers and κ for the classes (sets) of natural numbers. Among other symbols, there are

- a constant 0 of sort ι,
- a function symbol S of arity $\langle \iota, \iota \rangle$,
- a predicate symbol N of arity $\langle \iota \rangle$,
- and a predicate symbol $\in$ of arity $\langle \iota, \kappa \rangle$.

Among other computation rules, there is a rule

$$N(y) \longrightarrow \forall c\ (0 \in c \Rightarrow \forall x\ (x \in c \Rightarrow S(x) \in c) \Rightarrow y \in c)$$

1. Let c be a class, π be a proof of the proposition $0 \in c$, and π' be a proof of the proposition $\forall x\ (x \in c \Rightarrow S(x) \in c)$.

   Build a proof of
   $$S(0) \in c$$
   build a proof of
   $$S(S(0)) \in c$$
   build a proof of
   $$S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(0))))))))))))))))))))) \in c$$

2. Build a proof of
   $$N(0)$$
   build a proof of
   $$N(S(0)))$$
   build a proof of
   $$N(S(S(0)))$$
   build a proof of
   $$N(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(0))))))))))))))))))))))$$

3. Let c be a class, π be a proof of the proposition $0 \in c$, π' be a proof of the proposition $\forall x\ (x \in c \Rightarrow S(x) \in c)$, and ρ a proof of the proposition $N(y)$.

   Build a proof of
$$y \in c$$

4. Let c be a class, π be a proof of the proposition $0 \in c$, and π' be a proof of the proposition $\forall x\ (x \in c \Rightarrow S(x) \in c)$.

   Build a proof of
$$\forall y\ (N(y) \Rightarrow y \in c)$$

5. Use this proof with the term

$$S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(S(0)))))))