

# On the critical pairs of a rewrite system for vector spaces

Pablo Arrighi\* and Gilles Dowek†

In this note we prove that all the critical pairs of a rewrite system for vector spaces close. In this rewrite system the symbols  $+$  (for scalars),  $\times$ , and  $\cdot$  (for vectors) are associative and commutative. This result is used in [2] to prove the confluence of the rewrite system defining the *Lineal* linear-algebraic  $\lambda$ -calculus. This rewrite system is formed of the rules

$$\begin{array}{llll}
 (1) & \mathbf{0} + \alpha & \longrightarrow & \alpha \\
 (1e) & \mathbf{0} + \alpha + \chi & \longrightarrow & \alpha + \chi \\
 (2) & \mathbf{0} \times \alpha & \longrightarrow & \mathbf{0} \\
 (2e) & \mathbf{0} \times \alpha + \chi & \longrightarrow & \mathbf{0} + \chi \\
 (3) & \mathbf{1} \times \alpha & \longrightarrow & \alpha \\
 (3e) & \mathbf{1} \times \alpha + \chi & \longrightarrow & \alpha + \chi \\
 (4) & \alpha \times (\beta + \gamma) & \longrightarrow & (\alpha \times \beta) + (\alpha \times \gamma) \\
 (4e) & \alpha \times (\beta + \gamma) + \chi & \longrightarrow & (\alpha \times \beta) + (\alpha \times \gamma) + \chi \\
 (5) & \mathbf{u} + \mathbf{0} & \longrightarrow & \mathbf{u} \\
 (5e) & \mathbf{u} + \mathbf{0} + \mathbf{x} & \longrightarrow & \mathbf{u} + \mathbf{x} \\
 (6) & \mathbf{0} \cdot \mathbf{u} & \longrightarrow & \mathbf{0} \\
 (7) & \mathbf{1} \cdot \mathbf{u} & \longrightarrow & \mathbf{u} \\
 (8) & \alpha \cdot \mathbf{0} & \longrightarrow & \mathbf{0} \\
 (9) & \alpha \cdot (\beta \cdot \mathbf{u}) & \longrightarrow & (\alpha \times \beta) \cdot \mathbf{u} \\
 (10) & \alpha \cdot (\mathbf{u} + \mathbf{v}) & \longrightarrow & \alpha \cdot \mathbf{u} + \alpha \cdot \mathbf{v} \\
 (f1) & \alpha \cdot \mathbf{u} + \beta \cdot \mathbf{u} & \longrightarrow & (\alpha + \beta) \cdot \mathbf{u} & (*) \\
 (f1e) & \alpha \cdot \mathbf{u} + \beta \cdot \mathbf{u} + \mathbf{x} & \longrightarrow & (\alpha + \beta) \cdot \mathbf{u} + \mathbf{x} & (*) \\
 (f2) & \alpha \cdot \mathbf{u} + \mathbf{u} & \longrightarrow & (\alpha + 1) \cdot \mathbf{u} & (*) \\
 (f2e) & \alpha \cdot \mathbf{u} + \mathbf{u} + \mathbf{x} & \longrightarrow & (\alpha + 1) \cdot \mathbf{u} + \mathbf{x} & (*) \\
 (f3) & \mathbf{u} + \mathbf{u} & \longrightarrow & (1 + 1) \cdot \mathbf{u} & (*) \\
 (f3e) & \mathbf{u} + \mathbf{u} + \mathbf{x} & \longrightarrow & (1 + 1) \cdot \mathbf{u} + \mathbf{x} & (*)
 \end{array}$$

(\*) the term  $\mathbf{u}$  is closed and normal.

The rules 1, 1e, 2, 2e, 3, 3e, 4, and 4e form the system  $S_{0ext}$ , the rules 5, 5e, 6, 7, 8, 9, and 10 form the system  $E_{ext}$ , and the rules f1, f1e, f2, f2e, f3, and f3e for the system  $F_{ext}$ .

The system  $S_{0ext} \cup E_{ext}$  is non conditional, so we can check automatically

---

\*Université de Grenoble, Laboratoire LIG, 220 rue de la Chimie, 38400 Saint-Martin d'Hère, France, [pablo.arrighi@imag.fr](mailto:pablo.arrighi@imag.fr).

†INRIA, 23 avenue d'Italie, CS 81321, 75214 Paris Cedex 13, France, [gilles.dowek@inria.fr](mailto:gilles.dowek@inria.fr).

that its critical pairs close, for instance using the system CIME [1]. Thus, we do not need to check that the critical pairs where both rules are in  $S_{0_{ext}} \cup E_{ext}$  close. We have generated the other critical pairs, where one rule is in  $S_{0_{ext}} \cup E_{ext}$  and the other in  $F_{ext}$  or where both rules are in  $F_{ext}$ , again using CIME. We got 251 distinct critical pairs, 81 of these critical pairs do not verify the condition (\*), we show that the 170 other close. There are basically two cases. In the first, we have a critical pair, e.g.

$$\mathbf{0} + \alpha.\mathbf{u} \longleftarrow \mathbf{0}.\mathbf{u} + \alpha.\mathbf{u} \longrightarrow (0 + \alpha).\mathbf{u}$$

and we close it, in this example on  $\alpha.\mathbf{u}$ . In the second we have a critical pair, e.g.

$$\alpha.\mathbf{u} + \alpha.\mathbf{v} + \beta.(\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) \longrightarrow (\alpha + \beta).(\mathbf{u} + \mathbf{v})$$

and from the fact that this critical pair opens, we deduce that some terms, in this example  $\mathbf{u} + \mathbf{v}$ , must be closed and normal, then we deduce that some of their subterms, in this example  $\mathbf{u}$  and  $\mathbf{v}$ , are closed and normal, and this is used to close the critical pair, in this case on  $(\alpha + \beta).\mathbf{u} + (\alpha + \beta).\mathbf{v}$ .

### **f1 vs. 6 (1 pair)**

$$\mathbf{0} + \alpha.\mathbf{u} \longleftarrow \mathbf{0}.\mathbf{u} + \alpha.\mathbf{u} \longrightarrow (0 + \alpha).\mathbf{u}$$

closes on  $\alpha.\mathbf{u}$ .

### **f1 vs. 7 (1 pair)**

$$\alpha.\mathbf{u} + \mathbf{u} \longleftarrow \mathbf{1}.\mathbf{u} + \alpha.\mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u}$ .

### **f1 vs. 8 (1 pair)**

$$\mathbf{0} + \alpha.\mathbf{0} \longleftarrow \alpha.\mathbf{0} + \beta.\mathbf{0} \longrightarrow (\alpha + \beta).\mathbf{0}$$

closes on  $\mathbf{0}$ .

### **f1 vs. 9 (1 pair)**

$$(\alpha \times \gamma).\mathbf{u} + \beta.(\gamma.\mathbf{u}) \longleftarrow \alpha.(\gamma.\mathbf{u}) + \beta.(\gamma.\mathbf{u}) \longrightarrow (\alpha + \beta).(\gamma.\mathbf{u})$$

$\gamma.\mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  is closed and normal, closes on  $(\alpha \times \gamma + \beta \times \gamma).\mathbf{u}$ .

### **f1 vs. 10 (2 pairs)**

$$\alpha.\mathbf{u} + \alpha.\mathbf{v} + \beta.(\mathbf{u} + \mathbf{v}) \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) \longrightarrow (\alpha + \beta).(\mathbf{u} + \mathbf{v})$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \beta).\mathbf{u} + (\alpha + \beta).\mathbf{v}$ .

$$\alpha.((\beta + \gamma).\mathbf{u}) \leftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u}) + \alpha.(\gamma.\mathbf{u})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u}$ .

### **f2 vs. 5 (2 pairs)**

$$\alpha.\mathbf{0} \leftarrow \mathbf{0} + \alpha.\mathbf{0} \longrightarrow (1 + \alpha).\mathbf{0}$$

closes on  $\mathbf{0}$ .

$$\alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{u} \leftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

### **f2 vs. 5e (2 pairs)**

$$\alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{u} \leftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

$$\alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \leftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u} + \mathbf{v})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{0}$  is never closed and normal.

### **f2 vs. 6 (1 pair)**

$$\mathbf{0} + \mathbf{u} \leftarrow \mathbf{0}.\mathbf{u} + \mathbf{u} \longrightarrow (0 + 1).\mathbf{u}$$

closes on  $\mathbf{u}$ .

### **f2 vs. 7 (1 pair)**

$$\mathbf{u} + \mathbf{u} \leftarrow 1.\mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).\mathbf{u}$$

closes on  $(1 + 1).\mathbf{u}$ .

### **f2 vs. 8 (1 pair)**

$$\mathbf{0} + \mathbf{0} \leftarrow \mathbf{0} + \alpha.\mathbf{0} \longrightarrow (1 + \alpha).\mathbf{0}$$

closes on  $\mathbf{0}$ .

## f2 vs. 9 (1 pair)

$$(\alpha \times \beta).\mathbf{u} + \beta.\mathbf{u} \longleftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} \longrightarrow (1 + \alpha).(\beta.\mathbf{u})$$

$\beta.\mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  is closed and normal, closes on  $(\alpha \times \beta + \beta).\mathbf{u}$ .

## f2 vs. 10 (4 pairs)

$$\alpha.\mathbf{u} + \alpha.\mathbf{v} + \mathbf{u} + \mathbf{v} \longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\mathbf{u} + \mathbf{v})$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + 1).\mathbf{v}$ .

$$\alpha.((1 + \beta).\mathbf{u}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u}) + \alpha.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha).\mathbf{u}$ .

$$\alpha.((1 + \beta).(\mathbf{u} + \mathbf{v})) \longleftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v}) \longrightarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{u}) + \alpha.\mathbf{v}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + (\alpha \times \beta + \alpha).\mathbf{v}$ .

$$\alpha.((1 + \beta).(\mathbf{u} + \mathbf{v})) \longleftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v}) \longrightarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \alpha.\mathbf{u}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + (\alpha \times \beta + \alpha).\mathbf{v}$ .

## f3 vs. 5 (2 pairs)

$$\mathbf{0} \longleftarrow \mathbf{0} + \mathbf{0} \longrightarrow (1 + 1).\mathbf{0}$$

closes on  $\mathbf{0}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{u} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{0} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

## f3 vs. 5e (3 pairs)

$$\mathbf{0} + \mathbf{u} + \mathbf{u} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{0} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

$$\mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\mathbf{0} + \mathbf{u} + \mathbf{v})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{0}$  is never closed and normal.

$$\mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).(\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{0}$  is never closed and normal.

### f3 vs. 10 (5 pairs)

$$\alpha.((1+1).\mathbf{u}) \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) \longrightarrow \alpha.\mathbf{u} + \alpha.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha).\mathbf{u}$ .

$$\alpha.((1+1).(\mathbf{u} + \mathbf{v})) \leftarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v}) \longrightarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v}) + \alpha.\mathbf{v}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + (\alpha + \alpha).\mathbf{v}$ .

$$\alpha.((1+1).(\mathbf{u} + \mathbf{v})) \leftarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v}) \longrightarrow \alpha.(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{v} + \mathbf{v})$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + (\alpha + \alpha).\mathbf{v}$ .

$$\alpha.((1+1).(\mathbf{u} + \mathbf{v})) \leftarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v}) \longrightarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.\mathbf{u}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + (\alpha + \alpha).\mathbf{v}$ .

$$\alpha.((1+1).(\mathbf{u} + \mathbf{v} + \mathbf{w})) \leftarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w}) \longrightarrow \alpha.(\mathbf{u} + \mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{v} + \mathbf{w} + \mathbf{w})$$

$\mathbf{u} + \mathbf{v} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + (\alpha + \alpha).\mathbf{v} + (\alpha + \alpha).\mathbf{w}$ .

### f1e vs. 5 (2 pairs)

$$\alpha.\mathbf{u} + \beta.\mathbf{u} \leftarrow \mathbf{0} + \alpha.\mathbf{u} + \beta.\mathbf{u} \longrightarrow \mathbf{0} + (\alpha + \beta).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u}$ .

$$\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \leftarrow \mathbf{0} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \longrightarrow \mathbf{0} + (\alpha + \beta).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u} + \mathbf{v}$ .

### f1e vs. 5e (3 pairs)

$$\alpha.\mathbf{u} + \beta.\mathbf{u} \leftarrow \mathbf{0} + \alpha.\mathbf{u} + \beta.\mathbf{u} \longrightarrow \mathbf{0} + (\alpha + \beta).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u}$ .

$$\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \leftarrow \mathbf{0} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \longrightarrow \mathbf{0} + (\alpha + \beta).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u} + \mathbf{v}$ .

$$\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} \leftarrow \mathbf{0} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} \longrightarrow \mathbf{0} + (\alpha + \beta).\mathbf{u} + \mathbf{v} + \mathbf{w}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u} + \mathbf{v} + \mathbf{w}$ .

### fle vs. 6 (1 pair)

$$\mathbf{0} + \alpha.\mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0}.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longrightarrow (\mathbf{0} + \alpha).\mathbf{u} + \mathbf{v}$$

closes on  $\alpha.\mathbf{u} + \mathbf{v}$ .

### fle vs. 7 (1 pair)

$$\alpha.\mathbf{u} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{1}.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longrightarrow (\mathbf{1} + \alpha).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\mathbf{1} + \alpha).\mathbf{u} + \mathbf{v}$ .

### fle vs. 8 (1 pair)

$$\mathbf{0} + \beta.\mathbf{0} + \mathbf{u} \longleftarrow \alpha.\mathbf{0} + \beta.\mathbf{0} + \mathbf{u} \longrightarrow (\alpha + \beta).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

### fle vs. 9 (1 pair)

$$(\alpha \times \gamma).\mathbf{u} + \beta.(\gamma.\mathbf{u}) + \mathbf{v} \longleftarrow \alpha.(\gamma.\mathbf{u}) + \beta.(\gamma.\mathbf{u}) + \mathbf{v} \longrightarrow (\alpha + \beta).(\gamma.\mathbf{u}) + \mathbf{v}$$

$\gamma.\mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  is closed and normal, closes on  $(\alpha \times \gamma + \beta \times \gamma).\mathbf{u} + \mathbf{v}$ .

### fle vs. 10 (8 pairs)

$$\alpha.\mathbf{u} + \alpha.\mathbf{v} + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} \longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} \longrightarrow (\alpha + \beta).(\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \beta).\mathbf{u} + (\alpha + \beta).\mathbf{v} + \mathbf{w}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) \longrightarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u}) + \alpha.\mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) \longrightarrow \alpha.(\gamma.\mathbf{u} + \mathbf{v}) + \alpha.(\beta.\mathbf{u})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \alpha.(\gamma.\mathbf{u})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} + \mathbf{w}) \longrightarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) + \alpha.\mathbf{w}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v} + \alpha.\mathbf{w}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} + \mathbf{w}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \alpha.(\gamma.\mathbf{u} + \mathbf{w})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v} + \alpha.\mathbf{w}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} + \mathbf{w}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{w}) + \alpha.(\gamma.\mathbf{u} + \mathbf{v})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v} + \alpha.\mathbf{w}$ .

$$\alpha.((\beta + \gamma).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} + \mathbf{w}) \longrightarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{w}) + \alpha.\mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha \times \gamma).\mathbf{u} + \alpha.\mathbf{v} + \alpha.\mathbf{w}$ .

## f2e vs. 5 (4 pairs)

$$\alpha.\mathbf{u} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{u} + \mathbf{u} \longrightarrow \mathbf{0} + (1 + \alpha).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u}$ .

$$\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longrightarrow \mathbf{0} + (1 + \alpha).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u} + \mathbf{v}$ .

$$\alpha.\mathbf{0} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{0} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

$$\alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

## f2e vs. 5e (12 pairs)

$$\alpha.\mathbf{u} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{u} + \mathbf{u} \longrightarrow \mathbf{0} + (1 + \alpha).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u}$ .

$$\alpha.\mathbf{0} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{0} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

$$\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longrightarrow \mathbf{0} + (1 + \alpha).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u} + \mathbf{v}$ .

$$\alpha.\mathbf{0} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.\mathbf{0} + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).\mathbf{0} + \mathbf{u} + \mathbf{v}$$

closes on  $\mathbf{u} + \mathbf{v}$ .

$$\alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

$$\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \longrightarrow \mathbf{0} + (1 + \alpha).(\mathbf{u} + \mathbf{v})$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1 + \alpha).\mathbf{u} + (1 + \alpha).\mathbf{v}$ .

$$\alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{u} \longrightarrow \mathbf{0} + (1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{w}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{w}$ .

$$\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow \mathbf{0} + (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1 + \alpha).\mathbf{u} + (1 + \alpha).\mathbf{v} + \mathbf{w}$ .

$$\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{u} + \mathbf{v} \longrightarrow \mathbf{0} + (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1 + \alpha).\mathbf{u} + (1 + \alpha).\mathbf{v} + \mathbf{w} + \mathbf{x}$ .

$$\alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u}) + \mathbf{v} + \mathbf{w}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{0}$  is never closed and normal.

$$\alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{0}$  is never closed and normal.

$$\alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \alpha.(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\mathbf{0} + \mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{0}$  is never closed and normal.

## f2e vs. 6 (1 pair)

$$\mathbf{0} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0}.\mathbf{u} + \mathbf{u} + \mathbf{v} \longrightarrow (\mathbf{0} + \mathbf{1}).\mathbf{u} + \mathbf{v}$$

closes on  $\mathbf{u} + \mathbf{v}$ .



### f2e vs. 7 (1 pair)

$$\mathbf{u} + \mathbf{u} + \mathbf{v} \longleftarrow 1.\mathbf{u} + \mathbf{u} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1 + 1).\mathbf{u} + \mathbf{v}$ .

### f2e vs. 8 (1 pair)

$$\mathbf{0} + \mathbf{0} + \mathbf{u} \longleftarrow \mathbf{0} + \alpha.\mathbf{0} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

### f2e vs. 9 (1 pair)

$$(\alpha \times \beta).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\beta.\mathbf{u}) + \mathbf{v}$$

$\beta.\mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  is closed and normal, closes on  $(\alpha \times \beta + \beta).\mathbf{u} + \mathbf{v}$ .

### f2e vs. 10 (14 pairs)

$$\alpha.\mathbf{u} + \alpha.\mathbf{v} + \mathbf{u} + \mathbf{v} + \mathbf{w} \longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} + \mathbf{w} \longrightarrow (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1 + \alpha).\mathbf{u} + (1 + \alpha).\mathbf{v} + \mathbf{w}$ .

$$\alpha.((1 + \beta).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{u}) + \alpha.\mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((1 + \beta).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) \longrightarrow \alpha.(\mathbf{v} + \mathbf{u}) + \alpha.(\beta.\mathbf{u})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((1 + \beta).\mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \alpha.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.((1 + \beta).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \alpha.\mathbf{w}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha).\mathbf{u} + \alpha.\mathbf{v} + \alpha.\mathbf{w}$ .

$$\alpha.((1 + \beta).\mathbf{u} + \mathbf{v} + \mathbf{w}) \longleftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{u}) \longrightarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{w} + \mathbf{u})$$



$\mathbf{v}$  is required to be closed and normal for this critical pair to open, closes on  $(1+1).\mathbf{v} + \mathbf{u}$ .

$$\mathbf{0} + \mathbf{u} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} \longrightarrow (1+1).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1+1).(\mathbf{0} + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{v} + \mathbf{0}$  is never closed and normal.

### **f3e vs. 5e (17 pairs)**

$$\mathbf{u} + \mathbf{u} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{u} \longrightarrow \mathbf{0} + (1+1).\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(1+1).\mathbf{u}$ .

$$\mathbf{0} + \mathbf{u} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} \longrightarrow (1+1).\mathbf{0} + \mathbf{u}$$

closes on  $\mathbf{u}$ .

$$\mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow \mathbf{0} + (1+1).\mathbf{v} + \mathbf{u}$$

$\mathbf{v}$  is required to be closed and normal for this critical pair to open, closes on  $(1+1).\mathbf{v} + \mathbf{u}$ .

$$\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow \mathbf{0} + (1+1).(\mathbf{u} + \mathbf{v})$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1+1).\mathbf{u} + (1+1).\mathbf{v}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} \longrightarrow (1+1).\mathbf{0} + \mathbf{u} + \mathbf{v}$$

closes on  $\mathbf{u} + \mathbf{v}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1+1).(\mathbf{0} + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{v} + \mathbf{0}$  is never closed and normal.

$$\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow \mathbf{0} + (1+1).\mathbf{w} + \mathbf{u} + \mathbf{v}$$

$\mathbf{w}$  is required to be closed and normal for this critical pair to open, closes on  $(1+1).\mathbf{w} + \mathbf{u} + \mathbf{v}$ .

$$\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow \mathbf{0} + (1+1).(\mathbf{v} + \mathbf{w}) + \mathbf{u}$$

$\mathbf{v} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{v}$  and  $\mathbf{w}$  are closed and normal, closes on  $(1+1).\mathbf{v} + (1+1).\mathbf{w} + \mathbf{u}$ .

$$\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow \mathbf{0} + (1+1).(\mathbf{u} + \mathbf{v} + \mathbf{w})$$

$\mathbf{u} + \mathbf{v} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(1+1).\mathbf{u} + (1+1).\mathbf{v} + (1+1).\mathbf{w}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1+1).(\mathbf{0} + \mathbf{w}) + \mathbf{u} + \mathbf{v}$$

does not verify the conditions because  $\mathbf{w} + \mathbf{0}$  is never closed and normal.

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1+1).(\mathbf{0} + \mathbf{v} + \mathbf{w}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{v} + \mathbf{w} + \mathbf{0}$  is never closed and normal.

$$\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longrightarrow \mathbf{0} + (1+1).(\mathbf{w} + \mathbf{x}) + \mathbf{u} + \mathbf{v}$$

$\mathbf{w} + \mathbf{x}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{w}$  and  $\mathbf{x}$  are closed and normal, closes on  $(1+1).\mathbf{w} + (1+1).\mathbf{x} + \mathbf{u} + \mathbf{v}$ .

$$\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longrightarrow \mathbf{0} + (1+1).(\mathbf{v} + \mathbf{w} + \mathbf{x}) + \mathbf{u}$$

$\mathbf{u} + \mathbf{v} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(1+1).\mathbf{v} + (1+1).\mathbf{w} + (1+1).\mathbf{x} + \mathbf{u}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longrightarrow (1+1).(\mathbf{0} + \mathbf{w} + \mathbf{x}) + \mathbf{u} + \mathbf{v}$$

does not verify the conditions because  $\mathbf{w} + \mathbf{x} + \mathbf{0}$  is never closed and normal.

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longrightarrow (1+1).(\mathbf{0} + \mathbf{v} + \mathbf{w} + \mathbf{x}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{v} + \mathbf{w} + \mathbf{x} + \mathbf{0}$  is never closed and normal.

$$\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{y} + \mathbf{y} \longleftarrow \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{y} + \mathbf{y} \longrightarrow \mathbf{0} + (1+1).(\mathbf{w} + \mathbf{x} + \mathbf{y}) + \mathbf{u} + \mathbf{v}$$

$\mathbf{w} + \mathbf{x} + \mathbf{y}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{w}$ ,  $\mathbf{x}$ , and  $\mathbf{y}$  are closed and normal, closes on  $(1+1).\mathbf{w} + (1+1).\mathbf{x} + (1+1).\mathbf{y} + \mathbf{u} + \mathbf{v}$ .

$$\mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{y} + \mathbf{y} \longleftarrow \mathbf{0} + \mathbf{0} + \mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{y} + \mathbf{y} \longrightarrow (1+1).(\mathbf{0} + \mathbf{w} + \mathbf{x} + \mathbf{y}) + \mathbf{u} + \mathbf{v}$$

does not verify the conditions because  $\mathbf{w} + \mathbf{x} + \mathbf{y} + \mathbf{0}$  is never closed and normal.

### f3e vs. 10 (17 pairs)

$$\alpha.((1+1).\mathbf{v} + \mathbf{u}) \longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) \longrightarrow \alpha.(\mathbf{v} + \mathbf{v}) + \alpha.\mathbf{u}$$

$\mathbf{v}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha).\mathbf{v} + \alpha.\mathbf{u}$ .

$$\alpha.((1+1).\mathbf{v} + \mathbf{u}) \longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) \longrightarrow \alpha.(\mathbf{u} + \mathbf{v}) + \alpha.\mathbf{v}$$

$\mathbf{v}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha).\mathbf{v} + \alpha.\mathbf{u}$ .

$$\alpha.((1+1).\mathbf{w} + \mathbf{u} + \mathbf{v}) \longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) \longrightarrow \alpha.(\mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.\mathbf{u}$$



$\mathbf{w} + \mathbf{x}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{w}$  and  $\mathbf{x}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{w} + (\alpha + \alpha).\mathbf{x} + \alpha.\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.\left((1+1).\left(\mathbf{w}+\mathbf{x}\right)+\mathbf{u}+\mathbf{v}\right) \longleftarrow \alpha.\left(\mathbf{u}+\mathbf{v}+\mathbf{w}+\mathbf{w}+\mathbf{x}+\mathbf{x}\right) \longrightarrow \alpha.\left(\mathbf{u}+\mathbf{w}\right)+\alpha.\left(\mathbf{v}+\mathbf{w}+\mathbf{x}+\mathbf{x}\right)$$

$\mathbf{w} + \mathbf{x}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{w}$  and  $\mathbf{x}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{w} + (\alpha + \alpha).\mathbf{x} + \alpha.\mathbf{u} + \alpha.\mathbf{v}$ .

$$\alpha.\left((1+1).\left(\mathbf{v}+\mathbf{w}+\mathbf{x}\right)+\mathbf{u}\right) \longleftarrow \alpha.\left(\mathbf{u}+\mathbf{v}+\mathbf{v}+\mathbf{w}+\mathbf{w}+\mathbf{x}+\mathbf{x}\right) \longrightarrow \alpha.\left(\mathbf{u}+\mathbf{v}+\mathbf{v}+\mathbf{w}\right)+\alpha.\left(\mathbf{w}+\mathbf{x}+\mathbf{x}\right)$$

$\mathbf{v} + \mathbf{w} + \mathbf{x}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{x}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{v} + (\alpha + \alpha).\mathbf{w} + (\alpha + \alpha).\mathbf{x} + \alpha.\mathbf{u}$ .

$$\alpha.\left((1+1).\left(\mathbf{v}+\mathbf{w}+\mathbf{x}\right)+\mathbf{u}\right) \longleftarrow \alpha.\left(\mathbf{u}+\mathbf{v}+\mathbf{v}+\mathbf{w}+\mathbf{w}+\mathbf{x}+\mathbf{x}\right) \longrightarrow \alpha.\left(\mathbf{u}+\mathbf{w}+\mathbf{x}+\mathbf{x}\right)+\alpha.\left(\mathbf{v}+\mathbf{v}+\mathbf{w}\right)$$

$\mathbf{v} + \mathbf{w} + \mathbf{x}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{x}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{v} + (\alpha + \alpha).\mathbf{w} + (\alpha + \alpha).\mathbf{x} + \alpha.\mathbf{u}$ .

$$\alpha.\left((1+1).\left(\mathbf{w}+\mathbf{x}+\mathbf{y}\right)+\mathbf{u}+\mathbf{v}\right) \longleftarrow \alpha.\left(\mathbf{u}+\mathbf{v}+\mathbf{w}+\mathbf{w}+\mathbf{x}+\mathbf{x}+\mathbf{y}+\mathbf{y}\right) \longrightarrow \alpha.\left(\mathbf{u}+\mathbf{w}+\mathbf{w}+\mathbf{x}\right)+\alpha.\left(\mathbf{v}+\mathbf{x}+\mathbf{y}+\mathbf{y}\right)$$

$\mathbf{w} + \mathbf{x} + \mathbf{y}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{w}$ ,  $\mathbf{x}$ , and  $\mathbf{y}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{w} + (\alpha + \alpha).\mathbf{x} + (\alpha + \alpha).\mathbf{y} + \alpha.\mathbf{u} + \alpha.\mathbf{v}$ .

### **f1 vs. f3 (1 pair)**

$$(\alpha + \alpha).\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} \longrightarrow (1 + 1).\left(\alpha.\mathbf{u}\right)$$

closes on  $(\alpha + \alpha).\mathbf{u}$ .

### **f2 vs. f1e (2 pairs)**

$$(1 + \alpha).\left(\beta.\mathbf{u} + \gamma.\mathbf{u}\right) \longleftarrow \alpha.\left(\beta.\mathbf{u} + \gamma.\mathbf{u}\right) + \beta.\mathbf{u} + \gamma.\mathbf{u} \longrightarrow (\beta + \gamma).\mathbf{u} + \alpha.\left(\beta.\mathbf{u} + \gamma.\mathbf{u}\right)$$

does not verify the conditions because  $\beta.\mathbf{u} + \gamma.\mathbf{u}$  is never closed and normal.

$$(1 + \alpha).\left(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}\right) \longleftarrow \alpha.\left(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}\right) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \longrightarrow (\beta + \gamma).\mathbf{u} + \alpha.\left(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}\right) + \mathbf{v}$$

does not verify the conditions because  $\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}$  is never closed and normal.

### **f2 vs. f2e (2 pairs)**

$$(1 + \beta).\left(\alpha.\mathbf{u} + \mathbf{u}\right) \longleftarrow \alpha.\mathbf{u} + \beta.\left(\alpha.\mathbf{u} + \mathbf{u}\right) + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.\left(\alpha.\mathbf{u} + \mathbf{u}\right)$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \beta).\left(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}\right) \longleftarrow \alpha.\mathbf{u} + \beta.\left(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}\right) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.\left(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}\right) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}$  is never closed and normal.

### **f2 vs. f3e (2 pairs)**

$$(1 + \alpha).(\mathbf{u} + \mathbf{u}) \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).\mathbf{u} + \alpha.(\mathbf{u} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

### **f3 vs. f1e (3 pairs)**

$$(1 + 1).(\alpha.\mathbf{u} + \beta.\mathbf{u}) \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} \longrightarrow (\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \beta.\mathbf{u}$  is never closed and normal.

$$(1+1).(\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}) \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}$  is never closed and normal.

$$(1 + 1).(\alpha.\mathbf{u} + \mathbf{v}) \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (\alpha + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{v}$$

$\alpha.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + (1 + 1).\mathbf{v}$ .

### **f3 vs. f2e (6 pairs)**

$$(1 + 1).(\alpha.\mathbf{u} + \mathbf{u}) \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{u}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + 1).(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}) \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{u}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}$  is never closed and normal.

$$(1+1).(\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{u}) \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u})$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{u}$  is never closed and normal.

$$(1+1).(\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{u}) \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{v}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{u}$  is never closed and normal.

$$\begin{aligned} & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{v}) \\ & \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{v}$  is never closed and normal.

$$(1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v})$$

$$\longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow$$

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{u}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v}$  is never closed and normal.

### **f3 vs. f3e (1 pair)**

$$(1 + 1).(\mathbf{u} + \mathbf{v}) \longleftarrow \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{u} + \mathbf{v} + \mathbf{v}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(1 + 1).\mathbf{u} + (1 + 1).\mathbf{v}$ .

### **f1e vs. fle (8 pairs)**

$$(\alpha + \gamma).\mathbf{u} + \beta.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} \longrightarrow (\alpha + \beta).\mathbf{u} + \gamma.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} \longrightarrow (\alpha + \beta).\mathbf{u} + \gamma.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} \longrightarrow (\alpha + \gamma).\mathbf{u} + \beta.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u}$ .

$$(\alpha + \gamma).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \longrightarrow (\alpha + \beta).\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u} + \mathbf{v}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \longrightarrow (\alpha + \beta).\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u} + \mathbf{v}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \longrightarrow (\alpha + \gamma).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \gamma).\mathbf{u} + \mathbf{v}$ .

$$(\beta + \delta).\mathbf{v} + \alpha.\mathbf{u} + \gamma.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{u} + \delta.\mathbf{v} \longrightarrow (\alpha + \gamma).\mathbf{u} + \beta.\mathbf{v} + \delta.\mathbf{v}$$



$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + \gamma).\mathbf{u} + (\beta + \delta).\mathbf{v}$ .

$$(\beta + \delta).\mathbf{v} + \alpha.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{u} + \delta.\mathbf{v} + \mathbf{w} \rightarrow (\alpha + \gamma).\mathbf{u} + \beta.\mathbf{v} + \delta.\mathbf{v} + \mathbf{w}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + \gamma).\mathbf{u} + (\beta + \delta).\mathbf{v} + \mathbf{w}$ .

## f1e vs. f2e (14 pairs)

$$(\alpha + \beta).\mathbf{u} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{u} \rightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + 1).\mathbf{u}$ .

$$(\gamma + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u}) \leftarrow \alpha.(\beta.\mathbf{u}) + \gamma.\mathbf{u} + \beta.\mathbf{u} \rightarrow (1 + \alpha).(\beta.\mathbf{u}) + \gamma.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u}) \leftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} + \gamma.\mathbf{u} \rightarrow (1 + \alpha).(\beta.\mathbf{u}) + \gamma.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u}$ .

$$(\alpha + \beta).\mathbf{u} + \mathbf{v} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{u} \rightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + 1).\mathbf{u} + \mathbf{v}$ .

$$(\beta + \gamma).\mathbf{v} + \alpha.\mathbf{u} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{v} + \mathbf{u} \rightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{v}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1).\mathbf{u} + (\beta + \gamma).\mathbf{v}$ .

$$(\gamma + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u}) + \gamma.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u}) + \gamma.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + \mathbf{v}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u}) + \gamma.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + \mathbf{v}$ .

$$(\gamma + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + (\alpha + 1).\mathbf{v}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + (\alpha + 1).\mathbf{v}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u}) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \gamma.\mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\beta.\mathbf{u} + \gamma.\mathbf{u}$  is never closed and normal.

$$(\beta + \gamma).\mathbf{v} + \alpha.\mathbf{u} + \mathbf{w} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{v} + \mathbf{w} + \mathbf{u} \rightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{v} + \gamma.\mathbf{v} + \mathbf{w}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(1 + \alpha).\mathbf{u} + (\beta + \gamma).\mathbf{v} + \mathbf{w}$ .

$$(\gamma + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u} + \beta.\mathbf{u} + \mathbf{w} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u} + \mathbf{w}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + (\alpha + 1).\mathbf{v} + \mathbf{w}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{w} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \mathbf{v}) + \gamma.\mathbf{u} + \mathbf{w}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + \gamma).\mathbf{u} + (\alpha + 1).\mathbf{v} + \mathbf{w}$ .

$$(\beta + \gamma).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{w} + \mathbf{v} \rightarrow (1 + \alpha).(\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

does not verify the conditions because  $\beta.\mathbf{u} + \gamma.\mathbf{u} + \mathbf{v}$  is never closed and normal.

## **f1e vs. f3e (14 pairs)**

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} \rightarrow (1 + 1).(\alpha.\mathbf{u}) + \beta.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha + \beta).\mathbf{u}$ .

$$(\alpha + \beta).\mathbf{u} + \beta.\mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} \rightarrow (1 + 1).(\beta.\mathbf{u}) + \alpha.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \beta).\mathbf{u}$ .

$$(\alpha + \alpha).\mathbf{u} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \rightarrow (1 + 1).(\alpha.\mathbf{u}) + \mathbf{v}$$

closes on  $(\alpha + \alpha).\mathbf{u} + \mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \mathbf{v} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v} \rightarrow (1 + 1).\mathbf{v} + \alpha.\mathbf{u} + \beta.\mathbf{u}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \rightarrow (1 + 1).(\alpha.\mathbf{u}) + \beta.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha + \beta).\mathbf{u} + \mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \rightarrow (1 + 1).(\beta.\mathbf{u}) + \alpha.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + \beta).\mathbf{u} + \mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v} \rightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u}$$

$\alpha.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha + \beta).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{v} \rightarrow (1 + 1).(\beta.\mathbf{u} + \mathbf{v}) + \alpha.\mathbf{u}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \beta + \beta).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} \rightarrow (1 + 1).(\alpha.\mathbf{u} + \beta.\mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \beta.\mathbf{u}$  is never closed and normal.

$$(\alpha + \beta).\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \rightarrow (1 + 1).\mathbf{w} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  and  $\mathbf{w}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \rightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{w}) + \beta.\mathbf{u} + \mathbf{v}$$

$\alpha.\mathbf{u} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + \alpha + \beta).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(\alpha + \beta).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \rightarrow (1 + 1).(\beta.\mathbf{u} + \mathbf{w}) + \alpha.\mathbf{u} + \mathbf{v}$$

$\beta.\mathbf{u} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + \beta + \beta).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(\alpha + \beta).\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \beta.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \rightarrow (1 + 1).(\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{w}) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{w}$  is never closed and normal.

$$(\alpha + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \rightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{w}) + \mathbf{v}$$

$\alpha.\mathbf{u} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + \alpha).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

## f2e vs. f2e (30 pairs)

$$(1 + \beta).\mathbf{u} + \alpha.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + 1).\mathbf{u}$ .

$$(1 + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u}) \longleftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} + \mathbf{u} \longrightarrow (1 + \alpha).(\beta.\mathbf{u}) + \mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + 1).\mathbf{u}$ .

$$(1 + \beta).(\alpha.\mathbf{u}) + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u}) + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u})$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha + 1).\mathbf{u}$ .

$$(1 + \beta).\mathbf{v} + \alpha.\mathbf{u} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{v} + \mathbf{v}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1).\mathbf{u} + (\beta + 1).\mathbf{v}$ .

$$(1 + \beta).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \beta.\mathbf{u} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \beta + 1).\mathbf{u} + \mathbf{v}$ .

$$(1 + \beta).(\mathbf{v} + \mathbf{u}) + \alpha.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.(\mathbf{v} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\mathbf{v} + \mathbf{u}) + \mathbf{v}$$

$\mathbf{v} + \mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \beta + 1).\mathbf{u} + (\beta + 1).\mathbf{v}$ .

$$(1 + \beta).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} \longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.\mathbf{v} + \mathbf{u} + \mathbf{v} \longrightarrow (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \beta.\mathbf{v}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + \beta + 1).\mathbf{v}$ .

$$(1 + \beta).\mathbf{u} + \alpha.(\beta.\mathbf{u}) + \mathbf{v} \longleftarrow \alpha.(\beta.\mathbf{u}) + \beta.\mathbf{u} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).(\beta.\mathbf{u}) + \mathbf{v} + \mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \beta + 1).\mathbf{u} + \mathbf{v}$ .

$$(1 + \beta).(\alpha.\mathbf{u}) + \mathbf{v} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u}) + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha \times \beta + \alpha + 1).\mathbf{u} + \mathbf{v}$ .

$$(1 + \beta).(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{v}$$

$\alpha.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \alpha + 1).\mathbf{u} + (\beta + 1).\mathbf{v}$ .

$$(1 + \beta).(\alpha.\mathbf{u} + \mathbf{u}) + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1+\beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{u}) + \beta.\mathbf{u} + \mathbf{v} + \mathbf{u} \rightarrow (1+\alpha).(\beta.\mathbf{u} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\beta.\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1+\beta).(\mathbf{u} + \mathbf{v}) + \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) \leftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$(1+\beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u} + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{u}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + 1).\mathbf{u} + (\alpha + 1).\mathbf{v}$ .

$$(1+\beta).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v})$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$(1+\beta).\mathbf{v} + \alpha.\mathbf{u} + \mathbf{w} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \beta.\mathbf{v} + \mathbf{w} + \mathbf{v} + \mathbf{u} \rightarrow (1+\alpha).\mathbf{u} + \beta.\mathbf{v} + \mathbf{w} + \mathbf{v}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1).\mathbf{u} + (\beta + 1).\mathbf{v} + \mathbf{w}$ .

$$(1+\beta).(\mathbf{v} + \mathbf{u}) + \alpha.\mathbf{u} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \beta.(\mathbf{v} + \mathbf{u}) + \mathbf{w} + \mathbf{v} + \mathbf{u} \rightarrow (1+\alpha).\mathbf{u} + \beta.(\mathbf{v} + \mathbf{u}) + \mathbf{w} + \mathbf{v}$$

$\mathbf{v} + \mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \beta + 1).\mathbf{u} + (\beta + 1).\mathbf{v} + \mathbf{w}$ .

$$(1+\beta).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.\mathbf{v} + \mathbf{w} + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\mathbf{u} + \mathbf{v}) + \beta.\mathbf{v} + \mathbf{w}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + \beta + 1).\mathbf{v} + \mathbf{w}$ .

$$(1+\beta).(\mathbf{w} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{w} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{w} + \mathbf{v}) + \mathbf{w}$$

$\mathbf{w} + \mathbf{v}$  and  $\mathbf{u} + \mathbf{v}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + \beta + 1).\mathbf{v} + (\beta + 1).\mathbf{w}$ .

$$(1+\beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v}) + \beta.\mathbf{u} + \mathbf{w} + \mathbf{u} + \mathbf{v} \rightarrow (1+\alpha).(\beta.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u}$$

$\beta.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \beta + 1).\mathbf{u} + (\alpha + 1).\mathbf{v} + \mathbf{w}$ .

$$(1+\beta).\mathbf{u} + \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \beta.\mathbf{u} + \mathbf{w} + \mathbf{v} + \mathbf{u} \rightarrow (1+\alpha).(\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \mathbf{w}$$

does not verify the conditions because  $\beta.\mathbf{u} + \mathbf{v} + \mathbf{u}$  is never closed and normal.

$$(1+\beta).(\mathbf{u} + \mathbf{v}) + \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w}$$

$$\begin{aligned} &\longleftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} &(1 + \beta).(\mathbf{u} + \mathbf{v}) + \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{w} \\ &\longleftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{w} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}$  is never closed and normal.

$$(1 + \beta).(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}$$

$\alpha.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha \times \beta + \alpha + 1).\mathbf{u} + (\beta + 1).\mathbf{v} + \mathbf{w}$ .

$$(1 + \beta).(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \mathbf{w} \longleftarrow \alpha.\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \mathbf{w} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + \alpha).\mathbf{u} + \beta.(\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}) + \mathbf{w} + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}$  is never closed and normal.

$$\begin{aligned} &(1 + \beta).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} + \mathbf{u} \\ &\longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} \end{aligned}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} &(1 + \beta).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{u} \\ &\longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{w} \end{aligned}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} &(1 + \beta).(\mathbf{w} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{x} + \mathbf{u} \\ &\longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{w} + \mathbf{v}) + \mathbf{x} + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\mathbf{w} + \mathbf{v}) + \mathbf{x} + \mathbf{w} \end{aligned}$$

$\mathbf{w} + \mathbf{v}$  and  $\mathbf{u} + \mathbf{v}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + \beta + 1).\mathbf{v} + (\beta + 1).\mathbf{w} + \mathbf{x}$ .

$$\begin{aligned} &(1 + \beta).(\mathbf{u} + \mathbf{v}) + \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{x} + \mathbf{v}) + \mathbf{w} + \mathbf{x} \\ &\longleftarrow \alpha.(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{x} + \mathbf{v}) + \beta.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{x} + \mathbf{v} \longrightarrow \\ &\quad (1 + \alpha).(\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{x} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\beta.(\mathbf{u} + \mathbf{v}) + \mathbf{x} + \mathbf{v}$  is never closed and normal.

$$(1 + \beta).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{x} + \mathbf{u}$$

$$\longleftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{x} + \mathbf{w} + \mathbf{u} + \mathbf{v} \longrightarrow$$

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \beta.(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{x} + \mathbf{w}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}$  is never closed and normal.

## f2e vs. f3e (46 pairs)

$$(1 + \alpha).\mathbf{u} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).\mathbf{u} + \alpha.\mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1 + 1).\mathbf{u}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} \longleftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\alpha.\mathbf{u}) + \mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha + 1).\mathbf{u}$ .

$$(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1 + 1).\mathbf{u} + \mathbf{v}$ .

$$(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \alpha.\mathbf{u} + \mathbf{u}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longrightarrow (1 + 1).(\alpha.\mathbf{u}) + \mathbf{v} + \mathbf{u}$$

$\mathbf{u}$  is required to be closed and normal for this critical pair to open, closes on  $(\alpha + \alpha + 1).\mathbf{u} + \mathbf{v}$ .

$$(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{v} + \mathbf{u}) + \alpha.\mathbf{u}$$

$\mathbf{v} + \mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1 + 1).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{v}) + \mathbf{u}$$

$\alpha.\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + \alpha + 1).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} \longleftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).\mathbf{u} + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{v} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + 1 + 1).\mathbf{v}$ .

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{v}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).\mathbf{w} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u}$$

$\mathbf{u}$  and  $\mathbf{w}$  are required to be closed and normal for this critical pair to open, closes on  $(\alpha + 1).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + \alpha).\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{w} + \mathbf{u}) + \alpha.\mathbf{u} + \mathbf{v}$$

$\mathbf{w} + \mathbf{u}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + 1 + 1).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{u} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{w}) + \mathbf{v} + \mathbf{u}$$

$\alpha.\mathbf{u} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + \alpha + 1).\mathbf{u} + \mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + \alpha).\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} \leftarrow \alpha.\mathbf{u} + \alpha.\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\alpha.\mathbf{u} + \mathbf{w} + \mathbf{u}) + \mathbf{v}$$

does not verify the conditions because  $\alpha.\mathbf{u} + \mathbf{w} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{w} \leftarrow \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} \longrightarrow (1 + 1).(\mathbf{w} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v}$$



does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{w} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{u}) + \alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{u}) + \mathbf{w} + \mathbf{u}) + \mathbf{v} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{u}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u}$$

$\mathbf{u} + \mathbf{v}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$  and  $\mathbf{v}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + 1 + 1).\mathbf{v} + \mathbf{w}$ .

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{w} + \mathbf{w} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\mathbf{w} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u}$$

$\mathbf{u} + \mathbf{v}$  and  $\mathbf{w} + \mathbf{v}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + 1 + 1).\mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + \alpha).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v} \leftarrow \alpha.(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} + \mathbf{u}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{v} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{u} + \mathbf{w} + \mathbf{w} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{w} + \mathbf{w} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\mathbf{w} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{u} \leftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).(\mathbf{w} + \mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v})$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{v}) + \mathbf{w} + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{w} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{w} + \mathbf{w} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{v}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{v}) + \mathbf{u} + \mathbf{v}) + \mathbf{w} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{v}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{v} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow \\ & (1 + 1).(\mathbf{v} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{v} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow \\ & (1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{v} + \mathbf{w}) + \mathbf{u} \end{aligned}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{v} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{x} + \mathbf{x} + \mathbf{v} + \mathbf{v} \longrightarrow \\ & (1 + 1).(\mathbf{x} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} \end{aligned}$$

$\mathbf{u} + \mathbf{v}$  and  $\mathbf{x} + \mathbf{v}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{x}$  are closed and normal, closes on  $(\alpha + 1).\mathbf{u} + (\alpha + 1 + 1).\mathbf{v} + \mathbf{w} + (1 + 1).\mathbf{x}$ .

$$\begin{aligned} & (1 + \alpha).(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{x} + \mathbf{x} + \mathbf{v} \\ \leftarrow & \alpha.(\mathbf{u} + \mathbf{v}) + \alpha.(\mathbf{u} + \mathbf{v}) + \mathbf{w} + \mathbf{u} + \mathbf{x} + \mathbf{x} + \mathbf{v} + \mathbf{v} \longrightarrow \end{aligned}$$



does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{x} + \mathbf{v}$$

$$\longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{u} + \mathbf{x} + \mathbf{x} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow$$

$$(1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{v} + \mathbf{w}) + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{y} + \mathbf{y} + \mathbf{v}$$

$$\longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{u} + \mathbf{y} + \mathbf{y} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow$$

$$(1 + 1).(\mathbf{y} + \mathbf{v} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

$$(1 + \alpha).(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{y} + \mathbf{y} + \mathbf{v}$$

$$\longleftarrow \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{x} + \mathbf{u} + \mathbf{y} + \mathbf{y} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow$$

$$(1 + 1).(\alpha.(\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}) + \mathbf{y} + \mathbf{v} + \mathbf{w}) + \mathbf{x} + \mathbf{u}$$

does not verify the conditions because  $\mathbf{u} + \mathbf{v} + \mathbf{w} + \mathbf{w}$  is never closed and normal.

### **f3e vs. f3e (5 pairs)**

$$(1 + 1).\mathbf{u} + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} \longrightarrow (1 + 1).\mathbf{v} + \mathbf{u} + \mathbf{u}$$

$\mathbf{u}$  and  $\mathbf{v}$  are required to be closed and normal for this critical pair to open, closes on  $(1 + 1).\mathbf{u} + (1 + 1).\mathbf{v}$ .

$$(1 + 1).\mathbf{v} + \mathbf{u} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).\mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v}$$

$\mathbf{v}$  and  $\mathbf{w}$  are required to be closed and normal for this critical pair to open, closes on  $\mathbf{u} + (1 + 1).\mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + 1).(\mathbf{v} + \mathbf{w}) + \mathbf{u} \longleftarrow \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).\mathbf{w} + \mathbf{u} + \mathbf{v} + \mathbf{v}$$

$\mathbf{v} + \mathbf{w}$  is required to be closed and normal for this critical pair to open, thus  $\mathbf{v}$  and  $\mathbf{w}$  are closed and normal, closes on  $\mathbf{u} + (1 + 1).\mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + 1).(\mathbf{u} + \mathbf{w}) + \mathbf{v} + \mathbf{v} \longleftarrow \mathbf{u} + \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} \longrightarrow (1 + 1).(\mathbf{v} + \mathbf{w}) + \mathbf{u} + \mathbf{u}$$

$\mathbf{u} + \mathbf{w}$  and  $\mathbf{v} + \mathbf{w}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are closed and normal, closes on  $(1 + 1).\mathbf{u} + (1 + 1).\mathbf{v} + (1 + 1).\mathbf{w}$ .

$$(1 + 1).(\mathbf{v} + \mathbf{x}) + \mathbf{u} + \mathbf{w} + \mathbf{w} \longleftarrow \mathbf{u} + \mathbf{v} + \mathbf{v} + \mathbf{w} + \mathbf{w} + \mathbf{x} + \mathbf{x} \longrightarrow (1 + 1).(\mathbf{w} + \mathbf{x}) + \mathbf{u} + \mathbf{v} + \mathbf{v}$$

$\mathbf{v} + \mathbf{x}$  and  $\mathbf{w} + \mathbf{x}$  are required to be closed and normal for this critical pair to open, thus  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{x}$  are closed and normal, closes on  $\mathbf{u} + (1 + 1).\mathbf{v} + (1 + 1).\mathbf{w} + (1 + 1).\mathbf{x}$ .

## Acknowledgments

The authors would like to thank Evelyne Contejean for her help in using CIME for this proof.

## References

- [1] *The CiME Rewrite Tool*, <http://cime.lri.fr/>.
- [2] P. Arrighi and G. Dowek, *Lineal: A linear-algebraic  $\lambda$ -calculus*.