Two consequences of the hypothesis that we are within the world

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Abstract. If we assume that we are within the world, then freedom is a consequence of determinism, and predictability of the future is not.

The hypothesis that we are within the world

In the introduction of *A philosophical Essay on Probabilities* Pierre-Simon de Laplace imagines "An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed"[2]. Before discussing whether, for such an intellect, something or nothing would be uncertain, we can discuss, even if Laplace does not, where such an intellect could be located: inside the world or outside it? As this intellect is as vast as the world itself – that is, it contains as much information – it could not be a proper part of the world, unless both are infinite. Laplace's description indeed suggests that this intellect is located outside the world and looks at the world in the same way the chemist looks at her test tubes: from outside.

The classical interpretations of quantum physics – such as the Copenhagen interpretation – are consistent with this viewpoint that, like Laplace's intellect, we are outside the test tube, as they put observers and measuring instruments outside the observed system. Moreover, the physical laws that apply to the system – such as the Schrödinger equation – do not apply to the measuring instruments and to the observers, that have the privilege to evolve in a non deterministic way and to reduce the state vector of the system instantaneously.

This is consistent with idealism that puts the soul outside the material world: even if we claim not to subscribe to this viewpoint, we are used to consider the interactions between ourselves and the world, as if we were not part of it.

If we want to free ourselves from this viewpoint, and investigate, in contrast, the hypothesis that we are within the world, we have first to describe the world, for instance as a set of cells exchanging

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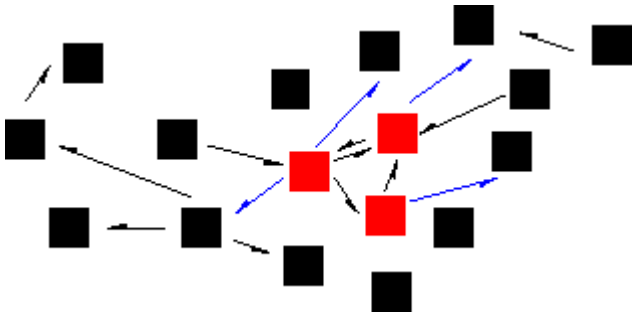
2 *Pierre Simon of Laplace, Essai philosophique sur les probabilites, cinquième edition, Bachelier, 1825, p. 3.*

information and evolving according to the received information, before isolating a subset *x* of this set: the I.



The world described as a set of cells, represented by squares,
that exchange information, represented by arrows.
The subset *x*, of cells colored in red, is the I.

This hypothesis that we are within the world has two unexpected consequences: freedom is a consequence of determinism and predictability of the future is not.

Freedom as a consequence of determinism

Determinism. Determinism is the thesis that the state of the world at any date in the future is a function of its current state. It implies that all the actions we will perform tomorrow are already determined, already written. This thesis seems to allow no room for free action. We can, therefore, easily understand that many were relieved, when quantum physics replaced this hypothesis by its exact negation, randomness, according to which the state of the world at any date in the future is not a function of its current state, that seems more compatible with our idea of freedom. This link between randomness and freedom is deeply rooted in our culture. The dictionaries, for instance, define, among other meanings, freedom as the undetermined character of the human will.

In our discourse on randomness and freedom, the word “randomness” is relatively well defined as this thesis that the state of the world at any future date is not a function of its current state. We can of course criticize this definition, or try to clarify the meaning of some words that occur in it, but we have to acknowledge that it exists. The word “freedom”, in contrast, is much less well defined. So this investigation on the relation between randomness and freedom seems to encourage us, more than everything, to try to define more precisely this notion of “freedom”.

Of course, when seeking for a precise definition of a common word, we are led to unravel several of its meanings. For instance, defining precisely the notion of weight led us to distinguish weight from

mass. So, seeking for a definition of the word “freedom”, we must *a priori* accept that our definition may cover some meanings of this word, but not all.

Proposing to define the notion of freedom as the undetermined character of will, the dictionaries exhibit a link between the notion of freedom and that of will, that suggests that, before defining this notion of “freedom”, we might need to first define that of “will”.

Will. Defining the notion of will requires first to decide to make it a primitive of a derived notion. If we want to make this notion primitive, we must introduce, in our language, a modality “*x* wants *A*”, where *x* is a person and *A* a statement, and provide rules or axioms that express the meaning of this modality. With a person *x* = Alice and the statement *A* = “Alice goes to Jerusalem tomorrow”, we can build the statement “Alice wants to go to Jerusalem tomorrow”. And we then can ask if we must state, or not, an axiom expressing that if Alice wants to go to Jerusalem tomorrow and if she can, then she will, that is if we want, or not, to state an axiom expressing that her will to go to Jerusalem constrains her to go, if she can.

If, in contrast, we want to make this notion derived, we must define it from others. In this case, it seems that we can define this modality from a more primitive one “*x* can *A*” and define the modality “*x* wants *A*” as the implication “if *x* can *A* then *A*”. Thus, the statement “Alice wants to go to Jerusalem tomorrow” is, by definition, synonymous of “If Alice can go to Jerusalem tomorrow, then she will”.

Making, this way, will a derived notion has the advantage of solving the problem of the constraining character of will: if Alice wants to go to Jerusalem tomorrow and she can, then she will. But, Alice is not constrained by her will to do anything. However, if she can go to Jerusalem tomorrow, and she does not, it means, by definition, that she did not want to. She may have believed that she did, she may have said that she did, but she did not.

This problem of the constraining character of the will seems to be a consequence of the choice to make will an primitive notion. And it disappears if we make it derived.

If we assume the rules of classical logic and that the statement “if *A* then *x* can *A*” always hold, then will and possibility play symmetric roles: the statement “*x* can *A*” is equivalent to the statement “if *x* wants *A* then *A*”. The proof of “if *x* wants *A* then *A*” from “*x* can *A*” is simple: it suffices to prove that if “*x* can *A*” and “*x* wants *A*” then *A*, which is a consequence of the definition of “*x* wants *A*”. The converse seems to require to use the excluded middle: if we assume “if *x* wants *A* then *A*”, then from the hypothesis “not *A*” we can deduce “not *x* wants *A*” hence “*x* can *A*”. From the hypothesis

A, we can also deduce "*x* can *A*" using the fact that the statement "if *A* then *x* can *A*" always holds. Thus possibility happens to coincide with the accomplishment of the will.

This definition has, in contrast, a bizarre consequence: if *A* is necessary, then "*x* wants *A*" also. For instance, the statement "Alice wants that 2 + 2 = 4" always holds. We can avoid this adding in the definition of the statement "*x* wants *A*", that *A* is contingent for *x*, that is "*x* can (not *A*)", but we shall not need this here.

Possibility. Thus, to define the modality "*x* wants *A*", we need to define this more primitive modality "*x* can *A*". This modality is related to the modality of possibility, "◊", but it is different. It is indeed not very different to say "it is possible that Alice goes to Jerusalem tomorrow" or "Alice can go to Jerusalem tomorrow". In contrast, it is very different to say "it is possible that lightning strikes at Alice's house tomorrow" and "Alice can make lightning strike at her house tomorrow", except, of course, if Alice is the master of lightning. How can we define this modality "*x* can *A*"?

The modality of possibility "◊" can be defined in terms of possible words, "it is possible that *A*" meaning that *A* holds in at least one possible world. The modality "*x* can *A*" not only means that *A* is true in at least one possible world, but also that *x* can make such a world happen. Thus, it seems that we must describe the way *x* acts in order to make happen a world in which *A* holds. And this description of the way *x* acts on the world is very different if we consider that *x* is inside the world, or outside.

If we consider that *x* is within the world, that is that she is the subset of cells in red on the picture above, then we can describe her action as the information, in blue on the picture, emitted by *x*, in the direction of the rest of the world. We then can say that the statement "*x* can *A*" holds if there exits some information that, emitted by *x*, would drive the world in a state in which *A* holds. For instance, if Alice has the key that corresponds to this lock, she can open this door. But if she is put in a dungeon, without a key, she cannot open the door of the dungeon: whatever information she emits in the direction of the world, the door will remain closed. This first attempt of a definition still requires to be made more precise. For instance, is it the case that Alice can open the door of her dungeon if she has the key that opens the door of the dungeon, but ignores that it does. Yet, it will suffice here.

We have described the world as a set of cells exchanging information and evolving according to the received information. This evolution can be deterministic, or not. In the first case, we can describe it by a function *f* that maps the current state *e* of a cell and the information *i* it receives to the state *f*(*e*,*i*) of this cell at a given future date. In the second, by a function *f* that maps the current sate *e* of

a cell and the information i it receives to a set $f(e,i)$ of the possible states of this cell at the future date.

A set always being the image of a function, a random phenomenon can always be decomposed into a deterministic phenomenon and a source of randomness. In the second case, we can therefore also, describe this evolution by a function g that maps the current sate e of the cell, the information i it receives, and an element ω of a source of randomness Ω, to a unique state $g(e,i,\omega)$. It is also possible to equip this set of possible states, or the source of randomness Ω, with a probability measure, but this will not be needed here.

If we add another function describing the information emitted by a cell, in function of its state, we can then define, in the deterministic case, a function that maps the current state of the world to state at a future date, and in the random case, a function that maps the current state of the world and an element of a source of randomness, to its state at the future date.

More interestingly, as we have isolated a person x from the rest of the world, we can define, in the deterministic case, a function F that maps the current state E of the rest of the world and the information I it receives from x to its state $F(E,I)$ at a future date, and in the random case, a function G that maps the current state of the rest of the world E, the information I it receives from x and an element ω of a source of randomness Ω to its state $G(E,I,\omega)$ at a future date.

We can then define the modality "x can A", in the deterministic case, as the fact that in the current state of the rest of the world, there exists a piece of information I such that A holds in the state of the rest of the world $F(E,I)$ and, in the non deterministic one, as the fact that in the current state of the rest of the world, there exists a piece of information I such that for all ω, A holds in the state of the rest of the world $G(E,I,\omega)$.

This modality, "x can A" is alethic or deontic, depending on whether the evolution rules of the cells according to the received information are the laws of physic or juridic rules. Depending on the way these rules are defined, Alice can or cannot drive against traffic in a one way street.

From possibility to freedom. We can, finally, sketch a definition of the notion of freedom: a person x is freer than a person y when everything y can, x also can. For instance, you are freer listening to this talk than a prisoner in a dungeon: you can – at least in the alethic meaning of the world –, leave if you are bored, go have a coffee and come back... all these things a prisoner cannot.

As the statement “*x* can *A*” is equivalent to the statement “if *x* wants *A* then *A*”, freedom can also be defined as the accomplishment of will: *x* is freer than *y* if each time the will of *y* is accomplished that of *x* also.

Freedom, randomness, determinism. The question that we are interested is then: is *x* freer in a random world or in a deterministic one? In other words: can *x* more in a random world or in a deterministic one? In other words: is the will of *x* more often accomplished in a random world or in a deterministic one?

I am afraid that our definitions lead us to the exact negation of the thesis we have sketched in the introduction, as *x* can more in a deterministic world, than in a random one.

For instance, we all have had, at a moment in our life, a lamp with a contact failure, a sporadic wifi router, a fussy printer, or a car that started every other morning... It is correct that the future is less determined, less written, when a lamp has a contact failure than when it does not. Each time we press the switch, two states of the world, very different one from the other, can happen. But does this make us freer?

It seems not, because our will to switch on the lamp to read, or to switch it off to sleep is less often accomplished, than if the lamp functioned as expected. We can less in a world where the lamps are random than in a world where they are deterministic.

A second example: I have once been told – I do not know if this story is true or not, but it does not matter here – that an airline had decided to sell plane tickets to a random destination. The passengers were invited to go to the airport at a given time and discovered the destination of their flight. We can feel some joy to discover there that randomness has gratified us a with a ticket to the middle of nowhere, rather than to Jerusalem, and this explains why some people play roulette in casinos or buy lottery tickets, but this changes nothing to the fact that our will to go to Jerusalem is more often accomplished, if we can chose our destination, that if it is chosen randomly.

Fortunately, what quantum physics teaches us is precisely that the world is not too much random. Measuring alternatively the spin of an electron according to a horizontal and a vertical axis indeed produces a random sequence. But because of the law or large numbers, this randomness mostly disappears, at ordinary scales. This explains that we can build planes that take us to Jerusalem when we want to go to Jerusalem, and to Paris when we want to go to Paris. This explains that we can build lamps that we can switch on when we want to read and off when we want to sleep. This explains that we can play roulette, when we want to, and not when we do not.

It thus seems that we must conclude that our freedom is more a consequence of the determinism than of the randomness of the evolution of the rest of the world.

We must however also examine the idea that the freedom of *x* would come from the randomness, not of the evolution of the rest of the world, but of the set of cells that we have isolated under the name *x*. In this case, the possibility for *x* to accomplish some of her wills would be provided by the determinism of the rest of the world, but the wills of *x*, that it the information she emits in direction of the rest of the world, would be random. With the definition of freedom sketched above, *x* would have a form of freedom, as some of her wills would be accomplished. But the fact that these wills are random would neither add nor remove anything from this freedom.

Is this a limitation of our definition of freedom? We do not think so. Indeed, I would probably not be freer, in the common sense of the word if, tomorrow, a random process in my brain decided me to become the follower of some new age guru or to spend my days reading tabloids. I do not exclude the possibility to become tomorrow the follower of some new age guru or to spend my days reading tabloids. I feel completely free to do so, and both projects may seduce me. But I do not see as a form of freedom that this decision would be a consequence of the fact that the spin, according to some axis, of some electron, in some of my neurons randomly took the value -1/2 or 1/2.

It thus seems that we must conclude that our freedom is more a consequence of the determinism, than of the randomness of the evolution of the world, including the I.

What if we were not within the world? In the description of the world above, we have isolated a person *x* from the rest of the world and the future state of the rest of the world is, in the deterministic case, a function *F* of its current state *E* and of the information *I* it receives from *x* and, in the random case, a function *G* of its current state *E*, of the information *I* it receives from *x* and of an element ω of a source of randomness Ω.

As we have said, the hypothesis that *x* is part of the world is not universally accepted: idealism does not see the person *x* as a part of the material word, but as an entity outside it. In the vocabulary specific to idealism, we must not use the expression “the rest of the world” but simply “the world”, as it is not possible to remove *x* from the material world it does not belong to.

The analytic viewpoint likes to see philosophical controversies as mere misunderstandings on the meaning of words. We could here suggest that difference between the idealist and materialist viewpoints is due to a simple misunderstanding on the meaning of the word “world”, that means, in the vocabulary of idealism, what the phrase “the rest of the world” means in that of materialism. We

shall not however go that far, as, although it is accurate, this difference does not exhaust the differences between these viewpoints, in particular on their hypotheses about the homogeneity between *x* and the rest of the world.

Using the vocabulary of idealism, we thus can say that the future state of the "world" $F(E,I)$ or $G(E,I,\omega)$ is a function of its current state *E*, of the information *I* it receives from *x* and, possibly, of an element ω of a source of randomness Ω. The future state of the "world" – that is of the rest of the world –, is not a function of its state *E* only, but also of the action *I* of *x* and, possibly, of an element ω of a source of randomness. The variables *I* and ω describe the sources that influence the evolution of the "world" – that is the rest of the world. For instance, the fact that a lamp, with a contact failure, turns on or not does not depend on its current state *E* only, but also on the action *I* of *x*, that presses the switch or not, and on the element ω of the source of randomness Ω describing the contact failure.

We can note that, in such a framework, the two descriptions that we qualified as "deterministic" and "random", become both "random", because, in both cases, the future state of the "world" – that is of the rest of the world –, is not a function of its current state only. However, even in such a framework, it seems useful to distinguish the two sources of "randomness": the actions *I* of *x* and, if we may say, the "pure" source of randomness ω, for instance to distinguish the action of the person *x* that presses the switch from the contact failure.

Indeed, the analysis that we have carried on above, transposes to the vocabulary of idealism and leads to the conclusion that the freedom of *x* does not come from the fact that the future state of the "world" – that is of the rest of the world – depends on the element ω of the source of pure randomness Ω, but that it depends on the action *I* of *x*. Our freedom is indeed a consequence of the randomness of the evolution of the "world" – that is of the rest of the world –, but of a very special form of "randomness": not pure randomness, but the actions of the person *x* that presses the switch, and that is located, by construction, outside the world.

We could go one step further than idealism, and identify the variable ω with the variable *I*, that is drop the variable ω, that is see, in all forms of randomness, the influence *I* of the consciousness of a person, located outside the world. We would then get the interpretations of quantum physics in which, when we measure the spin of an electron according some axis, non only we can obtain the values -1/2 or 1/2, but, moreover, the result of this measurement is determined by the consciousness of the observer. Even if they go rarely to such a caricature, such extreme interpretations of quantum physics have blossomed, from time to time, in its history, in particular when not only the measuring instrument is held responsible for the reduction of the state vector, but also the observer and her

consciousness. We can call globally these interpretations “the Cordoba interpretations” of quantum physics, even if there are differences between the points of view of the speakers of the Cordoba conference.

In such a framework, and it seems in such a framework only, it becomes possible to say that our freedom is a consequence of the randomness of measurement in quantum physics.

These interpretations however give rise to many well-known difficulties: the absence, in the subjective experience of many of us, of situations where we have deliberately chosen the value -1/2 or 1/2 as result of a measurement, the result of such a measurement when two observers chose different values, the extension of this ability to reduce the state vector to apes, cats, octopuses, and roses, the origin of the reduction of the state vector before the emergence of life…

These hypotheses that our consciousness is outside the world and that the manifestations of randomness in the world are the expression of our wills, are respectable. It seems possible – even if this is not what I am doing, because of the difficulties sketched above – to assume them and deduce that our freedom is a consequence of the randomness of the evolution of the world. In contrast, it seems difficult to assume both that we are within the world and that our freedom is a consequence of the randomness of the evolution of the world.

Predictability of the future is not a consequence of determinism

In the text cited above, Laplace also suggests that determinism implies predictability: “if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the world and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.”

Another consequence of the hypothesis that we are part of the world is that determinism does not imply predictability anymore.

It seems that Laplace himself had seen the problem as he stresses that in order to be able to predict the future, the intellect should be “vast enough to submit these data to analysis”. And, according to Laplace, this is because we are not such a vast enough intellect, that we need probabilities.

Even if we assume that the state of the world tomorrow is a function of its state today, and even if we assume that this function is computable, it is bold to deduce that we can predict the state of the world tomorrow, because the prefix “pre” of the verb “predict” means to say, before tomorrow, the

state of the world tomorrow. However, nothing seems to allow us to assume that we can, from within the world, compute today what the world will be tomorrow.

This question of the link between the time of the evolution of a system and the time needed to simulate this evolution is at the origin of Feynman's idea of a quantum computer. More recently, Stephen Wolfram has proposed to give the name “computational irreducibility” to this property of a phenomenon, not to have a simulation shorter than its evolution.

The evolution of the world is an example of a computationally irreducible phenomenon. Indeed, imagine an intellect, for instance a demon or a computer, able to predict today the state of the world tomorrow, and tomorrow the state of the world the day after tomorrow, and so on, and that would display each prediction as an image.

Then, if we assume that this intellect is within the world, it must itself appear on the image displaying the world tomorrow, displaying there the image of the world the day after tomorrow… The image of the world tomorrow would then contain the image of the world the day after tomorrow, and the day after… This way, predicting the state of the world tomorrow would require to also predict that of the world the day after tomorrow, and the day after, and all the days in the future. This would require an infinite amount of computation, impossible to execute in a finite time.

Thus both the theses that freedom is a consequence of randomness, and that determinism implies the predictability of the future, presuppose that we are outside the world. If we assume instead that we are within the world, then freedom is a consequence of determinism, and predictability of the future is not.