MPRI 2-7-1
Foundations of proof systems

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1 hour and a half.
All documents can be used.

1

[4 points]

Let $\Gamma = A, A \Rightarrow B, B \Rightarrow C, C \Rightarrow D$ and the Natural deduction proof

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, B \vdash C \Rightarrow D} \quad \dfrac{\dfrac{}{\Gamma, B \vdash B \Rightarrow C} \quad \dfrac{}{\Gamma, B \vdash B}}{\Gamma, B \vdash C}}{\Gamma, B \vdash D}}{\Gamma \vdash B \Rightarrow D} \quad \dfrac{\dfrac{}{\Gamma \vdash A \Rightarrow B} \quad \dfrac{}{\Gamma \vdash A}}{\Gamma \vdash B}}{\Gamma \vdash D}$$

Is this proof cut free?
What is the proof obtained by eliminate cuts in this proof?

2

[2 points]

Give an example of a proof containing a cut, such that eliminating this cut creates another cut.

3

[6 points]

Give a proof or a counter-model in constructive predicate logic of the following propositions

$$(\forall x\ P(x)) \Rightarrow P(0)$$

$$P(0) \Rightarrow (\forall x\ P(x))$$

$$(\exists y \forall x\ R(x,y)) \Rightarrow (\forall x \exists y\ R(x,y))$$

$$(\forall x \exists y\ R(x,y)) \Rightarrow (\exists y \forall x\ R(x,y))$$

4

[2 points] Give a proof or a counter-model in constructive predicate logic of the following propositions

$$(\neg\neg P) \Rightarrow P$$

5

[3 points]

Let π_1 be a proof in Arithmetic, presented in Deduction modulo, of the proposition $0 = 0$ and π_2 be a proof of $\forall z\ (N(z) \Rightarrow z+0 = z \Rightarrow S(z+0) = S(z))$. Give a proof of the proposition

$$\forall x\ (N(x) \Rightarrow x + 0 = x)$$

6

[3 points]

Give a proof in Simple type theory, presented in Deduction modulo, of the proposition

$$\exists p\ \varepsilon(p)$$