

MPRI 2-7-1
Fondements des systèmes de preuves

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Thursday, November 23rd, 2016

1 hour and a half.
All documents can be used.

1

(4 points)

Let P be a proposition symbol (that is, a predicate symbol of arity 0).

- (a) Give a proof of the proposition $P \Rightarrow P$.
- (b) Express this proof as a term of simply typed λ -calculus.
- (c) How many normal closed simply typed terms of type $P \Rightarrow P$ are there? Why?

2

(5 points)

- (a) Give two propositions A and B , such that the sequent $\vdash \forall x (A \vee B)$ has a constructive proof.
- (b) Let C and D be two propositions and π be a constructive cut-free proof of the sequent $\vdash \forall x (C \vee D)$. Show that this proof ends with two introduction rules.
- (c) Show that if the sequent $\vdash \forall x (C \vee D)$ has a constructive proof then so does the sequent $\vdash (\forall x C) \vee (\forall x D)$.

3

(4 points) Let P and Q be two proposition symbols (that is, predicate symbols of arity 0).

- (a) Give a model where $P \vee Q$ is not valid.
- (b) Give a model where $\neg P \wedge \neg Q$ is not valid.
- (c) Give a model where neither of these propositions is valid.

4

(6 points)

- (a) Give a term in Gödel system T expressing a function f such that $f(0) = 2$ and $f(1) = 0$.
- (b) Give a term in Gödel system T expressing a function g such that $g(0) = 1$, $g(1) = 2$, and $g(2) = 0$.
- (c) Give a term in Gödel system T expressing a function h such that $h(n)$ is $n \bmod 3$, the remainder of the division of n by 3.

5

(1 points)

- (a) Give a term in the Calculus of constructions of type

$$\forall X (X \Rightarrow (X \Rightarrow X) \Rightarrow X)$$