1 hour and a half.
All documents can be used.

1

(4 pts)
(a) Let $P$ be a proposition symbol (i.e. a predicate function or arity 0). Give a proof in natural deduction of the proposition $P \Rightarrow (P \Rightarrow P)$.
(b) Express this proof by a closed term in Simply typed lambda calculus.
(c) What is the type of this term?

2

(6 pts)
(a) In Deduction modulo, we consider the theory formed with the rewrite rule

$$P \rightarrow (P \Rightarrow \bot)$$

Give a proof of the proposition $\bot$ in this theory.
(b) Is this proof cut free?
(c) Is this theory super-consistent?
(d) Is it consistent?
3

(4 pts)
(a) Give a proof in Simple type theory of the proposition

$$\forall x \in (\exists x (x)) \Rightarrow \bot$$

(b) Express this proof by a term in the \(\lambda\text{-modulo}\) calculus.

4

(6 pts)

(a) Show that a normal term in Simply typed lambda-calculus has either the form \(\lambda x : A\ u\) or \((x\ u_1 \ldots\ u_n)\).

(b) Let \(P\) and \(Q\) be two atomic types. Show that there is no normal term of type \(P\) well-typed in the context \(y : (P \Rightarrow Q) \Rightarrow Q\).

(c) Show that there is no term of type \(P\) well-typed in the context \(y : (P \Rightarrow Q) \Rightarrow Q\).

(d) Show that there is no closed term of type \(((P \Rightarrow Q) \Rightarrow Q) \Rightarrow P\).

(e) Show that there is no normal term of type \(P\) well-typed in the context \(y : (P \Rightarrow Q) \Rightarrow P\).

(f) Show that there is no closed term of type \(((P \Rightarrow Q) \Rightarrow P) \Rightarrow P\).