

MPRI 2-7-1
Fondements des systèmes de preuves

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Thursday, March 1st, 2012

1 hour and a half.
All documents can be used.

1

(4 pts)

- (a) Let P be a proposition symbol (i.e. a predicate function or arity 0). Give a proof in natural deduction of the proposition $P \Rightarrow (P \Rightarrow P)$.
- (b) Express this proof by a closed term in Simply typed lambda calculus.
- (c) What is the type of this term?

2

(6 pts)

- (a) In Deduction modulo, we consider the theory formed with the rewrite rule

$$P \longrightarrow (P \Rightarrow \perp)$$

Give a proof of the proposition \perp in this theory.

- (b) Is this proof cut free?
- (c) Is this theory super-consistent?
- (d) Is it consistent?

3

(4 pts)

- (a) Give a proof in Simple type theory of the proposition

$$(\forall x \varepsilon(x)) \Rightarrow \perp$$

- (b) Express this proof by a term in the $\lambda\Pi$ -modulo calculus.

4

(6 pts)

- (a) Show that a normal term in Simply typed lambda-calculus has either the form $\lambda x : A u$ or $(x u_1 \dots u_n)$.
- (b) Let P and Q be two atomic types. Show that there is no normal term of type P well-typed in the context $y : (P \Rightarrow Q) \Rightarrow Q$.
- (c) Show that there is no term of type P well-typed in the context $y : (P \Rightarrow Q) \Rightarrow Q$.
- (d) Show that there is no closed term of type $((P \Rightarrow Q) \Rightarrow Q) \Rightarrow P$.
- (e) Show that there is no normal term of type P well-typed in the context $y : (P \Rightarrow Q) \Rightarrow P$.
- (f) Show that there is no closed term of type $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$.