Concept Alignement by Optimizing Gödel Translations

Internship Proposal

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Context. Dedukti is a proof checker that is primarily geared towards proof-assistant interoperability, making it effectively possible to share theorems and their proofs among different assistants, like Coq and Isabelle/HOL. In order to make classical proofs (that supports the use of the excluded middle) interoperable with constructive systems, an ecumenical logic has been defined in Dedukti [BDG+21], that is a logic which mixes classicism and constructivism, and where each connective can be chosen to be of classical or constructive nature.

Double-negation translations and their extensions, e.g. A-translation, are used to translate proofs from classical logics to constructive ones. This is done explicitely in ecumenical logic through the choice of the constructiveness of the connectives.

One of the challenges, when embedding a classical proof of *Theorem* $\mathcal{T}$ into a proof of *Theorem* $|\mathcal{T}|^{\neg}$ in ecumenical logic, is to preserve as much as possible the identity $|\mathcal{T}|^{\neg} = \mathcal{T}$, so as to keep the connectives and quantifiers constructive and to have a perfect alignment of definitions and theorems.

To this aim, since the seminal work of Kolmogorov and Gödel-Gentzen, aggressive optimizations of the number of negations introduced have been defined

- by Fréderic Gilbert [Gil15], with an optimality proof,
- appealing to focusing techniques [LM13].
- by Gilles Dowek [Dow16] (and further improvements [BH13]), where the ocurrences of connectives may be individually chosen to be classical or intuitionistic.

This last work is the core of the mechanism of ecumenical logic.

Objectives. The goal of this internship is to improve the definition of the classical connectives of ecumenical logic, by following the line pioneered by Gilbert. In particular, classical connectives can become functions, acting on their subformulas. This requires a theoretical study, and, secondarily, its implementation.

This work will then be extended in two directions : a switch to A-translation and Friedman's trick, and a more in-depth analysis of the remaining non-constructive connectives.

Prerequisites. This internship is mostly self-contained. It requires notions of theoretical Computer Science or a good mathematical backgound and, ideally, the knowledge of at least one functional programming language.

References

[BDG+21] Frédéric Blanqui, Gilles Dowek, Émilie Grienenberger, Gabriel Hondet, and François Thiré. Some axioms for mathematics. In Naoki Kobayashi, editor, *6th International Conference on Formal Structures for Computation and Deduction, FSCD 2021, July 17-24, 2021, Buenos Aires, Argentina (Virtual Conference)*, volume 195 of *LIPIcs*, pages 20:1–20:19. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021.

[BH13] Mélanie Boudard and Olivier Hermant. Polarizing double-negation translations. In Ken McMillan, Aart Middeldorp, and Andreï Voronkov, editors, *LPAR*, volume 8312 of *LNCS ARCoSS*, pages 182–197. Springer, 2013.

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[Gil15] Frédéric Gilbert. A lightweight double-negation translation. In Ansgar Fehnker, Annabelle McIver, Geoff Sutcliffe, and Andrei Voronkov, editors, *20th International Conferences on Logic for Programming, Artificial Intelligence and Reasoning - Short Presentations, LPAR 2015, Suva, Fiji, November 24-28, 2015.*, volume 35 of *EPiC Series in Computing*, pages 81–93. EasyChair, 2015.

[LM13] Chuck Liang and Dale Miller. Kripke semantics and proof systems for combining intuitionistic logic and classical logic. *Ann. Pure Appl. Log.*, 164(2):86–111, 2013.