**Exercises related to the previous session**

**Exercise 1.** Let $T^* = \{ A_1 \equiv C_1, \ldots, A_m \equiv C_m \}$ be an $\mathcal{ALC}$ TBox satisfying the following properties.

- Every $A_i$ is a concept name, and $A_i \equiv C_i$ is an abbreviation for $A_i \sqsubseteq C_i$ and $C_i \sqsubseteq A_i$.
- For all $i, j \in [1, m]$, if $A_j$ occurs in $C_i$, then $j > i$.
- If $i \neq j \in [1, m]$, then $A_i$ and $A_j$ are syntactically distinct.

Such a TBox $T^*$ is called acyclic.

1. Briefly define an acyclic graph from $T^*$, which would justify the terminology “$T^*$ is acyclic”.

2. Given an interpretation $\mathcal{I}$, show that there exists an interpretation $\mathcal{J}$ such that $\mathcal{J} \models T^*$, the interpretations of the role names and concept names different from $\{A_1, \ldots, A_m\}$ are identical in $\mathcal{I}$ and $\mathcal{J}$.

3. Design an algorithm that takes as input a knowledge base $\mathcal{K} = (T, A)$ with acyclic $T$ and returns an ABox $A'$ such that $\mathcal{K}$ is consistent iff $(\emptyset, A')$ is consistent. The proof for the soundness of the algorithm is not required.

4. Explain why your algorithm terminates and analyse its computational complexity.

**Exercise 2.** (Exponential-size interpretations) Define a family of concepts $(C_n)_{n \geq 1}$ such that each $C_n$ is of polynomial size in $n$ (for a fixed polynomial), $C_n$ is satisfiable, and the interpretations satisfying $C_n$ have at least $2^n$ individuals in its domains.
**Exercises related to today session**

Exercise 3. Let us consider the translation map $t$ into first-order logic. Let $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ be an interpretation.

1. Let $C$ be a complex concept in $\mathcal{ALC}$. Show that for all $a \in \Delta^\mathcal{I}$, we have $a \in C^\mathcal{I}$ iff $\mathcal{I}, \rho[x \leftarrow a] \models t(C, x)$ where $\rho$ is a first-order assignment.

2. Show that $\mathcal{I} \models K$ iff $\mathcal{I} \models t(K)$.

Exercise 4. (Model-checking in PTIME) Let $\mathcal{I}$ be an interpretation with finite domain and $C$ be an $\mathcal{ALC}$ concept. Show that the algorithm seen in the lecture to compute $C^\mathcal{I}$ indeed runs in polynomial time.

Exercise 5. Let us consider an alternative notion of size for a concept, say $dsize(C) \overset{\text{def}}{=} \text{card}(\text{sub}(C))$ (“DAG size”).

1. Design a family of concepts $(C_n)_{n \geq 1}$ such that $dsize(C_n)$ is in $O(n)$ and $\text{size}(C_n)$ is in $O(2^n)$.

2. Show that the satisfiability problem for $\mathcal{ALC}$ concepts when the size of $C$ is measured with $dsize(C)$ can be solved in polynomial space too.