Exercise 1. Consider the CGS below with two agents, two actions $a, b$ and the propositional variables $p, q, r$.

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| s1 | p, q |
| s2 | (a, b) |
| s3 | r, q |
| s4 | (b, b) |
| s5 | (a, b) |
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Determine which statements below hold.

1. $M, s_1 \models p \land \langle\langle 1 \rangle\rangle X q$
2. $M, s_1 \models \langle\langle 1 \rangle\rangle (p \lor r) U \neg q$
3. $M, s_1 \models \langle\langle 1 \rangle\rangle F \neg \langle\langle 2 \rangle\rangle X \neg p$
4. $M, s_1 \models \langle\langle 1 \rangle\rangle G p \land \langle\langle 2 \rangle\rangle G p \land \langle\langle 1, 2 \rangle\rangle F \neg p$
5. $M, s_2 \models \neg \langle\langle 1 \rangle\rangle X (q \land r) \land \neg \langle\langle 2 \rangle\rangle X p \land \neg \langle\langle 1, 2 \rangle\rangle X (p \lor r)$
6. $M, s_3 \models \langle\langle 1 \rangle\rangle G \langle\langle 1, 2 \rangle\rangle (\neg q U p)$

Exercise 2. Show that the following properties hold in ATL.

1. $\langle A \rangle X \varphi \land \langle A' \rangle X \varphi' \Rightarrow \langle A \cup A' \rangle X (\varphi \land \varphi')$ is valid when $A \cap A' = \emptyset$.
2. $\langle\langle 0 \rangle\rangle G (\psi \Rightarrow (\varphi \land \langle\langle A \rangle\rangle X \psi)) \Rightarrow \langle\langle 0 \rangle\rangle G (\psi \Rightarrow \langle A \rangle G \varphi)$.

Exercise 3. Show $\langle\langle A \rangle\rangle X \varphi \sqsupset_{\text{ATL}} \text{pre}(M, A, \varphi)$ (M is CGS, $A \subseteq \text{Agt}$, and $\varphi$ is an ATL formula). The definition of $\text{pre}(M, A, Z)$ is recalled below.

$\text{pre}(M, A, Z) \overset{\text{def}}{=} \{s \in S \mid \text{there is } f \in D_A(s) \text{ such that } \text{out}(s, f) \subseteq Z\}$

Exercise 4. Let $M$ be a CGS, $\varphi, \psi$ be ATL formulae and $A \subseteq \text{Agt}$. Show the following characterisations.

1. $\langle\langle A \rangle\rangle G \varphi \sqsupset_{\text{ATL}} \nu Z. (\langle\langle \varphi \rangle\rangle \sqcap \text{pre}(M, A, Z))$.
2. $\langle\langle A \rangle\rangle \varphi U \psi \sqsupset_{\text{ATL}} \mu Z. (\langle\langle \psi \rangle\rangle \sqcup (\langle\langle \varphi \rangle\rangle \sqcap \text{pre}(M, A, Z)))$. 


Exercise 5. Consider the concurrent game structure below with state space $S$.

1. Let $f_1: \{\text{Robot}_1\} \to \text{Act}$ be such that $f_1(\text{Robot}_1) = \text{push}$ and $f_2: \{\text{Robot}_2\} \to \text{Act}$ be such that $f_2(\text{Robot}_2) = \text{wait}$. Determine the sets $\text{out}(s_1, f_1)$ and $\text{out}(s_1, f_2)$.

2. Let $F_{\text{Robot}_1}$ be the positional strategy for $\text{Robot}_1$ such that $F_{\text{Robot}_1}(s_0) = \text{wait}$, $F_{\text{Robot}_1}(s_1) = \text{push}$, $F_{\text{Robot}_1}(s_2) = \text{wait}$. Then, determine the following sets of maximal computations

$$\text{Comp}(s_0, F_{\text{Robot}_1}), \text{Comp}(s_1, F_{\text{Robot}_1}), \text{Comp}(s_2, F_{\text{Robot}_1}).$$

3. Let $\text{Robot}_1$ adopt the following memoryful strategy $F_{\text{Robot}_1}^m$. Below, "$F(E) = a$" for a regular expression $E$, indicates that the value of $F$ for every element of $E$ is $a$. (In the rest of this exercise, we omit the joint actions in some $D_{\text{Agt}}(s)$)

$$F_{\text{Robot}_1}^m(\{s_0, s_1\}^*) = \text{wait}, \quad F_{\text{Robot}_1}^m(\{s_0, s_1\}^* s_2 S^*) = \text{push}.$$ 

That is, the strategy prescribes waiting until the state $s_2$ is visited, if ever, and then pushing forever.

(a) Determine the set of outcome plays starting from each of the following histories.

$$s_1 \quad s_2 \quad s_1, s_0 \quad s_1, s_0, s_1 \quad s_2, s_0, s_1 \quad s_2, s_1, s_0$$

(b) Which of these histories are consistent with the strategy $F_{\text{Robot}_1}^m$?