Exercise 1. Consider the CGS below with two agents, two actions \(a, b\) and the propositional variables \(p, q, r\).

Determine which statements below hold.

1. \(\mathcal{M}, s_1 \models p \land \langle 1 \rangle Xq\)
2. \(\mathcal{M}, s_2 \models \langle 1 \rangle (p \lor r) U \neg q\)
3. \(\mathcal{M}, s_1 \models \langle 1 \rangle F \neg \langle 2 \rangle X p\)
4. \(\mathcal{M}, s_1 \models \langle 1 \rangle G p \land \langle 2 \rangle G p \land \langle 1, 2 \rangle F \neg p\)
5. \(\mathcal{M}, s_2 \models \neg \langle 1 \rangle X (q \land r) \land \neg \langle 2 \rangle X p \land \neg \langle 1, 2 \rangle X (p \lor r)\)
6. \(\mathcal{M}, s_3 \models \langle 1 \rangle G \langle 1, 2 \rangle (\neg q U p)\)

Exercise 2. Let \(\mathcal{M} = (Agt, S, Act, act, \delta, L)\) be a concurrent game structure (CGS) \(A, A' \subseteq Agt\) be coalitions such that \(A \cap A' = \emptyset\), \(s \in S\) and \(\varphi, \varphi'\) be ATL formulae built over coalitions from \(Agt\).

1. Show that if \(\mathcal{M}, s \models (\langle A \rangle G \varphi) \land (\langle A' \rangle G \varphi')\) then \(\mathcal{M}, s \models \langle A \cup A' \rangle G (\varphi \land \varphi')\).
2. Is it always the case that if \(\mathcal{M}, s \models (\langle A \rangle F \varphi) \land (\langle A' \rangle F \varphi')\) then \(\mathcal{M}, s \models \langle A \cup A' \rangle F (\varphi \land \varphi')\)?
**Exercise 3.** (from exam 2021/2022) Given a concurrent game structure $M = (\text{Agt}, S, \text{Act}, \text{act}, \delta, L)$, coalitions $A \subseteq A' \subseteq \text{Agt}$ and a state $s \in S$, show that $M, s \models (\langle \emptyset \rangle X p \land \langle A \rangle X q) \Rightarrow \langle A' \rangle X (p \land q)$.

**Exercise 4.** Show that $\langle A \rangle X \varphi \land \langle A' \rangle X \varphi' \Rightarrow \langle A \cup A' \rangle X (\varphi \land \varphi')$ is valid when $A \cap A' = \emptyset$.

**Exercise 5.** Let $M$ be a CGS, $\varphi, \psi$ be ATL formulae and $A \subseteq \text{Agt}$. Show the following characterisations.

1. $\langle A \rangle G \varphi = \nu Z . (\langle \varphi \rangle Z \cap \text{pre}(M, A, Z))$.
2. $\langle A \rangle \varphi \psi = \mu Z . (\langle \psi \rangle Z \cup (\langle \varphi \rangle Z \cap \text{pre}(M, A, Z)))$.

**Exercise 6.** Consider the concurrent game structure below with state space $S$ and set of agents $\{\text{Robot}_1, \text{Robot}_2\}$.

![Diagram of concurrent game structure]

1. Let $\sigma_{\text{Robot}_1}$ be the positional strategy for $\text{Robot}_1$ such that $\sigma_{\text{Robot}_1}(s_0) = \text{wait}$, $\sigma_{\text{Robot}_1}(s_1) = \text{push}$, $\sigma_{\text{Robot}_1}(s_2) = \text{wait}$. Then, determine the following sets of maximal computations

$$\text{comp}(s_0, \sigma_{\text{Robot}_1}), \text{comp}(s_1, \sigma_{\text{Robot}_1}), \text{comp}(s_2, \sigma_{\text{Robot}_1}).$$

Use $\omega$-regular expressions to define such sets of computations.
2. Let $\text{Robot}_1$ adopt the following memoryful strategy $\sigma^m_{\text{Robot}_1}$. Below, "\(\sigma(E) = a\)" for a regular expression \(E\), indicates that the value of \(\sigma\) for every element of \(E\) is \(a\). So, \(a\) is the action chosen by $\text{Robot}_1$ (below, we do not use anymore the notation with the joint action \(f\))

\[
\sigma^m_{\text{Robot}_1}(\{s_0, s_1\}^+) = \text{wait} , \quad \sigma^m_{\text{Robot}_1}(\{s_0, s_1\}^*s_2s^*) = \text{push} .
\]

That is, the strategy prescribes waiting until the state \(s_2\) is visited, if ever, and then pushing forever. Define a Büchi automaton $B$ over the alphabet $\Sigma = \{s_1, s_2, s_3\}$ such that the language of $\omega$-words accepted by $B$ is the set of maximal computations $\text{comp}(s_1, \sigma^m_{\text{Robot}_1})$ (omitting the joint actions between two successive states). For instance, $B$ should accept the word $s_1s_0s_2^\omega$. 