Exercise 1. Let $\mathbb{M}$ be a CGS with resources, $s \in S$ and $\langle \langle A^b \rangle \rangle \Phi$ be a formula in ATL$(Agt, r)$ (ATL with $r$ resources). Show that the set $\{ b \in \mathbb{N}^r \mid \mathbb{M}, s \models \langle \langle A^b \rangle \rangle \Phi \}$ is upward-closed.

Exercise 2. Below, we present a CGS $\mathbb{M}$ with one resource and two agents, the transitions are labelled by pairs of actions with the respective weights. For instance, the total weight of the transition from $s_1$ to $s_2$ is $-2$.

1. Characterise the set $\{ n \in \mathbb{N} \mid \mathbb{M}, s_1 \models \langle \langle \{1\}^n \rangle \rangle (Gp_1 \lor Fp_2) \}$.

2. Characterise the set of natural numbers $n \in \mathbb{N}$ such that there is a positional and $n$-consistent strategy $\sigma$ w.r.t. $s_1$ for the coalition $\{1\}$ such that for all computations $\lambda \in \text{comp}(s_1, \sigma)$, we have $\mathbb{M}, \lambda \models (Gp_1) \lor (Fp_2)$.

Exercise 3. Design a CGS $\mathbb{M}$ with one resource, $s \in S$ and $b \in \mathbb{N}$ such that $\mathbb{M}, s \models \langle \langle A^b \rangle \rangle Fp$ but there is no positional $b$-consistent strategy $\sigma$ such that for all computations $\lambda \in \text{comp}(s, \sigma)$, we have $\mathbb{M}, \lambda \models Fp$.

Exercise 4. Let $\mathbb{M}$ be a CGS with one resource, $s \in S$, $A \subseteq Agt$ be a coalition, $b \in \mathbb{N}$ and $\sigma$ be a $b$-consistent strategy from $s$ for the coalition $A$ such that for all computations $\lambda \in \text{comp}(s, \sigma)$, we have $\mathbb{M}, \lambda \models Gp$.

1. Show that for all $\lambda \in \text{comp}(s, \sigma)$ with $RAV(b, \lambda) = (s_0, v_0) \rightarrow (s_1, v_1) \cdots (s_i, v_i) \cdots$, for all $i \geq 0$, we have $\mathbb{M}, s_i \models p$ and $v_i \geq 0$.

2. We assume a total ordering on the (finite) set $\bigcup_{s \in S} D_A(s)$ of joint actions with domain the coalition $A$ (ordering unspecified here). We write $D_A(s)$ to denote the set of joint actions for the coalition $A$ from the state $s$. Let us define a positional strategy $\sigma'$ as follows.
If a state $s'$ does not occur in any computation of $\text{comp}(s, \sigma)$, then $\sigma'(s') \overset{\text{def}}{=} \min D_A(s')$ (dummy value).

Now, assume that $s'$ occurs in some computation of $\text{comp}(s, \sigma)$. Let $\min_{s'}$ be

$$\min_{s'} \overset{\text{def}}{=} \min \{ v \mid \exists \lambda \in \text{comp}(s, \sigma) \text{ s.t. } (s', v) \text{ occurs in } RAV(b, \lambda) \}$$

Then,

$$\sigma'(s') \overset{\text{def}}{=} \min \{ (f_j)_{|A} \mid \exists \lambda = s_0 \overset{f_0}{\to} s_1 \cdots \in \text{comp}(s, \sigma) \text{ with } RAV(b, \lambda) = (s_0, v_0) \overset{j}{\to} (s_1, v_1) \cdots, j \in \mathbb{N} \text{ s.t. } (s_j, v_j) = (s', \min_{s'}) \}$$

$(f_j)_{|A}$ denotes the restriction of the (total) joint action $f_j$ to the agents in $A$. Considering the minimal value among all the $(f_j)_{|A}$’s is just a technical means to pick an arbitrary joint action. Show that all the states occurring in some computation in $\text{comp}(s, \sigma')$ satisfy the propositional variable $p$.

3. Show that for all computations $\lambda' = s_0' \overset{f_0}{\to} s_1' \cdots \in \text{comp}(s, \sigma')$ with $RAV(b, \lambda') = (s_0', v_0') \overset{j}{\to} (s_1', v_1') \cdots$, for all $i' \in \mathbb{N}$, there is a computation $\lambda = s_0 \overset{f_0}{\to} s_1 \cdots \in \text{comp}(s, \sigma)$ with $RAV(b, \lambda) = (s_0, v_0) \overset{i}{\to} (s_1, v_1) \cdots$ and $i \in \mathbb{N}$ such that $s_i' = s_i$ and $v_i' \geq v_i$.

4. Conclude $M, s \models \langle \langle A^b \rangle \rangle G p$ iff there is a positional $b$-consistent strategy $\sigma$ such that for all computations $\lambda \in \text{comp}(s, \sigma)$, we have $M, \lambda \models G p$.

**Exercise 5.** Complete the proof of undecidability of $\text{ATL}'(2)$ by showing that $M$ reaches the instruction $n$ iff $M, 1 \models \langle \{1\}^0 \rangle F p$. 

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