Counter Systems for Data Logics

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Models with Data

Ubiquity of data words
[Bouyer, IPL 02]

- Data word

$$\begin{matrix} a_1 & a_2 & a_3 & \cdots \\ d_1 & d_2 & d_3 & \cdots \end{matrix}$$

  - Each a_i belongs to a finite alphabet Σ.
  - Each d_i belongs to an infinite domain D.

- Timed word [Alur & Dill, TCS 94]

$$\begin{matrix} a & b & c & a & a & b \\ 0 & 0.3 & 1 & 2.3 & 3.5 & 3.51 \end{matrix}$$

- Runs from counter systems

$$\begin{matrix} q_0 & q_2 & q_3 & q_2 & q_3 & q_2 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{matrix}$$

- Integer arrays [Habermehl & Iosif & Vojnar, FOSSACS'08]

$$t[0] \quad t[1] \quad t[2] \quad t[3] \quad t[4] \quad t[5] \ldots$$

Finite alphabet and infinite domain

a	a	b	d	a	b
URL_1	URL_2	URL_1	URL_2	URL_3	URL_3

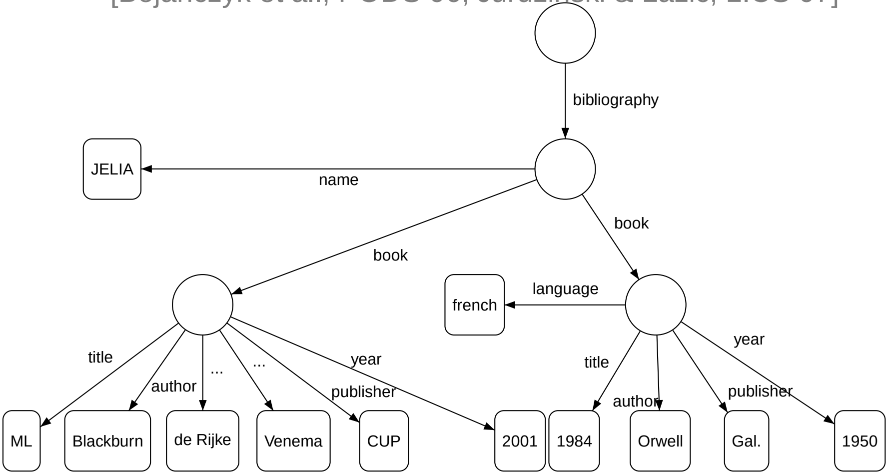
a	a	b	d	a	b
3	2.5	3	2.5	4	4

$a \quad a \quad b \quad d \quad a \quad b$

Data trees

Extension to data trees (XML documents with values).

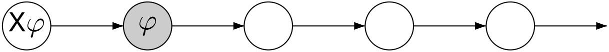
[Bojańczyk et al., PODS 06; Jurdziński & Lazić, LICS 07]



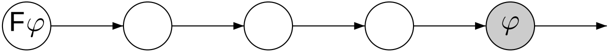
Formalisms for Data Words – Temporal Logics

Linear-time temporal operators

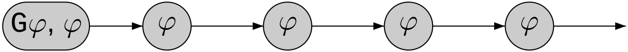
Xφ: next-time φ



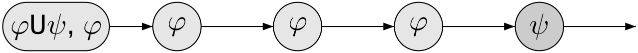
Fφ: sometimes φ



Gφ: always φ



φUψ: φ until ψ



A mechanism for handling data

- Case analyses in formulae are not sufficient with infinite domains.
- A register can store a data value and equality tests are performed between registers and current data values.
- Storing a value in a register:

$$\downarrow_r \varphi \stackrel{\text{def}}{=} \exists \mathsf{y}_r \ (\mathsf{y}_r = \mathsf{x}) \wedge \varphi$$

- Equality test between a register and a value: $\uparrow_r \stackrel{\text{def}}{=} \mathsf{y}_r = \mathsf{x}$. (in this talk, restriction to the simple equality tests)
- All data values at distinct positions are distinct:

$$\mathtt{G}(\downarrow_r \mathtt{XG}\neg \uparrow_r)$$

- Generalization with memory logics, e.g. memory bags have operations “register”, “forget” and “erase”.

[Mera, PhD thesis 09]

Freeze operator

- Freeze quantifier in hybrid logics.
  [Goranko 94; Blackburn & Seligman, JOLLI 95]

- Temporal semantics of imperative programs.
  [Manna & Pnueli, 1992]
  Program variable x never decreases below its initial value:

$$\exists y\ (x = y)\ \wedge\ \mathtt{G}(x \geq y)$$

- Freeze quantifier in real-time logics.
  [Alur & Henzinger, JACM 94]
  $y \cdot \varphi(y)$ binds the variable y to the current time t.

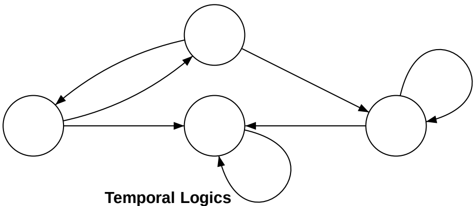
- Predicate λ-abstraction [Fitting, JLC 02].
  $\langle y \cdot \mathrm{F}\, P(y)\rangle(c)$: current value of constant c satisfies the predicate P.

- See also description logics over concrete domains.
  [Baader & Hanschke, IJCAI'91; Lutz, TOCL 04]

Hybrid logics as data logics

- Most standard models for modal logics are graphs in which nodes are labelled by propositional valuations.
- For a given formula, the set of propositional valuations is a finite alphabet.
- $\downarrow_y \varphi$: φ holds true in the variant model where proposition y is true only at the current state.
  [Goranko 94; Blackburn & Seligman, JOLLI 95].
- Models are enriched with node adresses.
- "Every reachable state can be visited infinitely often":

$AG \downarrow_y E\, XF\, y$



LTL with registers: $\mathrm{LTL}^{\downarrow}$

- $\mathrm{LTL}^{\downarrow}$ formulae:

$$\varphi \quad ::= \quad a \mid \uparrow_r \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \mathsf{U} \varphi \mid \mathsf{X}\varphi \mid \downarrow_r \varphi$$

  where $a \in \Sigma$ and $r \in \mathbb{N}^{+}$.

- Register valuation f: finite partial map from $\mathbb{N}^{+}$ to $\mathbb{N}$ $(= D)$.

- Models: finite or infinite data words over the alphabet Σ.

- Satisfaction relation:

$$\sigma, i \models_f \uparrow_r \quad \stackrel{\text{def}}{\Leftrightarrow} \quad r \in \mathrm{dom}(f) \text{ and } f(r) = d_i$$

$$\sigma, i \models_f \downarrow_r \varphi \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \sigma, i \models_{f[r \mapsto d_i]} \varphi$$

  (d_i: data value at position i)

- Unlike standard LTL, $\mathrm{LTL}^{\downarrow}$ can store a data value and perform equality tests.

Examples

- Nonce property: $\mathtt{G}(\downarrow_1 \mathtt{XG}\neg \uparrow_1)$.

$$\downarrow_1 \mathsf{X} \uparrow_1 \approx \mathsf{x} = \mathbf{X}\mathsf{x}$$

$$a \;\; a \;\; b \;\; d \;\; a \;\; b \;\; , 0 \not\models \mathsf{F}(a \wedge \downarrow_1 \mathsf{XF}(a \wedge \uparrow_1))$$

An another view on $\mathrm{LTL}^{\downarrow}$

- Standard LTL models are of the form $\mathbb{N} \rightarrow \mathcal{P}(\mathrm{PROP})$ for some countably infinite set PROP of atomic propositions.

- An LTL formula φ built over $\{\mathrm{p}_1, \ldots, \mathrm{p}_k\}$ constrains the models only for $\{\mathrm{p}_1, \ldots, \mathrm{p}_k\}$

- No LTL formula characterizes the class of models for which any two distinct positions have distinct valuations.

- $\mathrm{LTL}^{\downarrow}$ = extension of LTL (with standard models) where the registers store valuations in $\mathcal{P}(\mathrm{PROP} \setminus \mathrm{PROP}_k)$ and the alphabet is $\mathcal{P}(\mathrm{PROP}_k)$ with $\mathrm{PROP}_k = \{\mathrm{p}_1, \ldots, \mathrm{p}_k\}$.

Complexity of satisfiability problems

- Finitary and infinitary satisfiability problem for LTL are PSPACE-complete. [Sistla & Clarke, JACM 85]

- What about $\mathrm{LTL}^{\downarrow}$ with one register, with all registers etc.?

- Infinitary satisfiability problem for $\mathrm{LTL}^{\downarrow}$ restricted to X and F and to a single register is undecidable.

- Finitary satisfiability problem for $\mathrm{LTL}^{\downarrow}$ restricted to a single register is decidable but nonprimitive recursive.

  [Demri & Lazić, TOCL 09]

- Finitary satisfiability problem for $\mathrm{LTL}^{\downarrow}$ restricted to F and
  - to a single register is nonprimitive recursive too.
  - to two registers is undecidable.

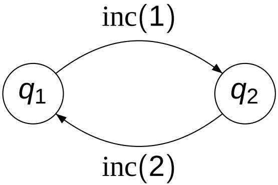
  [Figueira & Segoufin, MFCS'09]

- Nonprimitive recursiveness uses [Schnoebelen, IPL 02].

How Counter Systems Enter into the Play

Counter automata (CA)

- Counter system = finite-state automaton + counters.
- Counter: program variable interpreted by a non-negative integer.



- Counter automaton $\mathcal{S} = (Q, n, \delta)$
  - Finite set of control states Q.
  - Transitions in $\delta \subseteq Q \times \{\mathrm{zero}(i), \mathrm{inc}(i), \mathrm{dec}(i) : i \in [1, n]\} \times Q$.
  - Dimension n (number of counters).
- Runs of the form

$$\rho = \begin{matrix} q_0 \\ \vec{x}_0 \end{matrix} \rightarrow \begin{matrix} q_1 \ (\in Q) \\ \vec{x}_1 \ (\in \mathbb{N}^n) \end{matrix} \rightarrow \begin{matrix} q_2 \\ \vec{x}_2 \end{matrix} \rightarrow \ldots$$

Reachability problems

- Reachability problem:

  Input: counter automaton $\mathcal{S}$, $(q, \vec{0})$ and $(q', \vec{0})$.

  Question: is $(q, \vec{0}) \xrightarrow{*} (q', \vec{0})$?

- Control state reachability problem:

  Input: counter automaton $\mathcal{S}$, $(q, \vec{0})$ and q'.

  Question: is $(q, \vec{0}) \xrightarrow{*} (q', \vec{x'})$ for some $\vec{x'}$?

- Control state repeated reachability problem:

  Input: counter automaton $\mathcal{S}$, $(q, \vec{0})$ and q_f.

  Question: is there an infinite run from $(q, \vec{x})$ such that q_f is repeated infinitely often?

- Covering problem (extending control state reachability):

  Input: counter automaton $\mathcal{S}$, $(q, \vec{0})$ and $(q', \vec{x'})$.

  Question: is $(q, \vec{0}) \xrightarrow{*} (q', \vec{x''})$ with $\vec{x'} \preceq \vec{x''}$?

  ($\preceq$ is defined pointwise)

Counter automata generate data words

- A counter automaton and an initial configuration generate a set of runs viewed as data words with multiple data values.
- The finite alphabet is Q.
- Extension of freeze operators to $\downarrow_r^j$ and $\uparrow_r^j$ with $j \in [1, n]$.

Turing-completeness of Minsky machines

- A counter stores a single natural number.
- A Minsky machine can be viewed as a deterministic finite-state automaton with two counters.
- Operations on counters:
  - Check whether the counter is zero.
  - Increment the counter by one.
  - Decrement the counter by one if nonzero.
- Halting problem ($\approx$ control state reachability problem):

  input: a Minsky machine M;
  question: is the unique computation halts?
- The halting problem is undecidable and Minsky machines are Turing-complete. [Minsky, 67]

Reachability Problems for Gainy CA

Gainy counter automata

- Faulty systems perform errors such as losses or gains, e.g., see works on lossy channel systems.
  [Abdulla & Jonsson, IC 96]

- Three ways to model gainy counter automata:
  1. Standard CA (Q, n, δ) such that for $q \in Q$ and $i \in [1, n]$, $q \xrightarrow{\mathrm{inc}(i)} q \in \delta$.
  2. Alternative one-step relation: $(q, \vec{x}) \xrightarrow{t}_g (q', \vec{x'})$ iff there are $\vec{y}, \vec{y'}$ in $\mathbb{N}^n$ such that

     $$\vec{x} \preceq \vec{y} \ \text{ and } \ (q, \vec{y}) \xrightarrow{t} (q', \vec{y'}) \ (\text{exact step}) \ \text{ and } \ \vec{y'} \preceq \vec{x'}$$

  3. Gains occur in a lazy way: decrement on zero has no effect.

Benefits from Gainy CA

- Features:
  - Increment, decrement and zero-test.
  - Incrementation errors.

- Control state reachability problem is decidable but with a nonprimitive recursive complexity.
  See e.g., [Urquhart, JSL 99; Schnoebelen, IPL 02]

- Control state repeated reachability problem is undecidable.
  [Demri & Lazić, TOCL 09]
  (adapt a proof from [Ouaknine & Worrell, FOSSACS'06])

- These problems reduce to corresponding satisfiability problems for $\mathrm{LTL}^{\downarrow}$ restricted to X and F and to a single register.

Simulating Gainy CA

- Gainy CA $\mathcal{S}$ with initial configuration $(q_0, \vec{0})$.

- For $t \in \delta$, $\Sigma(t)$ denotes the instruction labelling it in $\Sigma = \{\mathrm{inc}(i), \mathrm{dec}(i), \mathrm{zero}(i) : i \in [1, n]\}$.

- Let us build φ in $\mathrm{LTL}^{\downarrow}$ s.t. φ is satisfiable iff $(\mathcal{S}, (q_0, \vec{0}))$ has an infinite run with q_f occurring infinitely often.

- φ is satisfiable only in models in which each position is labelled by a transition and by a value in $\mathbb{N}$.

- Infinite models of φ are of the form $(t_0, y_0), (t_1, y_1), (t_2, y_2), \cdots$ with $t_i \in \delta$ and $y_i \in \mathbb{N}$.

- For $I, J \in \mathbb{N}$, $I \sim J \overset{\text{def}}{\Leftrightarrow} y_I = y_J$.

Simulating gainy CA (II)

- Let us explain how the run from $(q_0, \vec{0})$ below is encoded.

$$(q_0, \vec{x_0}) \xrightarrow{a_0} (q_1, \vec{x_1}) \xrightarrow{a_1} \cdots \xrightarrow{a_{K-1}} (q_K, \vec{x_K}) \cdots$$

- Projection of the model over δ is

$$t_0 t_1 t_2 \cdots = q_0 \xrightarrow{a_0} q_1, q_1 \xrightarrow{a_1} q_2, \cdots$$

and q_f is repeated inifinitely often.

- Initial state is q_0:

$$\bigvee_{t = q_0 \xrightarrow{a} q} t$$

- The sequence of transitions respects δ:

$$\mathtt{G}\Big(\bigwedge_{t = q \xrightarrow{a} q' \in \delta} \big(t \Rightarrow \mathtt{X} \bigvee_{t' = q' \xrightarrow{a} q''} t'\big)\Big)$$

Simulating Gainy CA (III)

- Control state q_f is visited infinitely often: $\mathtt{GF} \bigvee_{t = q \xrightarrow{a} q_f} t$

- Each increment or decrement is associated to a unique value.

$$\mathrm{inc}(1) \quad \mathrm{inc}(1) \quad \mathrm{dec}(1) \quad \mathrm{inc}(1) \quad \mathrm{dec}(1) \quad \mathrm{dec}(1) \quad \mathrm{zero}(1)$$

- For $a \in \Sigma$, a is also used as a shortcut for $\bigvee_{t = q \xrightarrow{b} q' \in \delta,\ a = b} t$.

- For $i, j \in [1, n]$, there are no two positions for increments [resp. decrements] having the same value:

$$\mathtt{G}(\mathrm{inc}(i) \Rightarrow \neg(\downarrow_1 \mathtt{XF}(\uparrow_1 \wedge \mathrm{inc}(j)))) \quad \mathtt{G}(\mathrm{dec}(i) \Rightarrow \neg(\downarrow_1 \mathtt{XF}(\uparrow_1 \wedge \mathrm{dec}(j))))$$

Simulating Gainy CA (IV)

- The two next conditions are formulated in such a way to avoid using the until operator $\mathtt{U}$.

- For $i \in [1, n]$ and $J > I$, if $\Sigma(t_I) = \mathrm{inc}(i)$ and $\Sigma(t_J) = \mathrm{zero}(i)$, then there is no $K > J$ such that $\Sigma(t_K) = \mathrm{dec}(i)$ and $I \sim K$:

$$\mathtt{G}(\mathrm{inc}(i) \Rightarrow \downarrow_1 \neg(\mathsf{F}(\mathrm{zero}(i) \wedge (\mathsf{F}(\uparrow_1 \wedge \mathrm{dec}(i))))))$$

- For $i \in [1, n]$, if there are $J > I$ such that $\Sigma(t_I) = \mathrm{inc}(i)$ and $\Sigma(t_J) = \mathrm{zero}(i)$, then there is $K > I$ such that $\Sigma(t_K) = \mathrm{dec}(i)$ and $I \sim K$.

$$\mathtt{G}((\mathrm{inc}(i) \wedge \mathsf{F}\ \mathrm{zero}(i)) \Rightarrow \downarrow_1 (\mathsf{F}(\mathrm{dec}(i) \wedge \uparrow_1)))$$

- φ is satisfiable iff $(\mathcal{S}, (q_0, \vec{0}))$ has an infinite run such that q_f occurs infinitely often.

Gainy CA for $\mathrm{LTL}^{\downarrow}$ with one register !

- Control state repeated reachability problem for Gainy CA can be reduced to infinitary satisfiability for $\mathrm{LTL}^{\downarrow}$ restricted to one register. $\rightarrow$ undecidability

- Control state reachability problem for Gainy CA can be reduced to finitary satisfiability for $\mathrm{LTL}^{\downarrow}$ restricted to one register. $\rightarrow$ nonprimitive recursiveness

- In the finitary case, there is a converse reduction.

About nonprimitive recursiveness

- Control state reachability problem for gainy CA is nonprimitive recursive.
  See e.g., [Urquhart, JSL 99; Schnoebelen, MFCS'10]

- Ackermann function is nonprimitive recursive.
  (grows faster than any primitive recursive function)

- Decidable nonclassical logics with nonprimitive recursive complexity:
  - Products of modal logics with expanding domains by reduction from the reachability problem for lossy channel systems. [Gabelaia et al., APAL 06]
  - Relevance logic LR$+$ (and fragments) by introducing a branching extension of CA. [Urquhart, JSL 99]
  - Finitary Metric Temporal Logic MTL. [Ouaknine & Worrell, LICS'05]

Model-Checking Counter Automata

Motivations

- Model-checking with focus on data values

  1. To analyze runs of operational models with focus on data values (beyong control state reachability).
     E.g., “there is a value of counter 1 such that infinitely often counter 2 takes that value iff infinitely often counter 3 takes that value”:
     $$\mathtt{F} \downarrow_1^1 (\mathtt{GF} \uparrow_1^2 \Leftrightarrow \mathtt{GF} \uparrow_1^3)$$

  2. Model-checking rather than satisfiability.

- Current instance:

  - Operational models: classes of counter automata for which the reachability problem is decidable.

  - Most often, the reachability sets are definable in Presburger arithmetic (decidable first-order theory of $(\mathbb{N}, +)$) .

  - Specification language: LTL with registers.

Model-checking counter automata

- Infinitary model-checking problem $\mathrm{MC}^{\omega}(\mathrm{LTL}^{\downarrow})$:

  Input: CA $\mathcal{S} = (Q, n, \delta)$, configuration $(q, \vec{x}) \in Q \times \mathbb{N}^n$, and a sentence $\varphi \in \mathrm{LTL}^{\downarrow}$ over alphabet Q;

  Question: Is there an infinite run ρ such that $\rho, 0 \models \varphi$?

- Undecidability for nondeterministic one-counter automata:

  - $\mathrm{MC}^{<\omega}(\mathrm{LTL}_1^{\downarrow}(\mathsf{X}, \mathsf{F}))$ is Σ_1^0-complete.

  - $\mathrm{MC}^{\omega}(\mathrm{LTL}_1^{\downarrow}(\mathsf{X}, \mathsf{F}))$ is Σ_1^1-complete.

- Reachability sets are semilinear but universal problem for one-counter automata with alphabet is undecidable.

  [Ibarra, MST 79]

Reversal-bounded counter automata

- Reversal: Alternation from nonincreasing mode to nondecreasing mode and vice-versa.

- Sequence with 3 reversals:

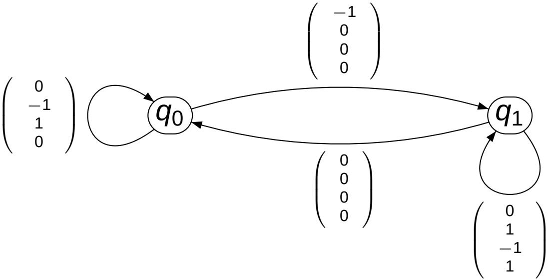
$$0011223334444\overline{3}33222\overline{3}33444455555\overline{4}$$

- Reversal-bounded counter automata: each run has a bounded number of reversals. [Ibarra, JACM 78]

- Reachability sets are effectively Presburger-definable. [Ibarra, JACM 78]

- Control state repeated reachability problem is decidable. [Dang & Ibarra & San Pietro, FST&TCS'01]

Vector addition systems with states (VASS)



- Succinct CA without zero-tests.
- Transitions of the form $q \xrightarrow{\vec{b}} q'$ with $\vec{b} \in \mathbb{Z}^n$, which is a shortcut for $\bigwedge_{i \in [1,n]} \mathsf{x}_i' = \mathsf{x}_i + \vec{b}(i)$.
- $\approx$ Petri nets (models of greater practical appeal).
- The reachability problem is decidable.

[Kosaraju, STOC'82; Mayr, SIAM 84]

Towards flatness

- $\mathrm{MC}^{\omega}(\mathrm{LTL}^{\downarrow})$ restricted to reversal-bounded CA and to formulae with at most one register is undecidable.

- Flat formulae: positive occurrence of $\varphi_1 \mathsf{U} \varphi_2$ implies $\downarrow$ does not occur in φ_1.

- $\neg(q \ \mathsf{U} \downarrow_1^1 \varphi)$ is not a flat formula.

- Flatness is a standard means to regain decidability for memoryful linear-time temporal logics.

Introducing parameters

- $\mathrm{MC}^{\omega}(\mathrm{LTL}^{\downarrow})$ restricted to reversal-bounded CA and to flat formulae is decidable. [Demri & Sangnier, FOSSACS'10]
- Decidability proof uses that the control state repeated reachability problem for parameterized reversal-bounded CA is decidable. [Ibarra et al., TCS 02]
- Transitions of the form $\mathit{add}(\mathrm{z})$ with parameter z.
- Reachability questions are relative to parameter values.

Formalisms for Data Words – First-Order Logics

First-order logic on data words

- Data word: nonempty finite sequence of pairs from $\Sigma \times \mathbb{N}$.

- Variable valuation *v* for a model σ: map from VAR to the positions of σ.

- Variables are interpreted as positions.

- Formulae of the logic $\mathrm{FO}^{\Sigma}(\sim, <, +1)$ (Σ is a finite alphabet)

  $\varphi ::= a(\mathsf{x}) \mid \mathsf{x} \sim \mathsf{y} \mid \mathsf{x} < \mathsf{y} \mid \mathsf{x} = \mathsf{y}+1 \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists \mathsf{x}\, \varphi$

- Last position is labelled by the letter $a \in \Sigma$:

$$\exists \mathsf{x}\ (\neg \exists \mathsf{y}\ \mathsf{x} < \mathsf{y}) \wedge a(\mathsf{x})$$

Data words as first-order structures

- Alphabet $\Sigma = \{a_1, \ldots, a_N\}$ and infinite domain $\mathbb{N}$.
- Data word $\sigma = (a_{i_1}, d_1) \cdots (a_{i_K}, d_K)$ is equivalent to

$$(\{1, \ldots, K\}, <, \sim, +1, P_1, \ldots, P_N)$$

  - For $j, j' \in \{1, \ldots, K\}$, $j \sim j'$ iff $d_j = d_{j'}$.
  - For $l \in \{1, \ldots, N\}$, $P_l \stackrel{\text{def}}{=} \{j \in \{1, \ldots, K\} : a_{i_j} = a_l\}$.
- First-order logic can be naturally interpreted over such structures.

Semantics

$$\sigma \models_v a(\mathsf{x}) \quad \overset{\text{def}}{\Leftrightarrow} \quad \Sigma(\mathsf{x}) = a \qquad \text{(letter at position } \mathsf{x}\text{)}$$

$$\sigma \models_v \mathsf{x} \sim \mathsf{y} \quad \overset{\text{def}}{\Leftrightarrow} \quad \mathbb{N}(\mathsf{x}) = \mathbb{N}(\mathsf{y}) \quad \text{(data at positions)}$$

$$\sigma \models_v \mathsf{x} < \mathsf{y} \quad \overset{\text{def}}{\Leftrightarrow} \quad v(\mathsf{x}) < v(\mathsf{y})$$

$$\sigma \models_v \mathsf{x} = \mathsf{y} + 1 \quad \overset{\text{def}}{\Leftrightarrow} \quad v(\mathsf{x}) = v(\mathsf{y}) + 1$$

$$\sigma \models_v \exists\, \mathsf{x}\, \varphi \quad \overset{\text{def}}{\Leftrightarrow} \quad \text{there is position } i \text{ s.t. } \sigma \models_{v[\mathsf{x} \mapsto i]} \varphi.$$

FO2 and VASS

- Theorem: Satisfiability problem for $\mathrm{FO2}(\sim, <, +1)$ is decidable. [Bojańczyk et al., LICS 06]

- Proof in two steps:
  - Satisfiability is first reduced to nonemptiness for data automata (undefined herein).
  - Nonemptiness for data automata is then reduced to the reachability problem for VASS.

- Theorem: There is a reduction from the reachability problem for VASS to finitary satisfiability for $\mathrm{FO2}(\sim, <, +1)$.

Fixing a few more things (proof)

- Instance: $\mathcal{S} = (Q, n, \delta)$, $(q_i, \vec{0})$, $(q_f, \vec{0})$.
- $\Sigma = Q \uplus \{\mathrm{inc}(i), \mathrm{dec}(i) : i \in [1, n]\}$.
  (below $a \in \{\mathrm{inc}(i), \mathrm{dec}(i) : i \in [1, n]\}$)
- The run $(q_0, \vec{x_0}) \xrightarrow{a_0} (q_1, \vec{x_1}) \xrightarrow{a_1} \cdots \xrightarrow{a_{K-1}} (q_K, \vec{x_K})$ encoded by a data word with projection $q_0 a_0 q_1 a_1 \cdots a_{K-1} q_K$.
- Run

$$\begin{array}{ccccccc} q_0 & q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \\ \binom{0}{0} & \binom{1}{0} & \binom{2}{0} & \binom{2}{1} & \binom{1}{1} & \binom{0}{1} & \binom{0}{0} \end{array}$$

  corresponds to data word

$$\begin{array}{ccccccccccccc} q_0 & \mathrm{inc}(1) & q_1 & \mathrm{inc}(1) & q_2 & \mathrm{inc}(2) & q_3 & \mathrm{dec}(1) & q_4 & \mathrm{dec}(1) & q_5 & \mathrm{dec}(2) & q_6 \\ \star & k_1 & \star & k_2 & \star & k_3 & \star & k_1 & \star & k_2 & \star & k_3 & \star \end{array}$$

Enforcing the projection

- φ_{proj}: conjunction of the formulae below.
  - The first letter is q_i:
    $$\exists \mathrm{x}\ (\neg \exists \mathrm{y}\ \mathrm{y} < \mathrm{x}) \wedge q_i(\mathrm{x})$$
  - The last letter is q_f:
    $$\exists \mathrm{x}\ (\neg \exists \mathrm{y}\ \mathrm{x} < \mathrm{y}) \wedge q_f(\mathrm{x})$$
  - Sequence of locations/actions respects graph of $\mathcal{S}$:
    $$\forall\, \mathrm{x}\ (\bigvee_{q \in Q} q(\mathrm{x})) \Rightarrow ((\neg \exists \mathrm{y}\ \mathrm{x} < \mathrm{y}) \vee$$
    $$\bigvee_{q \xrightarrow{a} q' \in \delta} (q(\mathrm{x}) \wedge (\exists \mathrm{y}\ \mathrm{y} = \mathrm{x} + 1 \wedge a(\mathrm{y})) \wedge$$
    $$(\exists \mathrm{y}\ \mathrm{y} = \mathrm{x} + 1 \wedge (\exists \mathrm{x}\ \mathrm{x} = \mathrm{y} + 1 \wedge q'(\mathrm{x})))))$$
- Observe the nice (and standard) recycling of variables.

Constraints on data values

- To encode counter values, each increment or decrement is attached to a datum.

- A desirable data word:

q_0	$\mathrm{inc}(1)$	q_1	$\mathrm{inc}(1)$	q_2	$\mathrm{inc}(2)$	q_3	$\mathrm{dec}(1)$	q_4	$\mathrm{dec}(1)$	q_5	$\mathrm{dec}(2)$	q_6
$\star$	k_1	$\star$	k_2	$\star$	k_3	$\star$	k_1	$\star$	k_2	$\star$	k_3	$\star$

- φ: conjunction of φ_{proj} and formulae below.

  - For $i, j \in [1, n]$, there are no two positions labelled by $\mathrm{inc}(i)$ and $\mathrm{inc}(j)$ having the same datum:

    $$\forall \mathrm{x}\ \mathrm{y}\ (\mathrm{x} < \mathrm{y} \wedge \mathrm{inc}(i)(\mathrm{x}) \wedge \mathrm{inc}(j)(\mathrm{y})) \Rightarrow \neg(\mathrm{x} \sim \mathrm{y}).$$

    (remember $\mathrm{inc}(i)$ and $\mathrm{dec}(i)$ are also letters in Σ)

  - Same with $\mathrm{dec}(i)$ and $\mathrm{dec}(j)$:

    $$\forall \mathrm{x}\ \mathrm{y}\ (\mathrm{x} < \mathrm{y} \wedge \mathrm{dec}(i)(\mathrm{x}) \wedge \mathrm{dec}(j)(\mathrm{y})) \Rightarrow \neg(\mathrm{x} \sim \mathrm{y}).$$

Constraints on data values (II)

- For $i \in [1, n]$, for every position labelled by $\mathrm{dec}(i)$, there is a past position labelled by $\mathrm{inc}(i)$ with the same data value:

$$\forall \mathsf{x}\ \mathrm{dec}(i)(\mathsf{x}) \Rightarrow (\exists\, \mathsf{y}\ (\mathsf{y} < \mathsf{x}) \wedge (\mathsf{x} \sim \mathsf{y}) \wedge \mathrm{inc}(i)(\mathsf{y}))$$

- Since in the final configuration, any counter value is zero, we impose that for $i \in [1, n]$, for every position labelled by $\mathrm{inc}(i)$, there is a future position labelled by $\mathrm{dec}(i)$ with same data value:

$$\forall \mathsf{x}\ \mathrm{inc}(i)(\mathsf{x}) \Rightarrow (\exists\, \mathsf{y}\ (\mathsf{x} < \mathsf{y}) \wedge (\mathsf{x} \sim \mathsf{y}) \wedge \mathrm{dec}(i)(\mathsf{y}))$$

- One can show $(q_f, \vec{0})$ is reachable from $(q_i, \vec{0})$ iff φ is satisfiable.

$\mathrm{FO3}(\sim, <, +1)$ is undecidable [Bojańczyk et al., LICS 06]

- Extend VASS with zero-tests.
- Nonemptiness problem (or equivalent control state reachability) is undecidable.
- Use the third variable to encode zero-tests:

$$\forall \mathsf{x}\ \mathrm{zero}(i)(\mathsf{x}) \Rightarrow$$

$$(\forall \mathsf{y}\ (\mathsf{y} < \mathsf{x} \wedge \mathrm{inc}(i)(\mathsf{y})) \Rightarrow \exists \mathsf{z}\ ((\mathsf{y} < \mathsf{z} < \mathsf{x}) \wedge \mathrm{dec}(i)(\mathsf{y}) \wedge (\mathsf{y} \sim \mathsf{z})))$$

Formalisms for Data Words – Automata

Specifying classes of data words

- Register automata
  - Register automata [Kaminski & Francez, TCS 94]
  - Data automata [Bouyer & Petit & Thérien, IC 03]
  - Machines for strings over infinite alphabets. [Neven & Schwentick & Vianu, TOCL 04]
  - See the survey [Segoufin, CSL'06]

- Many new classes
  - Class automata [Bojańzcyk & Lasota, LICS'10].
  - Variable automata. [Grumberg & Kupferman & Sheinval, LATA'10]
  - etc.
- Many other formalisms
  - Rewriting systems with data [Bouajjani et al., FCT'07]
  - Hybrid logics [Schwentick & Weber, STACS'07]
  - XPath on data trees.

Other relationships with counter automata

- Class counting automata counts the number of data values along a word. [Manuel & Ramanujan, RP'09]
  - Comparisons to constant values.
  - Nonemptiness reduces to the covering problem for VASS.
  - EXPSPACE upper bound with integers in unary.

- Automata for bounded-depth data trees:
  - Decidability of nonemptiness problem by reduction into priority CA. [Björklund & Bojańczyk, ICALP'07]
  - Nonemptiness for priority CA shown decidable in [Reinhardt, Hab. thesis 05].

- Safety fragment of $\mathrm{LTL}^{\downarrow}$ with one register on infinite data words. [Lazić, FST&TCS'06]
  - No U-subformulae with positive polarity.
  - EXPSPACE upper bound by introducing a subclass of gainy counter automata.

What about languages?

- Each formula defines a class of data words (those satisfied by the formula) and a class of words over a finite alphabet obtained by projection.
- Each data logic defines a class of languages made of words over a finite alphabet.
- Each counter automaton (with alphabet and augmented with initial and final control states) defines a class of languages made of words over a finite alphabet.
- $\mathrm{LTL}^{\downarrow}$ with a unique register is equivalent to gainy counter automata. [Demri & Lazić, TOCL 09]
- $\mathrm{FO2}(\sim, <, +1)$ is equivalent to VASS. [Bojańczyk et al., LICS'06]
- See new relationships in [Bojańczyk & Lasota, LICS'10].

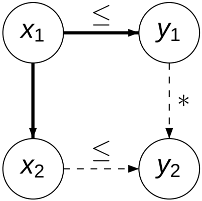
Perspectives

Branching extensions

- Data trees
  - See e.g., register automata for data trees in [Jurdzińsky & Lazić, LICS'07; Figueira, PODS'09]
- Branching VASS: computations are finite trees instead linear structures. [Verma & Goubault-Larrecq, DTMCS 05]
  - Reachability problem for BVASS can be reduced to satisfiability for FO2 over data trees. [Bojańczyk et al., PODS'06]
  - Decidability status of the reachability problem is open.
  - Covering problem and boundedness problem are 2EXPTIME-complete. [Demri et al., FST&TCS'09]

Well-structured transition systems

- Well-structured transition system $(S, \rightarrow, \leq)$
  [Finkel & Schnoebelen, TCS 01]
  - $\leq$ is a well-quasi-ordering: for any infinite sequence $\vec{x_0}, \vec{x_1}, \ldots$ in S, there are $i < j$ such that $\vec{x_i} \leq \vec{x_j}$.
  - $\rightarrow$ and $\leq$ are upward compatible:



- Most decidability proofs uses the well-structuredness of underlying transition systems.

- This is made explicit in [Figueira, ICDT'10].

- How far can we use well-structured transition systems to show decidability? What about computational complexity?

Conclusion

- Data logics/automata can benefit from counter systems.
- Verification of counter systems need data logics as specification languages.
- Relationships with other nonclassical logics: product of modal logics, relevance logics, memoryful temporal logics, hybrid logics, memory logics . . .
- Many open problems in decidability, complexity, expressive power of data logics.

  See e.g., recent [David & Libkin & Tan, LPAR'10]
- Study of new automata models:
  - Variable automata.

    [Grumberg & Kupferman & Sheinvald, LATA'10]
  - Register automata with guess and spread.

    [Figueira, ICDT'10]

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