A short introduction to
ATL-like logics with resources

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Logics for resource-bounded agents

- ATL-like logics with models where transitions have costs/rewards and resource requirements are expressed in the syntax.

- Model-checking problems for such logics are often undecidable as games on VASS are often undecidable.

- Many existing resource logics:
  - RBTL* [Bulling & Farwer, CLIMA X ’09]
  - QATL* [Bulling & Goranko, EPTCS 2013]
  - RB±ATL [Alechina et al., ECAI’14]
  - etc.

- Other logics for resource-bounded agents: step logic, justification logic, etc.
Concurrent game structures

\[ \text{Act} = \{a, b, c\} \]

\[ \text{Agt} = \{1, 2\} \]

\[ \text{S} = \{s_1, s_2, s_3, s_4\} \]

- **Action manager** \( \text{act} : \text{Agt} \times \text{S} \rightarrow \mathcal{P}(\text{Act}) \setminus \{\emptyset\} \).
  \[ \text{act}(1, s_3) = \{c\} \]

- **Transition function** \( \delta : \text{S} \times (\text{Agt} \rightarrow \text{Act}) \rightarrow \text{S} \).
  \[ \delta(s_4, [1 \mapsto c, 2 \mapsto c]) = s_3 \]

- **Labelling** \( L : \text{S} \rightarrow \mathcal{P}(\text{PROP}) \).
Basic concepts: joint actions and computations

- $f : A \rightarrow Act$: joint action by $A \subseteq Agt$ in $s$. 
  Proviso: for all $a \in A$, we have $f(a) \in act(a,s)$.

- $D_A(s)$: set of joint actions by $A$ in $s$.

\[
\text{out}(s, f) \overset{\text{def}}{=} \{ s' \in S \mid \exists g \in D_{Agt}(s) \text{ s.t. } f \sqsubseteq g \& s' = \delta(s,g) \}
\]

- Computation $\lambda = s_0 \xymatrix{ f_0 \ar[r] & s_1 \xymatrix{ f_1 \ar[r] & s_2 \cdots}$ such that for all $i$, we have $s_{i+1} \in \delta(s_i, f_i)$.

- Linear model $L(s_0) \rightarrow L(s_1) \rightarrow L(s_2) \cdots$. 
Basic concepts: strategies

- A strategy $F_A$ for $A$ is a map from the set of finite computations to the set of joint actions by $A$ such that
  $$F_A(s_0 \xrightarrow{f_0} s_1 \cdots \xrightarrow{f_{n-1}} s_n) \in D_A(s_n).$$

- $\lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \cdots$ respects $F_A \iff \forall i < |\lambda|,$
  $$s_{i+1} \in \text{out}(s_i, F_A(s_0 \xrightarrow{f_0} s_1 \cdots \xrightarrow{f_{i-1}} s_i))$$

- $\lambda$ respecting $F_A$ is maximal whenever $\lambda$ cannot be extended further while respecting the strategy.

- $\text{comp}(s, F_A)$: max. computations from $s$ respecting $F_A$. 

The logic ATL

\[ \phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \langle \langle A \rangle \rangle X \phi \mid \langle \langle A \rangle \rangle G \phi \mid \langle \langle A \rangle \rangle \phi U \phi \]

\[ p \in \text{PROP} \quad A \subseteq \text{Agt} \]

\[ \mathcal{M}, s \models p \quad \overset{\text{def}}{\iff} \quad s \in L(p) \]

\[ \mathcal{M}, s \models \langle \langle A \rangle \rangle X \phi \quad \overset{\text{def}}{\iff} \quad \text{there is a strategy } F_A \text{ s.t.}
\quad \text{for all } s_0 \xrightarrow{f_0} s_1 \ldots \in \text{comp}(s, F_A),
\quad \text{we have } \mathcal{M}, s_1 \models \phi \]

\[ \mathcal{M}, s \models \langle \langle A \rangle \rangle \phi_1 U \phi_2 \quad \overset{\text{def}}{\iff} \quad \text{there is a strategy } F_A \text{ s.t. for all}
\quad \lambda = s_0 \xrightarrow{f_0} s_1 \ldots \in \text{comp}(s, F_A),
\quad \text{there is some } i < |\lambda| \text{ s.t. } \mathcal{M}, s_i \models \phi_2 \text{ and}
\quad \text{for all } j \in [0, i - 1], \text{ we have } \mathcal{M}, s_j \models \phi_1. \]
Model-checking problem

- Model-checking problem for ATL:
  
  **Input:** $\phi$ in ATL, a finite CGS $\mathcal{M}$ and a state $s$,
  
  **Question:** $\mathcal{M}, s \models \phi$?

- Model-checking problem for ATL is \(P\)-complete.
  
  Labeling algorithm.  
  
  [Alur & Henzinger & Kupferman, JACM 2002]

- $\text{ATL}^* = \text{ATL} + \text{all path formulae à la CTL}^*$.

- Model-checking problem for $\text{ATL}^*$ is $2\text{EXPTIME}$-complete.
Resource-bounded concurrent game structures

- Number $r$ of resources/counters.

- Partial cost function $\text{cost} : S \times \text{Agt} \times \text{Act} \rightarrow \mathbb{Z}^r$.

- Action $\text{idle} \in \text{act}(a, s)$ with no cost.

- Given a joint action $f : A \rightarrow \text{Act}$,

$$\text{cost}_A(s, f) \overset{\text{def}}{=} \sum_{a \in A} \text{cost}(s, a, f(a))$$
\[\text{cost}(s_2, 1, a) = (1, 1, 1, 1) \quad \text{cost}(s_2, 2, a) = (-2, 1, -3, 1)\]

\[\text{cost}\{1, 2\}(s_2, [1 \mapsto a, 2 \mapsto a]) = (-1, 2, -2, 2)\]
**b-strategies**

▶ Initial budget \( b \in (\mathbb{N} \cup \{\omega\})^r \).

▶ \( \lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \ldots \) in \( \text{comp}(s, F_A) \) is \( b \)-consistent:
  
  ▶ \( v_0 \overset{\text{def}}{=} b \),

  ▶ \( v_{i+1} \overset{\text{def}}{=} v_i + \text{cost}_A(s_i, F_A(s_0 \xrightarrow{f_0} s_1 \ldots \xrightarrow{f_{i-1}} s_i)) \),

  ▶ for all \( i \), \( 0 \preceq v_i \).

Asymmetry between \( A \) and \((Agt \setminus A)\)

▶ \( \text{comp}(s, F_A, b) \): set of all the \( b \)-consistent computations.

▶ \( F_A \) is a \( b \)-strategy w.r.t. \( s \overset{\text{def}}{=} \)

\[
\text{comp}(s, F_A) = \text{comp}(s, F_A, b)
\]
The logic $\text{RB} \pm \text{ATL} (\mathit{Agt}, r)$ [Alechina et al., ECAI’14]

$$\phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \langle\langle A^b \rangle\rangle X \phi \mid \langle\langle A^b \rangle\rangle G \phi \mid \langle\langle A^b \rangle\rangle \phi U \phi$$

$$p \in \text{PROP} \quad A \subseteq \text{Agt} \quad b \in (\mathbb{N} \cup \{\omega\})^r$$

$$\mathcal{M}, s \models p \quad \iff \quad s \in L(p)$$

$$\mathcal{M}, s \models \langle\langle A^b \rangle\rangle X \phi \quad \iff \quad \text{there is a } \mathbf{b}\text{-strategy } F_A \text{ w.r.t. } s \text{ s.t. for all } s_0 \xrightarrow{f_0} s_1 \ldots \in \text{comp}(s, F_A), \text{ we have } \mathcal{M}, s_1 \models \phi$$

$$\mathcal{M}, s \models \langle\langle A^b \rangle\rangle \phi_1 U \phi_2 \quad \iff \quad \text{there is a } \mathbf{b}\text{-strategy } F_A \text{ w.r.t. } s \text{ s.t. for all } \lambda = s_0 \xrightarrow{f_0} s_1 \ldots \in \text{comp}(s, F_A), \text{ there is some } i < |\lambda| \text{ s.t. } \mathcal{M}, s_i \models \phi_2 \text{ and for all } j \in [0, i - 1], \text{ we have } \mathcal{M}, s_j \models \phi_1.$$
Alternative semantics

- In $\text{RB} \pm \text{ATL}$, $\text{comp}(s, F_A) = \text{comp}(s, F_A, b)$ implies the maximal computations are infinite.

- Infinite semantics: arbitrary strategy but quantifications over infinite computations only.

- Finite semantics: arbitrary strategy but quantifications over maximal computations only.
Resource-bounded reasoners for AI

▶ \(\text{RB}^{\pm}\text{ATL}\) is one of the logics for reasoning about resources. See papers in AAAI, IJCAI, ECAI, etc.

▶ Relationships with counter machines known for establishing undecidability or complexity lower bounds.

▶ Various flavours of resource-bounded logics exist: RBCL, RAL, PRB-ATL, etc.
Alternating VASS [Courtois & Schmitz, MFCS’14]

- Alternating VASS $\mathcal{A} = (Q, r, R_1, R_2)$:
  - $R_1$ is a finite subset of $Q \times \mathbb{Z}^r \times Q$. (unary rules)
  - $R_2$ is a finite subset of $\bigcup_{\beta \geq 2} Q^\beta$ (fork rules)

- Proof: tree labelled by elements in $Q \times \mathbb{N}^r$ following the rules in $\mathcal{A}$.

\[
\begin{aligned}
&\vdots (q_3, (4, 8)) (q_0, (0, 8)) \\
&\quad (q_2, (1, 5)) (q_1, (1, 5)) \\
&\quad (q_0, (1, 5)) (q_1, (2, 2)) \\
q_1 &\xrightarrow{(-1, +3)} q_0 \quad q_0 \rightarrow q_1, q_2 \quad q_2 \xrightarrow{(+3, +3)} q_3
\end{aligned}
\]
Decision problems

- State reachability problem for AVASS:
  
  **Input:** AVASS $A$, control states $q_0$ and $q_f$,

  **Question:** is there a finite proof of AVASS with root $(q_0, 0)$ and each leaf belongs to $\{q_f\} \times \mathbb{N}^r$?

- Non-termination problem for AVASS:
  
  **Input:** $A$, $q_0$,

  **Question:** is there a proof with root $(q_0, 0)$ and all the maximal branches are infinite?

VASS games with asymmetry between the two players
Main Correspondences

<table>
<thead>
<tr>
<th>RB±ATL</th>
<th>Alternating VASS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic in AI</td>
<td>Verification games</td>
</tr>
<tr>
<td>proponent restriction condition</td>
<td>updates in $R_1$ / no update in $R_2$</td>
</tr>
<tr>
<td>computation tree for $F_A$</td>
<td>proof</td>
</tr>
<tr>
<td>formulae in the scope of $\langle\langle A^b\rangle\rangle$</td>
<td>monotone objectives</td>
</tr>
</tbody>
</table>

- From RB±ATL model-checking to the state reachability and the non-termination problems for AVASS.
- From RB±ATL* model-checking to the parity games for AVASS.
- Parameters synthesis thanks to the computation of the Pareto frontier of parity games.

See [Abdulla et al., CONCUR’13]
Complexity of RB±ATL fragments

<table>
<thead>
<tr>
<th>$r \backslash \text{card}(Agt)$</th>
<th>arbitrary</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>arbitrary</td>
<td>$2\text{EXPTIME-c.}$</td>
<td>$2\text{EXPTIME-c.}$</td>
<td>EXPSPACE-c.</td>
</tr>
<tr>
<td>≥ 4</td>
<td>$\text{EXPTIME-c.}$</td>
<td>$\text{EXPTIME-c.}$</td>
<td>PSPACE-c.</td>
</tr>
<tr>
<td>2, 3</td>
<td>$\text{PSPACE-h. in EXPTIME}$</td>
<td>$\text{PSPACE-h. in EXPTIME}$</td>
<td>PSPACE-c.</td>
</tr>
<tr>
<td>1</td>
<td>in PSPACE</td>
<td>in PSPACE</td>
<td>PTIME-c.</td>
</tr>
</tbody>
</table>

Complexity characterisations established in

[Alechina et al., JCSS 2017; Alechina et al., RP’16; etc.]

based on the relationships with (A)VASS and results from

[Habermehl, ICATPN’97; Courtois & Schmitz, MFCS’14; Colcombet et al., LICS’17]
Parameterized $\text{RB} \pm \text{ATL}^*$: $\text{ParRB} \pm \text{ATL}^*$

- $b \in (\mathbb{N} \cup \{\omega\})'$ replaced by tuples of variables.
  
  $$\langle\langle\{1\}(x_1,x_2)\rangle\rangle \sqcap Uq_f \land \langle\langle\{2\}(x_2,x_3)\rangle\rangle \sqcap Uq'_f$$

- MC problem for ParRB$\pm$ATL*: compute the maps $v : \{x_1, \ldots, x_n\} \to (\mathbb{N} \cup \{\omega\})$ such that $\mathcal{M}, s \models v(\phi)$.

- Symbolic representation for such maps are computable.
Other temporal logics for AI

- TIME: International Symposium on Temporal Representation and Reasoning
  - Artificial Intelligence
  - Temporal Databases
  - Logic

- Interval temporal logics, ATL-like logics, temporal logics over concrete domains, etc.
Concluding remarks

- Formal relationships between resource-bounded logics and games on alternating VASS.

- Open problems:
  - Parameter synthesis.
  - Complexity for small fragments by bounding further the syntactic resources.
  - Alternative semantics for applications.