Resource-bounded ATL: 
The Quest for Tractable Fragments

F. Belardinelli $^1$  
S. Demri $^2$

$^1$Imperial College London, UK and Université d’Evry, France  
$^2$LSV, CNRS, ENS Paris-Saclay, Université Paris-Saclay, France

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Background

- ATL: logic to reason about the strategic abilities of agents in multi-agent systems.
  
  [Alur & Henzinger & Kupferman, JACM 2002]

- Concurrent game structures:

- Actions have no cost.

- Research question: Can we reason about resources efficiently?
Logics for resource-bounded agents

- ATL-like logics with models where transitions have costs or rewards and resource requirements stated in the syntax.

- Many existing resource logics:
  - RBTL* [Bulling & Farwer, CLIMA X ’09]
  - QATL* [Bulling & Goranko, EPTCS 2013]
  - RB±ATL [Alechina et al., ECAI’14]
  - etc.

- Model-checking problems for such logics are often undecidable as games on VASS are often undecidable.

- Other logics for resource-bounded agents: step logic, justification logic, etc.
Our contribution

- Motivation: looking for tractable fragments for the logic \( \text{RB} \pm \text{ATL} \) and variants.

- Model checking \( \text{RB} \pm \text{ATL} (\{1\}, 1) \) is \( \text{PTIME} \)-complete. Reasoning about a single resource in CTL comes at no extra computational complexity.

- Proof strategy: reduction to decision problems on VASS. We show that the control state reachability and non-termination problems for 1-VASS are in \( \text{PTIME} \).

- A quick comparison: model-checking problem for
  - \( \text{ATL} \) is \( \text{P} \)-complete.
  - \( \text{ATL}^* \) is \( 2\text{EXPTIME} \)-complete.
  - \( \text{RB} \pm \text{ATL} ([1, k], r) \) is \( 2\text{EXPTIME} \)-complete.
Motivating scenario

- A rover is exploring an unknown area.
- At any time it can move around or recharge its battery, but not at the same time.

- Moving around consumes one energy unit at every time step, whereas the rover can recharge of one energy unit at a time.
- Switching between modes also requires one energy unit.

- **Specification**: Is it always the case that, given an energy budget of $b$ units, the rover will be able to move?
Resource-bounded concurrent game structures

- Number \( r \) of resources/counters.

- Partial “cost” function \( \text{cost} : S \times \text{Agt} \times \text{Act} \rightarrow \mathbb{Z}^r \).

- Action \( \text{idle} \in \text{act}(a, s) \) with no cost.

- Given a joint action \( f : A \rightarrow \text{Act} \),

\[
\text{cost}_A(s, f) \overset{\text{def}}{=} \sum_{a \in A} \text{cost}(s, a, f(a))
\]
b-strategies

- Strategy \( F_A : s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \cdots \xrightarrow{f_{n-1}} s_n \xrightarrow{f} A \xrightarrow{} \text{Act} \).

- Initial budget \( b \in (\mathbb{N} \cup \{\omega\})^r \).

- \( \lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \cdots \) in \( \text{comp}(s, F_A) \) is \( b \)-consistent:
  - \( v_0 \overset{\text{def}}{=} b \),
  - \( v_{i+1} \overset{\text{def}}{=} v_i + \text{cost}_A(s_i, F_A(s_0 \xrightarrow{f_0} s_1 \cdots \xrightarrow{f_{i-1}} s_i)) \),
  - for all \( i \), \( 0 \leq v_i \).

- \( F_A \) is a \( b \)-strategy w.r.t. \( s \overset{\text{def}}{=} \) all the computations from \( s \) respecting \( F_A \) are \( b \)-consistent.
Resource-bounded ATL: RB$\pm$ATL

- RB$\pm$ATL: extension of ATL to reason about resources.
  [Alechina et al., ECAI’14]

- Formulae:
  \[ \phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \langle\langle A^b \rangle\rangle X \phi \mid \langle\langle A^b \rangle\rangle G \phi \mid \langle\langle A^b \rangle\rangle \phi U \phi \]
  \[ p \in \text{PROP} \quad A \subseteq \text{Agt} \quad b \in (\mathbb{N} \cup \{\omega\})^r \]

- $\langle\langle A^b \rangle\rangle \phi$: existential strategy quantifier with initial budget.

- Satisfaction relation:
  \[ M, s \models p \iff s \in L(p) \]
  \[ M, s \models \langle\langle A^b \rangle\rangle \psi \iff \text{there is a } b\text{-strategy } F_A \text{ w.r.t. } s \]
  \[ \text{s.t. } \forall \lambda = s_0 \xrightarrow{i_0} s_1 \ldots \in \text{comp}(s, F_A), \]
  \[ \lambda \text{ satisfies } \psi \]
One agent/resource and CTL temporal operators

- For $|Ag| = 1$, we obtain a resource-bounded version of CTL ($b \in \mathbb{N}$):

$$E^b \psi \overset{\text{def}}{=} \langle\langle\{1\}^b\rangle\rangle \psi \quad \langle\langle\{1\}\omega\rangle\rangle \psi \equiv E \psi \quad \langle\langle\emptyset^b\rangle\rangle \psi \equiv A \psi \quad \langle\langle\emptyset\omega\rangle\rangle \psi \equiv A \psi$$

- It is always the case that, given an energy budget of $b$ units, the rover will be able to move:

$$\approx \text{AG} \quad \neg\langle\langle\{\text{rover}\}\omega\rangle\rangle F \neg\langle\langle\{\text{rover}\}^b\rangle\rangle F \text{ mov} \quad (F \varphi \overset{\text{def}}{=} T U \varphi)$$

- We also consider $\langle\langle A^b \rangle\rangle \psi_1 R \psi_2$ with

$$\psi_1 R \psi_2 \overset{\text{def}}{=} \neg(\neg \psi_1 U \neg \psi_2) \quad (R : \text{release operator})$$
The model-checking problem for RB±ATL:

- **Input:** \( k, r \geq 1, \phi \) in RB±ATL([1, k], r), a finite RB±ATL([1, k], r) model \( \mathcal{M} \) and a state \( s \),
- **Question:** \( \mathcal{M}, s \models \phi ? \)

**Correspondences from** [Alechina et al., TCS 2018]

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- From RB±ATL model-checking to the state reachability and the non-termination problems for AVASS.
### Complexity of RB±ATL fragments

| \( r \times |Agt| \) | arbitrary          | 2          | 1          |
|-----------------|-------------------|------------|------------|
| arbitrary       | 2\text{EXPTIME-c.} | 2\text{EXPTIME-c.} | \text{EXPSPACE-c.} |
| \( \geq 4 \)    | \text{EXPTIME-c.}  | \text{EXPTIME-c.}  | \text{PSPACE-c.}  |
| \( 2, 3 \)      | \text{PSPACE-h. in EXPTIME} | \text{PSPACE-h. in EXPTIME} | \text{PSPACE-c.}  |
| 1               | in \text{PSPACE}  | in \text{PSPACE}  | \text{PTIME-c. (this paper)} |
|                 | \text{PTIME-hard (from ATL)} | \text{PTIME-hard}  |             |

Complexity characterisations established in

[Alechina et al., JCSS 2017; Alechina et al., TCS 2018; etc.]

based on the relationships with (A)VASS and results from

[Habermehl, ICATPN’97; Blondin et al., LICS’15; Colcombet et al., LICS’17]
Limitations and positive result

| $r \setminus |Agt|$ | arbitrary | 2 | 1 |
|----------------|--------|---|---|
| arbitrary            | 2EXPTIME-c. | 2EXPTIME-c. | EXPSPACE-c. |
| $\geq 4$              | EXPTIME-c. | EXPTIME-c. | PSPACE-c. |
| 2, 3                  | PSPACE-h. in EXPTIME | PSPACE-h. in EXPTIME | PSPACE-c. |
| 1                     | in PSPACE PTIME-hard (from ATL) | in PSPACE PTIME-hard | PTIME-c. (this paper) |

- **Limitations:**
  - Complexity spans from PTIME to 2EXPTIME.
  - Loose complexity bounds in several cases. (e.g., $r = 2, 3$ and $|Agt| \geq 2$).

- **Model checking $RB^{\pm ATL}(\{1\}, 1)$ is PTIME-complete.**
  - As hard as CTL: reasoning about resources comes at no extra computational complexity!
1-VASS in a nutshell

- Proving that the model-checking problem RB±ATL({1}, 1) is in PTIME done by solving decision problems for 1-VASS.

- Vector addition systems with states (VASS):

- Runs are sequences for the form

\[(q_0, x_0) \rightarrow (q_1, x_1) \cdots \rightarrow (q_i, x_i) \rightarrow \cdots \quad (x_j \geq 0)\]
Decision problems for VASS

- Control-state reachability problem (CREACH(1-VASS)):
  Input: 1-VASS $\mathcal{V}$, initial configuration $(q_0, x_0)$, target control state $q_f$.
  Question: Is there a finite run from $(q_0, x_0)$ to a configuration with state $q_f$?

- Non-termination problem (NONTER(1-VASS)):
  Input: 1-VASS $\mathcal{V}$, initial configuration $(q_0, x_0)$.
  Question: Is there an infinite run with initial configuration $(q_0, x_0)$?

- With arbitrary number of counters, both problems are known to be EXPSPACE-complete.

- We show that both CREACH(1-VASS) and NONTER(1-VASS) are in PTIME.
Proof ideas

- **CREACH(1-VASS) in PTIME**: $\exists x_f \text{ s.t. } (q_0, x_0) \xrightarrow{*} (q_f, x_f)$ iff either there is a finite run
  
  or there is a simple run leading to $q_f$.

- Similar to proof idea for boundedness problem in [Rosier & Yen, JCSS 1986], but actually we fixed that proof.

- **NONTER(1-VASS) in PTIME**: there is a non-terminating run from $(q_0, x_0)$ iff $\exists q, x$ s.t.

- Latest news: recent improvement to NC.
  
  [Almagor et al., arXiv’19]
A labelling algorithm leading to PTIME

- To decide whether $\mathcal{M}, s \models \phi$, we design a labelling algorithm that works bottom-up on the structure of $\phi$.

- Subformulae $\langle\langle\{1\}^b\rangle\rangle \psi_1 \mathsf{U} \psi_2$ are dealt with by solving $\mathsf{CREACH}(\mathcal{V}^M)$. \hspace{1cm} (\(b \in \mathbb{N}\))

- Subformulae $\langle\langle\{1\}^b\rangle\rangle \mathsf{G} \psi$ are dealt with by solving $\mathsf{NONTER}(\mathcal{V}^M)$. \hspace{1cm} (\(b \in \mathbb{N}\))

- Subformulae of $\langle\langle\{1\}^{\omega}\rangle\rangle \psi_1 \mathsf{U} \psi_2$ and $\langle\langle\{1\}^{\omega}\rangle\rangle \mathsf{G} \psi$ are reduced to instances of CTL model-checking.

- The whole procedure is in PTIME.
Conclusions and future work

- Model-checking problem for $\text{RB} \pm \text{ATL}(\{1\}, 1)$ is in $\text{PTIME}$. (binary encoding for integers, no succinct encoding for RB-CGS)

- Relationships between resource-bounded logics & VASS.

- Material in the paper untouched today:
  - $\text{RBTL} \approx$ the one-agent version $\text{RB} \pm \text{ATL}$ ($\{1\}, r$).
  - Results for $\text{RB} \pm \text{ATL}^*$ and comparison with those for $\text{RB} \pm \text{ATL}$.

- Future work:
  1. Budget synthesis: find a (minimal) budget $b$ such that $M, s \models \langle \langle A^b \rangle \rangle \psi$.
  2. Implementation in a model checking tool.
  3. Complexity of the model-checking problem for $\text{RB} \pm \text{ATL}((1, 2), 1)$?